10. Mechanical Properties of fluid.

The substance that can flow is assigned as fluid. Fluids possesses shear modulus due to its ability of compressibility.
The shearing stress of fluids is about million times smaller than that of solids.

Pressure :

If any object is immersed in liquid/fluid at rest, it experiences a force over its surface in contact with liquid. This force exerted by liquid is always perpendicular to the surface of fluid. This gives rise concept of liquid pressure.
The force acting normally to the given surface area of object is called as pressure.

If F = normal force
A = Area,

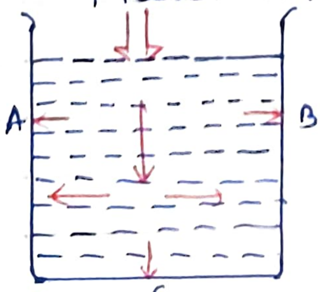
$$\therefore P_{avg} = \frac{Force}{Area} = \frac{F}{A}$$

In limiting value, $P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$

Pressure is scalar quantity.
SI unit is N/m^2 or pascal (Pa)
Dimensions are $[M^1 L^{-1} T^{-2}]$

Pascal's law :-

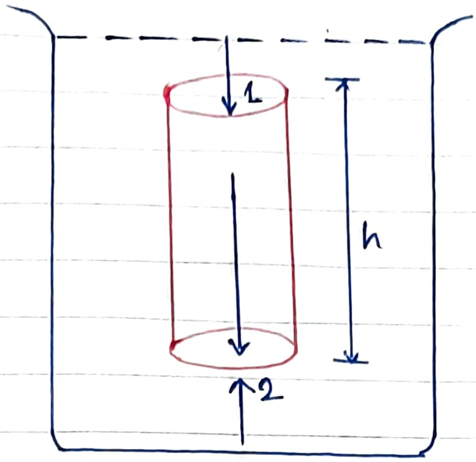
The pressure exerted at any part of fluid at rest is transmitted equally to all parts of fluid and to the walls of vessels.



i.e. According to Pascal's law, pressure at points A, B & C is same.
i.e. $P_A = P_B = P_C$

$$\frac{F_A}{A_1} = \frac{F_B}{A_2} = \frac{F_C}{A_3}$$

Variation of pressure with depth.



Consider a fluid at rest in vessel as shown in fig. Imagine that there is a cylinder of liquid is situated inside it, whose area of cross-section is 'A' and height is 'h'. The upper face of cylinder has pressure 'P_1'

$\therefore P_1 = F_1/A \Rightarrow F_1 = P_1 A$. Similarly lower face has pressure P_2 $\therefore F_2 = P_2 A$. Due to these forces resultant force acting on cylinder is $\Delta F = F_2 - F_1$

$$\therefore \Delta F = P_2 A - P_1 A = (P_2 - P_1) A \quad \text{—— (I)}$$

For equilibrium, this force should be balanced by gravitational force i.e. weight of cylinder

$$\therefore \Delta F = \text{weight of cylinder}$$

$$\therefore (P_2 - P_1) A = mg \quad \text{—— (II)}$$

If 'ρ' is density of fluid, then

$$\rho = \text{mass of cylinder} / \text{volume of cylinder}$$

$$\therefore \rho = m/V \Rightarrow m = \rho V = \rho A h \quad (\because V = Ah)$$

$$\therefore (P_2 - P_1) A = \rho A h g$$

$$\therefore P_2 - P_1 = \rho h g \quad \text{—— (III)}$$

This shows that pressure difference depends on height (vertical length) of object.

$$\therefore P_2 = P_1 + \rho h g \quad \text{—— (IV)}$$

If P_a is pressure contributed by atmosphere, then we can write

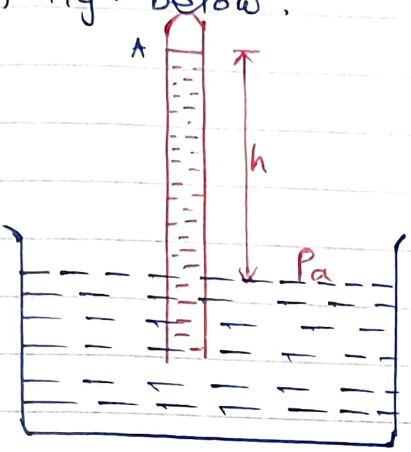
$$P = P_a + \rho h g$$

The difference is pressure due to fluid & atmospheric pressure at height 'h' is called as Gauge pressure.

