Laws of motion.

Push or pull is necessary to move object from one place to another. The external quantity that helps to perform activities like pulling a trolley, pushing cupboard required is force.

The physical quantity that can change the state of object is called as Force. (Here state means → state of rest to motion or vice versa.)

Aristotle's fallacy :-

Greek thinker, Aristotle observed the motion of objects under different conditions and concluded that 'An external force is required to keep an object in motion.'

The above statements hold good when we consider motion of objects/vehicles on ground under influence of frictional, gravitational forces. In some cases like uniform motion of car do not require force, so Aristotle's fallacy had some flaws & to overcome those flaws, concept of inertia is to be studied.

The law of inertia :-

Sir Galileo studied motion of objects on horizontal plane, inclined plane and observed that moving bodies on frictionless track/path do not have acceleration or retardation. He also observed that frictional forces can not be totally eliminated from motion. This leads the concept of inertia (to overcome frictional forces.)

Inertia is tendancy of object to remain in its state. Body can not change it's state untill external agency i.e. force acts on it.

Newton's 1st law of motion :
With the help of Galileo's concepts on inertia, Sir Isaac Newton laid the foundation of mechanics with three laws of motion.
statement: Every body continuous ~~to~~ be in state of ~~moti~~ rest or uniform motion along straight line unless an external unbalanced force acts on it.
Newton's 1st law signifies the importante of inertia.
eg. 1) to pull a trolley loaded with lugguage requires certain quantity of force ~~due~~ to overcome frictional force.
2) book kept on table will remain in state of rest untill external force changes its position.
If net external force acting on body is zero, its acceleration is zero.

Momentum :-
The quantity gained by body in motion is called as momentum. It is defined as the product of mass and velocity.
If 'm' is mass of object moving with velocity 'v', then momentum 'p' of body is,
$$p = mv.$$
SI unit is kg·m/s. It is vector quantity.
* Mass and velocity are important parameters to study effects of force acting on body.
eg. 1) bullet fired from gun has high velocity due to which it can penetrate human body easily.
2) a cricket ball travels faster after hitting by batsman.

Newton's 2nd law of motion :

The rate of change of momentum of a body is directly proportional to applied force and takes place in the direction of applied force.

$\therefore F \propto \frac{dp}{dt}$ [Where dp = Change in momentum, dt = small time interval.]

$\therefore F = k.\frac{dp}{dt}$ —— (1)

If velocity of body changes from v to $v+dv$ in time Δt, then the equation (1) can be written as,

$F = k.\frac{d}{dt}(mv)$

$= k.m.\frac{dv}{dt}$

$= k.m.a$ ($\because$ acceleration, $a = dv/dt$)

The constant of proportionality has value unity, $\therefore$ $\boxed{F = ma}$

SI unit of force is 'newton' (N).

Where $1\ N = 1\ kg \cdot 1\ m/s^2$

* Here if $F = 0$, then $a = 0$. $\Rightarrow$ This is analogous to 1st law.
* If the motion is studied using vectors then the components of forces can be given as,

$F_x = m.a_x$, $F_y = m.a_y$ & $F_z = m a_z$

* Equation $F = ma$ includes only external forces that are acting on body. It do not involves internal forces present in body.
* Impulse :- The product of force and time interval (i.e. momentum) is called impulse.

Large forces acting for short interval are impulsive

Newton's 3rd law of motion :

To every action, there is always an equal, opposite and instanteneous reaction.

- Forces always occurs in pair.
- Force acting on body 1 is equal in magnitude & opposite in direction on body 2
- Action reaction forces acts on different bodies.

$$\vec{F}_{12} = -\vec{F}_{21}$$

ie. force on body 1 by body 2 = −(force on body 2 by 1)

Conservation of momentum :-

Consider two colliding bodies 1 and 2 collides on each other and seperates in time Δt.

Let $\vec{F}_{12}$ = Force on body 1 by 2

$\vec{F}_{21}$ = Force on body 2 by 1

then by 3rd law,

$$\vec{F}_{12} = -\vec{F}_{21} \quad \text{——— (1)}$$

According to 2nd law

$$\vec{F}_{12} = \frac{\text{Change in momentum of body 1}}{\Delta t}$$

$$\therefore F_{12} \cdot \Delta t = P_1' - P_1$$

$$\& \quad F_{21} \cdot \Delta t = P_2' - P_2$$

∴ On substituting in eqn (1)

$$P_1' - P_1' = -(P_2' - P_2)$$

$$\therefore P_1' - P_1 = -P_2' + P_2$$

$$\therefore P_1' + P_2' = P_1 + P_2$$

Total momentum after collision = Total momentum before collision.

ie. The total momentum of an isolated system of interacting particles is conserved.

Equilibrium of a particle :-

An object is to be said in equilibrium if net external force on the particles is zero.

ie if F_1 & F_2 be the two forces acting on body then for equilibrium,

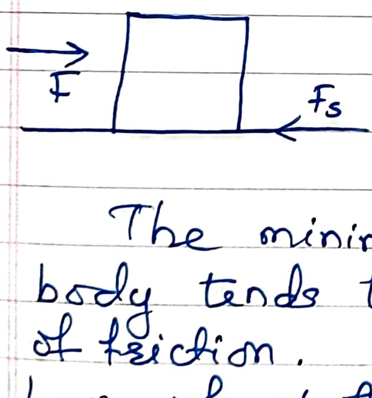
$$F_1 + F_2 = 0 \quad \text{or} \quad F_1 = -F_2$$

Common forces in mechanics :-

Tension acting on cables of elevator/lift, restoring force in a spring are some examples of common forces that are used in mechanics. Lets discuss some of them in detail.

