

# Algebraic Expressions and Identities

1.) Simplify:  $(2x^2 - 3xy^2)^2$

ANSWER:

Here we use identity.

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a = 2, b = 3xy^2$$

$$(2x^2 - 3xy^2)^2 = (2x^2)^2 + (-2 \times 2x^2 \times 3xy^2) + (3xy^2)^2$$

$$(2x^2 - 3xy^2)^2 = 4x^4 - 12x^3y^2 + 9x^2y^4$$

2.) Simplify:  $(25x^2 - 16y^2) \div (5x + 4y)$

ANSWER:

$$(25x^2 - 16y^2) = \text{we write this as, } a^2 - b^2$$

$$a = 5x \quad b = 4y$$

We know,

$$(a + b)(a - b) = a^2 - b^2$$

$$(5x + 4y)(5x - 4y) = (25x^2 - 16y^2)$$

$$(25x^2 - 16y^2) \div (5x + 4y) = (5x + 4y)(5x - 4y) \div (5x + 4y)$$

$$(25x^2 - 16y^2) \div (5x + 4y) = (5x - 4y)$$

3.) Simplify:  $(3a + 2b)(3a - 2b) - 5b^2$

ANSWER:

We know,

$$(a + b)(a - b) = a^2 - b^2$$

We put,  $a = 3a, b = 2b$

$$(3a + 2b)(3a - 2b) = 3a^2 - 2b^2$$

$$(3a + 2b)(3a - 2b) - 5b^2 = (3a)^2 - (2b)^2 - 5b^2$$

$$(3a + 2b)(3a - 2b) - 5b^2 = 9a^2 - 9b^2$$

**4.) Simplify:  $(4x - 3y)^2 + 24xy$**

ANSWER:

Here  $(4x - 3y)^2 + 24xy$

We use  $(a - b)^2 = a^2 - 2ab + b^2$

$a = 4, b = 3y$

$= (4x)^2 + (-2 \times 4x \times 3y) + (3y)^2$

$(4x - 3y)^2 = 16x^2 - 24xy + 9y^2$

$16x^2 - 24xy + 9y^2 + 24xy$

**$(4x - 3y)^2 + 24xy = 16x^2 + 9y^2$**

**5.) Simplify:  $9x^2 + 24xy + 16y^2 / 3x + 4y$**

ANSWER:

We know,

$(a + b)^2 = a^2 + 2ab + b^2$

$9x^2 + 24xy + 16y^2 = (3x + 4y)^2$

$9x^2 + 24xy + 16y^2 / 3x + 4y = (3x + 4y)^2 / 3x + 4y$

**$9x^2 + 24xy + 16y^2 / 3x + 4y = 3x + 4y$**

**6.) Simplify:  $(x^2 + 5x + 6) \div (x + 2)$**

ANSWER:

Here we use,

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$a = 2$  and  $b = 3$

$(x^2 + 5x + 6)$  we put this as  $(x + 2)(x + 3)$

$(x + 2)(x + 3) \div (x + 2)$

**$(x^2 + 5x + 6) \div (x + 2) = (x + 3)$**

**7.) Simplify:  $(6.25p^2 - 2.25q^2) \div (2.5p + 1.5q)$**

ANSWER:

We know,

$$(a + b)(a - b) = a^2 - b^2$$

We put,  $a = 6.25p$ ,  $b = 2.25q$

$$(6.25p^2 - 2.25q^2) = (6.25p + 2.25q)(6.25p - 2.25q)$$

$$(6.25p^2 - 2.25q^2) \div (2.5p + 1.5q) =$$

$$(6.25p + 2.25q)(6.25p - 2.25q) \div (2.5p + 1.5q)$$

$$= 2.5p - 1.5q$$

**8.) Simplify:  $(7x + 4y)^2 - 49x^2 - 16y^2$**

ANSWER:

$$(7x + 4y)^2 \text{ Here we use } (a + b)^2 = a^2 + 2ab + b^2$$

$$a = 7x \quad b = 4y$$

$$7x^2 + 2 \times 7x \times 4y + 4y^2$$

$$(7x + 4y)^2 = 49x^2 + 56xy + 16y^2$$

$$(7x + 4y)^2 - 49x^2 - 16y^2 = 49x^2 + 56xy + 16y^2 - 49x^2 - 16y^2$$

$$(7x + 4y)^2 - 49x^2 - 16y^2 = 56xy$$

**9.) Simplify:  $(x^2 + 10x + 25) \div (x + 5)$**

ANSWER:

$$(x^2 + 10x + 25) \text{ Here we use } (a + b)^2 = a^2 + 2ab + b^2$$

$$a = x \quad b = 5$$

$$(x^2 + 10x + 25) = (x + 5)^2$$

$$(x^2 + 10x + 25) \div (x + 5) = (x + 5)^2 \div (x + 5)$$

$$(x^2 + 10x + 25) \div (x + 5) = x + 5$$

**10.) Simplify:  $(a + b)(a^2 + b^2 - ab)$**

ANSWER:

We know, standard identity,

$$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$(a + b)(a^2 + b^2 - ab) = a^3 + b^3$$

11.) Simplify:  $(m^2 + mn + n^2)(m - n)$

ANSWER:

We know, standard identity

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Here,  $a = m$     $b = n$

$$(m^2 + mn + n^2)(m - n) = m^3 - n^3$$

12.) If  $a = 3$ ,  $b = 4$ , then find the value of  $(a + b)^2 + ab$

ANSWER:

Given,  $a = 3$ ,  $b = 4$  we put this value in  $(a + b)^2 + ab$

$$(a + b)^2 + ab = (3 + 4)^2 + (3 \times 4)$$

$$(a + b)^2 + ab = (9 + 24 + 16) + 12$$

$$(a + b)^2 + ab = 49 + 12$$

$$(a + b)^2 + ab = 61$$

13.) If  $m = -5$ ,  $n = 7$ ,  $a = 3$ ,  $b = 3$ , then find the value of  $(a - b)^3 + (m + n)^2$

ANSWER:

Here, we use

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

Here  $a = 3$  and  $b = 3$

$$(a - b)^3 = 3^3 - 3^3 - 3 \times 3 \times 3 (3 - 3)$$

$$(a - b)^3 = 0$$

$$(m + n)^2 = m^2 + 2mn + n^2$$

Here  $m = -5$ ,  $n = 7$

$$(m + n)^2 = -5^2 + 2 \times -5 \times 7 + 7^2$$

$$(m + n)^2 = 25 - 70 + 49$$

$$(m + n)^2 = 4$$

$$(a - b)^3 + (m + n)^2 = 0 + 4$$

$$(a - b)^3 + (m + n)^2 = 4$$

**14.) If  $p = 7$ ,  $q = -5$  then find the value of  $(q + p)^2 - qp$**

ANSWER:

$$(q + p)^2 - qp$$

Here we use,

$$(q + p)^2 = q^2 + 2qp + p^2$$

Here,  $p = 7$ ,  $q = -5$

$$(q + p)^2 = -5^2 + 2 \times -5 \times 7 + 7^2$$

$$(q + p)^2 = 25 - 70 + 49$$

$$(q + p)^2 = 4$$

$$(q + p)^2 - qp = 4 - (7 \times -5)$$

$$(q + p)^2 - qp = 39$$

**15.) If  $x = 10$ ,  $y = 17$  then find the value of  $(x - y)^2 + (x + y)$**

ANSWER:

$$(x - y)^2 + (x + y)$$

Here we use,

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x - y)^2 + (x + y) = x^2 - 2xy + y^2 + (x + y)$$

$$x = 10, y = 17$$

$$10^2 - 2 \times 10 \times 17 + 17^2 + (10 + 17)$$

$$100 - 340 + 289 + 27$$

$$(x - y)^2 + (x + y) = 76$$

**16.) If  $p = 1.5$ ,  $q = 0.5$  then find the value of  $(p + q)^2 - (2)^3$**

ANSWER:

$$(p + q)^2 - (2)^3$$

Here we use,

$$(p + q)^2 = p^2 + 2pq + q^2$$

$$(p + q)^2 - (2)^3 = p^2 + 2pq + q^2 - 8$$

$$p = 1.5, q = 0.5$$

$$p^2 + 2pq + q^2 - 8 = 1.5^2 + 2 \times 1.5 \times 0.5 + 0.5^2 - 8$$

$$= 2.25 + 1.5 + 0.25 - 8$$

$$= 4 - 8$$

$$(p + q)^2 - (2)^3 = -4$$

**17.) If  $a = 10$ ,  $b = -5$ ,  $c = 2$ , then find the value of  $(a + b + c)^3$**

ANSWER:

$$(a + b + c)^3$$

$$(10 - 5 + 2)^3$$

$$= (7)^3$$

$$(a + b + c)^3 = 343$$

**18.) If  $x = 100 / \sqrt{25 + (1^0 \times 0)}$  then find the value of  $x^2$**

ANSWER:

$$x = 100 / \sqrt{25 + (1^0 \times 0)}$$

We have to find value of  $x^2$

$$x^2 = (100 / \sqrt{25 + (1^0 \times 0)})^2$$

$$x^2 = 100^2 / 25$$

$$x^2 = 400$$

**19.) If  $a = 101^2 - 100^2$  then find the value of  $a - 1$**

ANSWER:

$$a = 101^2 - 100^2$$

We have to find  $a - 1$

We know,

$$(a + b)(a - b) = a^2 - b^2$$

We put,  $a = 101$ ,  $b = 100$

$$(101 + 100)(101 - 100) = 101^2 - 100^2$$

$$201 \times 1 = 101^2 - 100^2$$

$$101^2 - 100^2 = 201$$

**The value of  $a - 1 = 201 - 1 = 200$**

**20.) If  $m = 100^2 - 98^2$  then find the value of  $m + 4$**

ANSWER:

$$m = 100^2 - 98^2$$

$$(a + b)(a - b) = a^2 - b^2$$

We put,  $a = 100$ ,  $b = 98$

$$(100 + 98)(100 - 98) = 198 \times 2$$

$$100^2 - 98^2 = 396$$

**The value of  $m + 4 = 396 + 4 = 400$**

**21.) If  $a = 100$ ,  $b = 98$  then find the value of  $(a - b)^2 + (a + b)$**

ANSWER:

$$a = 100, b = 98$$

We have to find  $(a - b)^2 + (a + b)$

We know,

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a - b)^2 + (a + b) = a^2 - 2ab + b^2 + (a + b)$$

$$100^2 - 2 \times 100 \times 98 + 98^2 + (100 + 98)$$

$$(a - b)^2 + (a + b) = 201$$

**22.) Simplify:  $(\sqrt{25} - \sqrt{16})(\sqrt{16} - \sqrt{9}) + \sqrt{36} - \sqrt{16}$**

ANSWER:

We know,

$$\sqrt{25} = 5$$

$$\sqrt{9} = 3$$

$$\sqrt{16} = 4$$

$$\sqrt{36} = 6$$

$$(\sqrt{25} - \sqrt{16})(\sqrt{16} - \sqrt{9}) + \sqrt{36} - \sqrt{16} = (5-4) \times (4-3) + 6 - 4$$

$$(5-4) \times (4-3) + 6 - 4 = 1 \times 1 + 2$$

$$(\sqrt{25} - \sqrt{16})(\sqrt{16} - \sqrt{9}) + \sqrt{36} - \sqrt{16} = 3$$

**23.) Simplify:  $(2a - b)^2 - (a + 2b)^2 + 3b^2 + ab$**

ANSWER:

Here we use,

$$(2a - b)^2 = (a - b)^2 = a^2 - 2ab + b^2$$

$$a = 2a \text{ and } b = b$$

$$(2a - b)^2 = 4a^2 - 4ab + b^2$$

$$(a + 2b)^2 = (a + b)^2 = a^2 + 2ab + b^2$$

$$a = a \text{ and } b = 2b$$

$$(a + 2b)^2 = a^2 + 4ab + 4b^2$$

$$(2a - b)^2 - (a + 2b)^2 + 3b^2 + ab = 4a^2 - 4ab + b^2 - a^2 + 4ab + 4b^2 + 3b^2 + ab$$

$$= 4a^2 - 4ab + b^2 - a^2 + 4ab + 4b^2 + 3b^2 + ab$$

On solving,

$$(2a - b)^2 - (a + 2b)^2 + 3b^2 + ab = 3a^2 - 7ab$$

**24.) Simplify:  $(a + b)^2 + (a - b)^2 - ab$**

ANSWER:

Here we use,

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^2 + (a - b)^2 - ab = a^2 + 2ab + b^2 + a^2 - 2ab + b^2 - ab$$

$$(a + b)^2 + (a - b)^2 - ab = 2a^2 + 2b^2 - ab$$



**25.) Simplify:  $(a - b)(a + b) + (a + b)(a - b) + a^2 - b^2$**

ANSWER:

We know,

$$(a - b)(a + b) = a^2 - b^2$$

$$(a - b)(a + b) + (a + b)(a - b) + a^2 - b^2 = a^2 - b^2 + a^2 - b^2 + a^2 - b^2$$

$$a^2 - b^2 + a^2 - b^2 + a^2 - b^2 = 3a^2 - 3b^2$$

**26.) If  $(5)^2 - (4.9)^2 = x$ , find the value of  $(0.01 \times x)$**

ANSWER:

Here we use,

$$(a - b)(a + b) = a^2 - b^2$$

$$a = 5 \quad \text{and} \quad b = 4.9$$

$$(5 - 4.9) \times (5 + 4.9) = (5)^2 - (4.9)^2$$

$$x = 0.1 \times 9.9 = 0.99$$

$$\text{The value of } (0.01 \times x) = (0.01 \times 0.1 \times 9.9)$$

$$(0.01 \times x) = 0.0099$$

**28.) Simplify:  $(x + y)^2 - (x - y)^2 + x - y$**

ANSWER:

Here we use,

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^2 - (x - y)^2 + x - y = x^2 + 2xy + y^2 - x^2 - 2xy + y^2 + x - y$$

$$x^2 + 2xy + y^2 - x^2 - 2xy + y^2 + x - y = 4xy + x - y$$

$$(x + y)^2 - (x - y)^2 + x - y = 4xy + x - y$$

**29.) What should be subtracted from  $(a + b)^2$  to make it  $(a - b)^2$ ?**

ANSWER:

We know,

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

**4ab** subtracted from  $(a + b)^2$  to make it  $(a - b)^2$

**30.) What should be added in  $25x^2 + 16y^2$  to make it  $(5x + 4y)^2$ ?**

ANSWER:

We use

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(5x + 4y)^2 = 5x^2 + 2 \times 5x \times 4y + 4y^2$$

$$(5x + 4y)^2 = 25x^2 + 16y^2 + 40xy$$

**40xy** added in  $25x^2 + 16y^2$  to make it  $(5x + 4y)^2$

**31.)  $121m^2 - 100n^2$  should be divided by which expression to get  $11m + 10n$ ?**

ANSWER:

$$121m^2 - 100n^2 / x = 11m + 10n$$

By cross multiplication,

$$121m^2 - 100n^2 / 11m + 10n = x$$

$$(a - b)(a + b) = a^2 - b^2$$

$$121m^2 - 100n^2 = (11m + 10n)(11m - 10n)$$

On solving,

$$X = 11m - 10n.$$

**32.) Simplify:  $(2x + 3y)^2 + 24xy$**

ANSWER:

$$(2x + 3y)^2 + 24xy$$

Here we use,

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$a = 2x \quad b = 3y$$

$$(2x + 3y)^2 = 2x^2 + 2 \times 2x \times 3y + 3y^2$$

$$(2x + 3y)^2 = 4x^2 + 12xy + 9y^2$$

$$(2x + 3y)^2 + 24xy = 4x^2 + 12xy + 9y^2 + 24xy$$

$$(2x + 3y)^2 + 24xy = 4x^2 + 36xy + 9y^2$$

**33.) Simplify:  $(3x - 4y)^2 - 16y^2$**

ANSWER:

$$(3x - 4y)^2 - 16y^2$$

Here we use,

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a = 3x \quad b = 4y$$

$$(3x - 4y)^2 = 3x^2 - 2 \times 3x \times 4y + 4y^2$$

$$(3x - 4y)^2 = 9x^2 - 24xy + 16y^2$$

$$(3x - 4y)^2 - 16y^2 = 9x^2 - 24xy + 16y^2 - 16y^2$$

$$(3x - 4y)^2 - 16y^2 = 9x^2 - 24xy$$

**34.) Simplify:  $(2.5m - 0.5n)^2 + 2.5mn + 3.5mn$**

ANSWER:

$$(2.5m - 0.5n)^2 + 2.5mn + 3.5mn$$

Here we use,

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a = 2.5m \quad b = 0.5n$$

$$(2.5m - 0.5n)^2 = 2.5m^2 - 2 \times 2.5m \times 0.5n + 0.5n^2$$

$$(2.5m - 0.5n)^2 = 6.25m^2 - 2.5mn + 0.25n^2$$

$$(2.5m - 0.5n)^2 + 2.5mn + 3.5mn$$

$$= 6.25m^2 - 2.5mn + 0.25n^2 + 2.5mn + 3.5mn$$

$$(2.5m - 0.5n)^2 + 2.5mn + 3.5mn = 6.25m^2 + 3.5mn + 0.25n^2$$

**35.) What should be subtracted from  $(x + y)^2$  to get  $x^2 + y^2$ ?**

ANSWER:

$$(x + y)^2$$

We know,

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$x^2 + 2xy + y^2 - \text{Subtraction} = x^2 + y^2$$

$$\text{Subtraction} = x^2 + 2xy + y^2 - (x^2 + y^2)$$

$$\text{Subtraction} = 2xy$$

**2xy** subtracted from  $(x + y)^2$  to get  $x^2 + y^2$

**36.) The sides of a rectangle are 5x units and 7y units. Find the area and perimeter of the rectangle.**

ANSWER:

Given that, the sides of a rectangle are 5x units and 7y units

Area of Rectangle = Length x Breadth

$$\text{Area of Rectangle} = 5x \times 7y = 35xy$$

$$\text{Perimeter of Rectangle} = 2 \times (\text{Length} + \text{Breadth})$$

$$\text{Perimeter of Rectangle} = 2 \times (5x + 7y)$$

$$\text{Perimeter of Rectangle} = 10x + 14y$$

**37.) Find the value of 61 × 5.9**

ANSWER:

$$61 \times 5.9$$

We multiply without decimal point. After we give decimal point.

$$61 \times 59 = 3599$$

$$61 \times 5.9 = 359.9$$

**38.) Find the value of 302 × 298**

ANSWER:

$$302 \times 298$$

We write,

$$302 = (300 + 2)$$

$$298 = (300 - 2)$$

$$302 \times 298 = (300 + 2) \times (300 - 2)$$

We know,

$$(a - b)(a + b) = a^2 - b^2$$

$$a = 300 \quad \text{and} \quad b = 2$$

$$(300 + 2) \times (300 - 2) = 300^2 - 2^2$$

$$90000 - 4$$

$$302 \times 298 = 89996$$

**39.) If  $x = 2 + \sqrt{3}$  and  $y = 2 - \sqrt{3}$ , then find the value of  $(x - y)^2$ .**

ANSWER:

Given,

$$x = 2 + \sqrt{3} \quad \text{and} \quad y = 2 - \sqrt{3}$$

We have to find  $(x - y)^2$ .

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(2 + \sqrt{3} - 2 - \sqrt{3})^2 = (2 + \sqrt{3})^2 - (2 \times 2 - \sqrt{3} \times 2 + \sqrt{3}) + (2 - \sqrt{3})^2$$

$$(2 + \sqrt{3} - 2 - \sqrt{3})^2 = 4 + 3 - 11 + 1$$

$$(2 + \sqrt{3} - 2 - \sqrt{3})^2 = 12$$

**42.) If  $(53)^2 = (48)^2 + 5x$ , then find the value of  $x$ .**

ANSWER:

Given,

$$(53)^2 = (48)^2 + 5x$$

$$5x = (53)^2 - (48)^2$$

We use,

$$(a - b)(a + b) = a^2 - b^2$$

$$a = 53 \quad \text{and} \quad b = 48$$

$$(53 - 48) \times (53 + 48) = (53)^2 - (48)^2$$

$$5 \times 101 = (53)^2 - (48)^2$$

$$(53)^2 - (48)^2 = 505$$

$$5x = (53)^2 - (48)^2$$

$$5x = 505$$

$$x = 505 / 5$$

$$x = 101$$

43.) If  $a = 0.8$  and  $b = 0.5$ , then find the value of  $a^2 + b^2 + ab$ .

ANSWER:

Given,

$$a = 0.8 \text{ and } b = 0.5$$

We have to find  $a^2 + b^2 + ab$ .

$$a^2 + b^2 + ab = (0.8)^2 + (0.5)^2 + 0.8 \times 0.5$$

$$a^2 + b^2 + ab = 0.64 + 0.25 + 0.40$$

$$a^2 + b^2 + ab = 1.29$$

44.) If  $x = 5$  and  $b = 3.2$ , then find the value of  $x^2 + y^2 - xy$ .

ANSWER:

Given,

$$x = 5 \text{ and } b = 3.2$$

We have to find  $x^2 + y^2 - xy$ .

$$x^2 + y^2 - xy = 5^2 + (3.2)^2 - 5(3.2)$$

$$x^2 + y^2 - xy = 25 + 10.24 - 16$$

$$x^2 + y^2 - xy = 19.24$$

45.) If  $x = 3 + 2\sqrt{2}$  and  $y = 17 + 12\sqrt{2}$ , then find the value of  $\sqrt{y} - \sqrt{x}$

ANSWER:

Given,

$$x = 3 + 2\sqrt{2} \text{ and } y = 17 + 12\sqrt{2}$$

We have to find  $\sqrt{y} - \sqrt{x}$

$$\sqrt{x} = \sqrt{3 + 2\sqrt{2}}$$

$$\sqrt{y} = \sqrt{17 + 12\sqrt{2}}$$

$$\sqrt{y} - \sqrt{x} = \sqrt{17 + 12\sqrt{2}} - \sqrt{3 + 2\sqrt{2}}$$

On solving we get,

$$\sqrt{y} - \sqrt{x} = 2 + \sqrt{2}$$