Mercury barometer :

A device used to measure atmospheric pressure was discovered by Torricelli; is called as mercury barometer.

A long glass tube closed at one end and filled with mercury is inverted into trough of mercury is called mercury barometer. It is shown in fig. below.



At certain height of mercury in long pipe,

pressure inside column = pressure outside coloumn

$\therefore$ $P_a = h\rho g$ —— (1)

For 1 atmospheric pressure

$h = 76$ cm of mercury

$\rho = 13600$ g/cm^3

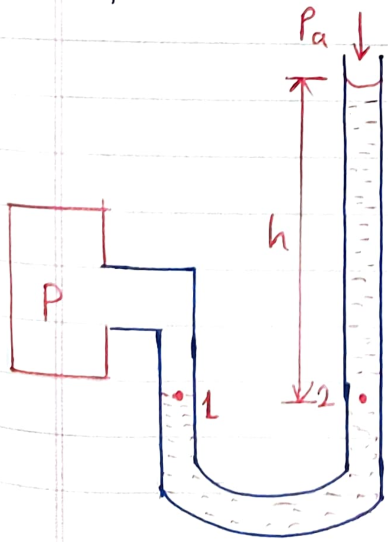
$g = 980$ cm/s^2

$\therefore$ 1 atm $= 1.013 \times 10^5$ N/m^2

* A pressure when height of mercury column is 1 mm is called 1 torr

$\therefore$ 1 torr $= 133$ N/m^2

Open tube manometer :



It is used to measure pressure diff. between two points. It consists of U-tube (one end - open to atmosphere and other connected to object whose pressure is to be measured) shown in fig.

When $P_1 = P_2$ then

$P - P_a = h\rho g$.

Applications of Pascal's law

1) Hydraulic Press : Used to press/crush objects
2) Hydraulic Lift : Used to lift heavy objects/vehicles
3) Hydraulic brakes : Used in vehicles to stop it.

These machines works on Pascal's law
i.e. Pressure at input piston = pressure at output piston.

$$\therefore P_I = P_O$$

$$\therefore F_1/A_1 = F_2/A_2$$

$$\therefore F_2 = \frac{F_1}{A_1} \times A_2$$

if $A_2 > A_1$ then $F_2 > F_1$

* Archemedes Principle :

If any object is immersed completely or partially in fluid, it exerts a buoyant force which is equal to weight of ~~object~~ fluid displaced by object.

$\therefore$ Buoyant force = weight of ~~object~~ fluid displaced

$$= m'g = \rho_{fluid} V_{fluid}\, g$$

Fluid dynamics :

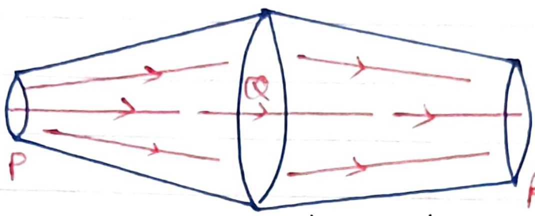
The branch of physics which deals with fluids in motion is called as fluid dynamics.
On the basis of velocity of flow, flows are divided into

1) streamline flow : The flow in which velocity of each layer during motion is constant throughout is called as steam line flow or laminar flow.
2) Turbulent flow : The flow in which

velocity of all layers is not constant, is known as turbulent flow.

* The tangent at any point on flow which gives direction of flow is called stream line.
  * No streamlines can intersect each other.

Equation of continuity :



Consider a streamline flow through the pipe of different cross-sections as shown in fig.