* Frictional force / Friction :-

The opposition to the relative motion between objects in contact is called friction. It occurs between irregularities present on the surface of objects.



Friction is divided into

(I) static friction.
(II) Kinetic friction.
(III) Rolling friction.

The minimum value of force for which a body tends to move is known as limiting force of friction.

- Laws of static friction :-

1. Limiting force of static friction (f_s) is directly proportional to the normal reaction.

$\therefore \quad f_s \propto N.$

$\therefore \quad f_s = \mu_s \cdot N.$

where μ_s is coeff. of static friction.

$\mu_s = f_s / N.$

In practice, $f_s \le \mu_s N.$

2. Limiting force of static friction is independent of the area of contact.
3. Limiting force of static friction depends upon nature of surface & materials in contact.

* If the frictional force opposes the relative motion between surfaces in contact then friction is said to be, kinetic friction & minimum force is called as limiting force of kinetic friction (f_K)
Hence $f_K \propto N \Rightarrow f_K = \mu_K N$.
where μ_K = coeff. of kinetic friction.

* If an object like ring, sphere, cylinder is rolling over plane horizontal or inclined surface then the friction is rolling friction.
Rolling friction is smallest among all types -
Ball bearings used in mechanics resemblance the existance of rolling friction.
Rolling friction reduces power dissipation in case of mechanics.

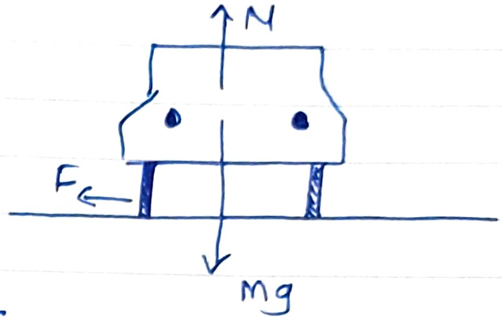
Circular motion :-
The motion of particles along circumference of circle is known as circular motion.
Centripetal force is necessary to produce it.
It is always acts along radius & directed towards centre of circle. It is given by

$$F_{CP} = \frac{mv^2}{r} \quad \text{or} \quad F_{CP} = m\omega^2 r.$$

Where m = mass of body
v = linear speed
r = Radius of circle.

* Motion of car on level road :-

Consider a car of mass 'm' is moving along curved horizontal road of radius of curvature R.



Let N = Normal reaction
= weight of ~~weigh~~ car
= mg —— ①

For equilibrium of car, necessary centripetal force is provided by frictional force between road & tyres of car.

$\therefore f \leq \mu_s N$.

$\therefore \frac{mv^2}{R} \leq \mu_s mg$ (from eqn 1)

$\therefore v^2 \leq \mu_s Rg$ —— ②

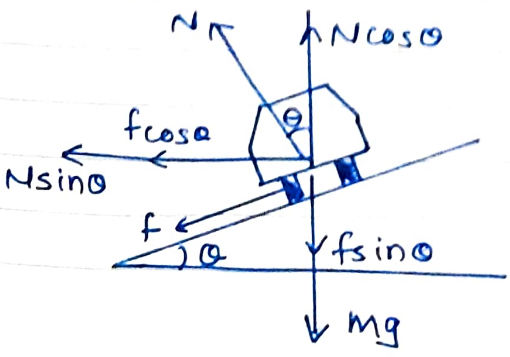
This is nothing but max^m speed at which car can be driven safely along curved road.

$\therefore V_{max} = \sqrt{\mu_s Rg}$ —— ③

From eq ③ we can conclude that maximum safety speed is independant of mass of vehicle.

* Motion of car on banked road :-

Elevation of outer edge of road over its inner edge is called as banking of road. The angle made by elevated road surface with horizontal road is called as angle of banking.



Consider vehicle of mass 'm' is moving along curved banked road of angle of banking 'θ' (as shown in fig.)

The weight of vehicle is balanced by component $N\cos\theta$ of normal reaction & $F\sin\theta$ of frictional force

$$\therefore \quad mg = N\cos\theta - f\sin\theta \quad \text{——} (1)$$

Also the necessary centripetal force is provided by $N\sin\theta$ & $f\cos\theta$.

$$\therefore \frac{mv^2}{R} = N\sin\theta + f\cos\theta \quad \text{——} (11)$$

dividing eqn (1) by (11) we get,

$$\frac{mv^2/R}{mg} = \frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}$$

$$\therefore v^2 = Rg\left[\frac{N\sin\theta + f\cos\theta}{N\cos\theta - f\sin\theta}\right]$$

for $v = v_{max}$, $f \le \mu_s N$.

$$\therefore V_{max}^2 = Rg\left[\frac{N\sin\theta + \mu_s N\cos\theta}{N\cos\theta - \mu_s N\sin\theta}\right]$$

$$= Rg\left[\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right]$$

$$= Rg\left[\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right]$$

$$\therefore V_{max} = \sqrt{Rg\left(\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right)} \quad \text{——} (3)$$

if $\mu_s = 0$ then $V = V_{optimum}$

$$\therefore V_{opt} = \sqrt{Rg\tan\theta}$$

$$\text{or} \quad \boxed{\tan\theta = \frac{v^2}{Rg}} \quad \text{——} (4)$$

Maximum safety speed of vehicle is independent of mass of vehicle.