Let A_1 = Area of cross section at pt. P
A_2 = Area of cross section at point Q
A_3 = Area of cross-section at point R.

let V_1, V_2, V_3 be the corresponding velocities of flow at points P, Q & R respectively.

let liquid/flow travels distance x_1 in time Δt at pt. P. $\therefore V_1 = x_1/\Delta t \Rightarrow x_1 = V_1 \Delta t$ —— (i)

Similarly, $V_2 = x_2/\Delta t \Rightarrow x_2 = V_2 \Delta t$ &
$V_3 = x_3/\Delta t \Rightarrow x_3 = V_3 \Delta t$ } (ii) be the displacements of particles in time interval Δt.

From conservation of mass, we can say that amount of particles travels in 'Δt' through points P, Q & R is same.

$$\therefore \Delta m_P = \Delta m_Q = \Delta m_R \quad \text{—— (iii)}$$

if ρ = density of fluid

$$\therefore \rho = \frac{\Delta m_P}{V} \Rightarrow \Delta m_P = \rho V$$
$$= \rho A_1 x_1 \quad (\because Vol^m = Area \times length)$$

$$\therefore \Delta m_P = \rho A_1 V_1 \Delta t \quad (\because \text{from eq}^n \text{ i})$$

& $\Delta m_Q = \rho A_2 V_2 \Delta t$, $\Delta m_R = \rho A_3 V_3 \Delta t$

$\therefore$ Eq^n (iii) becomes

$$\rho A_1 V_1 \Delta t = \rho A_2 V_2 \Delta t = \rho A_3 V_3 \Delta t \Rightarrow A_1 V_1 = A_2 V_2$$

i.e. Av = constant.

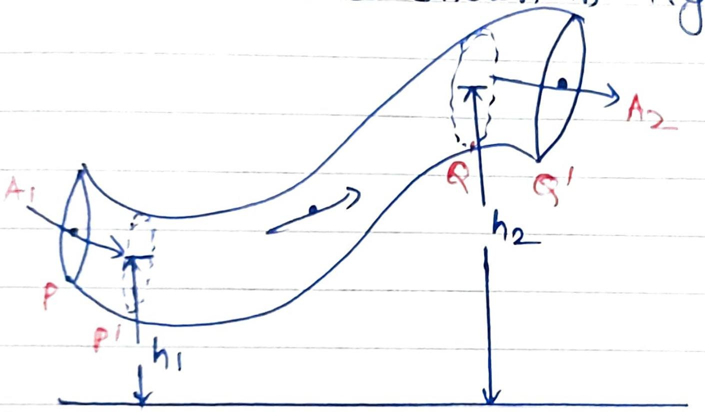
Bernoulli's Principle :-

For streamline flow, the sum of pressure, kinetic energy per unit volume and potential energy per unit volume at all points in flow remains constant.

i.e.

$$\text{Pressure} + \frac{\text{Kinetic energy}}{\text{Volume}} + \frac{\text{Potential energy}}{\text{Volume}} = \text{constant}$$

Proof : Consider streamline flow of fluid of density 'ρ' through the pipe of different cross-sections as shown in fig. below.



Let,

A_1 = Area of cross-section at pt PP'

A_2 = Area of cross-section at QQ'

v_1 = velocity of flow through PP'

v_2 = velocity of flow through QQ'

Then by equation of continuity

$$A_1 v_1 = A_2 v_2 \qquad \text{——— (1)}$$

Let fluid moves distance x_1 & x_2 from PP' & QQ' respectively in time Δt.

Then work done by force due to pressures P_1 & P_2 at points P & Q are

$$W_1 = P_1 A_1 x_1 \qquad \&$$
$$W_2 = P_2 A_2 x_2$$

But $x_1 = v_1 \Delta t$ & $x_2 = v_2 \Delta t$

$\therefore W_1 = P_1 A_1 v_1 \Delta t$ & $W_2 = P_2 A_2 v_2 \Delta t$

or $W_1 = P_1 \Delta V$ & $W_2 = P_2 \Delta V$

$(\because \Delta V = \text{Area} \times \text{distance} = Ax)$

The difference in work done from PP' to QQ' is

$$\Delta W = W_1 - W_2$$
$$= P_1 \Delta V - P_2 \Delta V$$
$$\Delta W = (P_1 - P_2)\Delta V \quad \text{--- (I)}$$

Also change in potential energy from QQ' to PP' is

$$\Delta U = \Delta m\, g\,(h_2 - h_1)$$
$$= \rho \Delta V \cdot g\,(h_2 - h_1) \quad (\because \Delta m = \rho/\Delta V) \text{ --- (II)}$$

finally, change in kinetic energy can be given as

$$\Delta KE = \frac{1}{2}\Delta m\,(V_2^2 - V_1^2)$$

$$= \frac{1}{2}\rho \cdot \Delta V \cdot g\,(V_2^2 - V_1^2) \text{ --- (III)}$$

Using Work-energy theorem, we can say

$$\Delta W = \Delta U + \Delta KE$$
$$\therefore (P_1 - P_2)\Delta V = \rho \Delta V \cdot g\,(h_2 - h_1) + \frac{1}{2}\rho \Delta V \cdot g\,(V_2^2 - V_1^2)$$
$$\therefore P_1 - P_2 = \rho g h_2 - \rho g h_1 + \frac{1}{2}\rho g V_2^2 - \frac{1}{2}\rho g V_1^2$$

$$\therefore P_1 + \rho g h_1 + \frac{1}{2}\rho g V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2$$

ie. $\boxed{P + \rho g h + \frac{1}{2}\rho V^2 = \text{constant}}$

Applications of Bernoulli's principle:

1) Speed of efflux:- (Torricelli's law)

The speed of flow of fluid through any opening (hole) is given by formula,

$$V = \sqrt{2gh + \frac{2(P - Pa)}{\rho}}$$

if $P = Pa$, then $P - Pa = 0$.

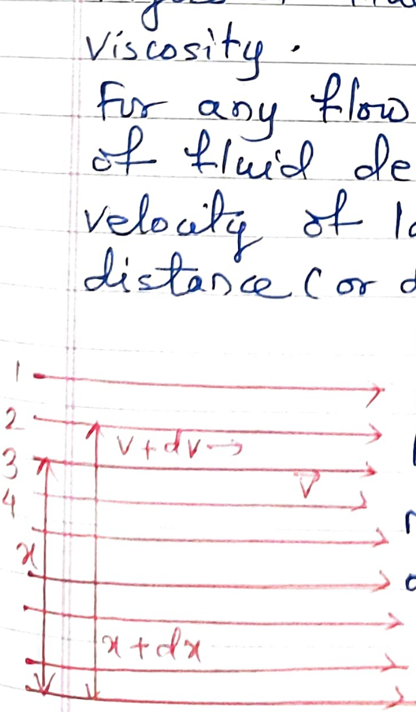
$$V = \sqrt{2gh}.$$

2) Venturimeter : It is a device use to measure speed of fluid. It works on principle of Bernoulli's equation.
The speed of fluid can be given as.

$$v = \sqrt{2\rho_m g h/\rho \left(\left(\frac{A}{a}\right)-1\right)}^{-1/2}$$

3) Dynamic lift :
The force acting on body in fluid is nothing but dynamic lift. Following are some important cases of dynamic lift
a) Ball moving without spin b) Ball moving with spin, c) Aerofoil or lift on aircraft wing.

Viscosity :
The friction between layers of fluid is nothing but viscosity.
The opposition to the relative motion between layers of fluid (or solid in fluid) is known as viscosity.
For any flow of fluid, velocities of inside layers of fluid decreases from top to bottom. ie. velocity of layers changes gradually with the distance (or depth of fluid)



Consider the layer 3, flowing with velocity 'v' at distance 'x' from bottom. Then layer 2 will have slightly more velocity Δv, ie. $v+dv$ when distance is increased by Δx, ie $x+dx$

$$\therefore \Delta v = v + dv - v = dv$$
$$\Delta x = x + dx - x = dx$$
$$\therefore \text{Velocity gradient} = \frac{\Delta v}{\Delta x} = \frac{dv}{dx}$$

Coefficient of viscosity (η)

The ratio of shearing stress to shearing strain is called as coefficient of viscosity.

$$\therefore \eta = \frac{Fl}{VA}$$

SI unit = $N.s/m^2$ or $Pa.s$

Dimensions = $[M^1 L^{-1} T^{-1}]$

Stoke's law :

Viscous force acting on spherical body falling freely in viscous fluid is directly proportional to

a) radius of body b) velocity of body

c) coefficient of viscosity of fluid.

$$\therefore f \propto r v \eta$$

$$\therefore f = 6\pi r v \eta$$

Using stoke's law, terminal velocity of body under fluid can be given as

$$V_t = \frac{2}{9} \frac{(\rho - \sigma) r^2 g}{\eta}$$

Reynold's number :

Reynold's number is used to study type of flow by defining term called critical velocity of fluid. It is given by

$$R = \frac{\rho V d}{\eta}$$

where ρ = density of fluid, v = velocity of fluid

d = diameter of pipe.

if a) $R < 1000$, flow is streamline

b) $R > 2000$, flow is turbulent

Surface tension (T)

The tendency of liquid surface due to which it behaves as stretched membrane is known as surface tension.

It is defined as the force acting normally to the length of object.

$\therefore$ Surface tension, $T = \dfrac{\text{Force}}{\text{Length}}$

$$T = F/L$$

SI unit is N/m

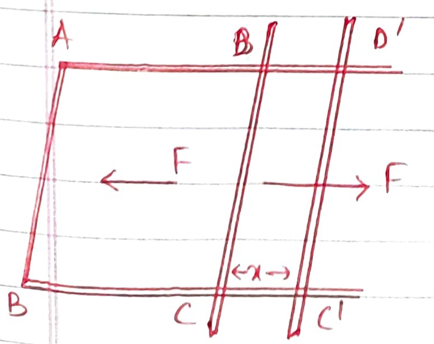
Dimensions = $[M^1 L^0 T^{-2}]$

Surface energy :

Energy stored in the molecules of free surface of liquid is known as surface energy.

Surface energy and surface tension :

Consider a rectangular frame of metal wire ABCD on which wire CD can slide without friction. Dip the frame in a soap solution, take out & held horizontally so that the soap film forms on frame as shown in fig.



Let T = Surface tension of film.

l = length of wire CD.

$F = 2Tl$. — ① (since film has 2 layers)

If we pull the wire CD by 'x' then work done,

$$dW = F\cdot x = 2Tlx = T(2lx)$$

$\therefore dW = T \cdot dA \quad (\because dA = 2lx)$ [increase in area]

This work is nothing but energy stored, called as surface energy $\therefore dU = T \cdot dA$

i.e. surface energy = surface tension × surface area.

Angle of contact :

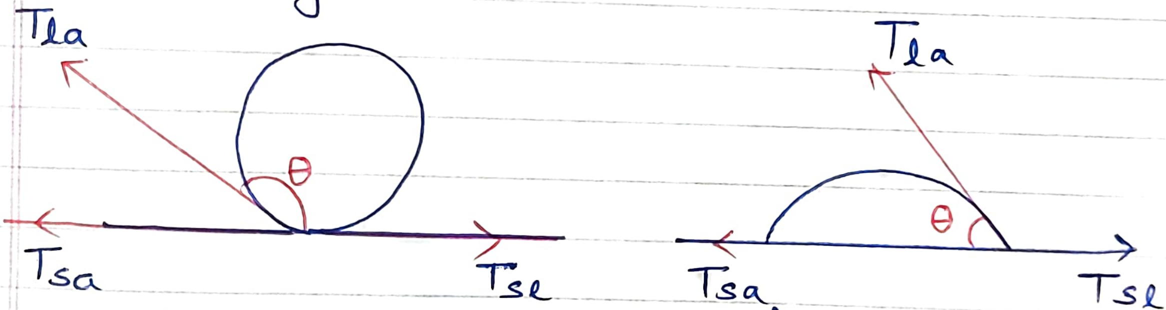
When a liquid is filled in any vessel, it forms meniscus due to which certain angle is produced by liquid surface with solid.

The angle between tangent drawn at the point of contact between liquid & surface of solid measured inside liquid (within liquid) is called as angle of contact. It denoted by 'θ'. Angle of contact decides the shape of drop of liquid, when falls on solid surface.

factors affecting angle of contact :

1) Impurities added in liquid.
2) Pressure above & below free surface of liquid
3) Temperature
4) Material ~~of~~ liquid - solid in contact.

Consider the liquid added on plane surface shown in fig.



Let T_{sa} = Force due to surface tension between solid - air interface

T_{sl} = Force due to surface tension between solid - liquid interface.

T_{al} = Force due to surface tension between air - liquid interface.

For equilibrium,

$$T_{sa} = T_{al}\cos\theta + T_{sl}$$

or $$\cos\theta = \frac{T_{sa} - T_{sl}}{T_{la}}$$

Case i) if $T_{se} > T_{sa}$ → angle of contact is obtuse

ii) if $T_{se} < T_{sa}$ → angle of contact is acute.

iii) if $T_{se} = T_{sa}$ → angle of contact is zero.

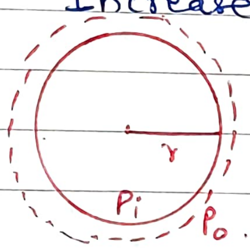
Drops and bubble :

Due to surface tension, liquid drops forms spherical shape. The pressure inside & outside these drops/ bubbles changes with radius of it.

Consider a liquid drop of radius 'r' such that P_i → pressure inside drop & P_o → pressure outside drop. Then the excess pressure $= P_i - P_o$

Suppose that the radius of drop is increased by Δr due to action of external force then

Increase in surface area $= A_2 - A_1$



$$= 4\pi(r+\Delta r)^2 - 4\pi r^2$$
$$= 4\pi r^2 + 8\pi r\Delta r + 4\pi \Delta r^2 - 4\pi r^2$$
$$dA = 8\pi r\Delta r \quad (\text{neglecting } \Delta r^2)$$

By def^n, work done,

$$dW = T \cdot dA = T \cdot 8\pi r \cdot \Delta r \quad \text{——— (A)}$$

Mechanical work done = Force × displacement

$$dW = (P_i - P_o)\, 4\pi r^2 \times \Delta r \quad \text{——— (B)}$$

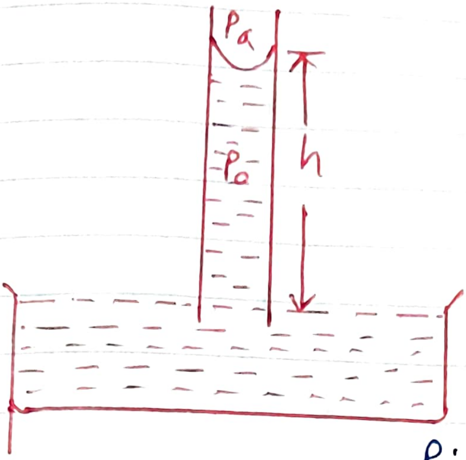
On equating eq^n (A) & (B) we get

$$(P_i - P_o) = \frac{2T}{r} \quad \text{——— (c)}$$

For Bubbles, $(P_i - P_o) = \dfrac{4T}{r}$ ——— (d)

Equation (c) & (d) are also known as Laplace's law of spherical membrane.

Capillary rise :-



The liquid meniscus formation depends upon the pressure inside & outside free surface of liquid.

When $P_a > P_o$, meniscus formed is concave.

$P_o > P_a$, meniscus formed is convex.

we have, excess pressure is

$$P_i - P_o = \frac{2T}{r} \cos\theta .$$

But $\Delta P = h\rho g$

$$\therefore h\rho g = \frac{2T}{r} \cos\theta \Rightarrow \boxed{T = \frac{h\rho r g}{2\cos\theta}}$$

if $\theta = 0^\circ \Rightarrow h\rho g = \frac{2T}{r}$

$$\boxed{\therefore h = \frac{2T}{\rho g r}}$$

The rise or fall of liquid level inside capillary tube when dipped in liquid is known as capillary action.

Detergents & surface tension:

Addition of detergent in water decreases surface tension of water so that water can sprayed through each fabric part easily.

Detergent molecules in water absorbs the stain, molecules of wax, oil from cloth & helps them to clean throughly.