Chapter 8. Matrices

Exercise 8.1

1.) Classify the following matrices:

i) $\begin{bmatrix} 2 & -1 \\ 5 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} 2 & 3 & -7 \end{bmatrix}$ iii) $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ iv) $\begin{bmatrix} 2 & -4 \\ 0 & 0 \\ 1 & 7 \end{bmatrix}$

v) $\begin{bmatrix} 2 & 7 & 8 \\ -1 & \sqrt{2} & 0 \end{bmatrix}$ vi) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

→ i) Given matrix is a square matrix of order 2.

ii) Given matrix is a row matrix of order (1×3).

iii) Given matrix is a column matrix of order (3×1).

iv) Given matrix is a matrix of order (3×2)

v) Given matrix is a matrix of order (2×3)

vi) Given matrix is a zero matrix of order (2×3).

2.) i) If a matrix has 4 elements, what are the possible order it can have?

ii) If a matrix has 8 elements, what are the possible order it can have?

→ i) It can have (1×4), (4×1) or (2×2) order.

ii) It can have (1×8), (8×1) or, (2×4) or (4×2) order.

3.) Construct a (2×2) matrix whose elements a_{ij} are given by

i) $a_{ij} = 2i - j$

ii) $a_{ij} = i \cdot j$

→ i) The required matrix be $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$.

ii) The required matrix be $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

4.) find the values of x & y if $\begin{bmatrix} 2x+y \\ 3x-2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$.

$\rightarrow$ Given two matrices are equal.

$\Rightarrow \begin{bmatrix} 2x+y \\ 3x-2y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

On comparing corresponding elements of matrix,

$2x+y=5$ —— ①

$3x-2y=4$ —— ②

① $\times 2 \Rightarrow$ $4x+2y=10$

$3x-2y=4$

$7x = 14$

$\boxed{x=2}$ put in ① $\Rightarrow$

$2x+y=5$

$2(2)+y=5$

$4+y=5$

$\boxed{y=1}$

Thus, the required values are $x=2$, $y=1$.

5.) find the value of x if $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$

$\rightarrow$ Given that, $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$

on compairing the elements which are corresponding,

$\Rightarrow$ $3x+y=1$ —① and $-y=2$

$2y-x=-5$

$\Rightarrow \boxed{y=-2}$ put in ①

$3x+y=1$

$3x-2=1$

$3x=3$ $\Rightarrow \boxed{x=1}$

Thus, the required values of x & y are $x=1$ & $y=-2$.

6.) If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, find the values of x and y.

$\rightarrow$ Given that, $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$

On compairing the corresponding elements,

$x+3=5$ $\quad$ $y-4=3$

$\boxed{x=2}$ $\quad$ $\boxed{y=7}$

Thus, the required values of $x=2$ & $y=7$

7) Find the values of x, y and z if $\begin{bmatrix} x+2 & 6 \\ 3 & 5z \end{bmatrix} = \begin{bmatrix} -5 & y^2+y \\ 3 & -20 \end{bmatrix}$

$\rightarrow$ Given that, $\begin{bmatrix} x+2 & 6 \\ 3 & 5z \end{bmatrix} = \begin{bmatrix} -5 & y^2+y \\ 3 & -20 \end{bmatrix}$

On compairing the corresponding elements of matrix,

$x+2=-5$ $\quad$ $5z=-20$ $\quad$ and $\quad$ $y^2+y=6$

$\boxed{x=-7}$ $\quad$ $\boxed{z=-4}$ $\quad$ $y^2+y-6=0$

$y^2+3y-2y-6=0$

$y(y+3)-2(y+3)=0$

$(y+3)(y-2)=0$

$\Rightarrow$ $y+3=0$ or $y-2=0$

$\boxed{y=-3}$ or $\boxed{y=2}$

Thus, the required values are $x=-7$, $y=-3,2$, $z=-4$.

8) Find the values of x, y, a & b if $\begin{bmatrix} x-2 & y \\ a+2b & 3a-b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$.

$\rightarrow$ Given that, $\begin{bmatrix} x-2 & y \\ a+2b & 3a-b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$

On compairing corresponding elements of matrix,

$x-2=3$ $\quad$ $\boxed{y=1}$ $\quad$ $a+2b=5$ & $3a-b=1$ — ①

$\Rightarrow \boxed{x=5}$ $\quad$ $6a-2b=2$

$7a=7$

$\boxed{a=1}$ put in ①

$\Rightarrow$ $3-b=1$

$\boxed{2=b}$

Thus, the required values of are $x=5$, $y=1$, $a=1$, $b=2$.

9.) Find the values of a, b, c and d if $\begin{bmatrix} a+b & 3 \\ 5+c & ab \end{bmatrix} = \begin{bmatrix} 6 & d \\ -1 & 8 \end{bmatrix}$.

$\rightarrow$ Given that, $\begin{bmatrix} a+b & 3 \\ 5+c & ab \end{bmatrix} = \begin{bmatrix} 6 & d \\ -1 & 8 \end{bmatrix}$

On compairing the corresponding elements of matrix,

$a+b=6$ — ① $\quad$ $5+c=-1$ $\quad$ $\boxed{d=3}$ $\quad$ $ab=8$

$\boxed{c=-6}$ $\quad$ $\boxed{a=8/b}$

put in ①

$\frac{8}{b}+b=6$

$8+b^2=6b$

$b^2-6b-8=0$

$b^2 - 6b - 8 = 0$

b^2

Now, $a+b=6$ and $ab=8$

We have, $(a-b)^2 = (a+b)^2 - 4ab$

$(a-b)^2 = 36-32 = 4 = (\pm 2)^2$

$\boxed{a-b=\pm 2}$

If $(a-b) = -2$ then $(a+b)=6$

$\Rightarrow (a-b)+(a+b) = 4$ $\Rightarrow a-b=-2$

$2a = 4$ $\quad 2-b=-2$

$\boxed{a=2}$ $\quad \boxed{4=b}$

Thus, the required values of a & b are found to be $a=2, b=4$.

Exercise 8.2

1.) Given that, $M=\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ and $N=\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$, find $M+2N$.

→ Given that, $M=\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ and $N=\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$

Then $M+2N = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 2+4 & 0+0 \\ 1-2 & 2+4 \end{bmatrix}$

$\boxed{M+2N = \begin{bmatrix} 6 & 0 \\ -1 & 6 \end{bmatrix}}$

2.) If $A=\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ find $(2A-3B)$.

→ Given that, $A=\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

Then, $2A-3B = 2\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

$$2A-3B = \begin{bmatrix} 4 & 0 \\ -6 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ -6 & 9 \end{bmatrix} = \begin{bmatrix} 4-0 & 0-3 \\ -6+6 & 2-9 \end{bmatrix}$$

$$\boxed{(2A-3B) = \begin{bmatrix} 4 & -3 \\ 0 & -7 \end{bmatrix}}$$

3) $A = \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix}$ and $B = \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$ find $\sin A(A) + \cos A(B)$.

→ Given that, $A = \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix}$ and $B = \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$

Then, $\Rightarrow \sin A(A) + \cos A(B) =$

$$= \sin A \begin{bmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{bmatrix} + \cos A \begin{bmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2 A & -\sin A \cdot \cos A \\ +\sin A \cdot \cos A & \sin^2 A \end{bmatrix} + \begin{bmatrix} \cos^2 A & \sin A \cdot \cos A \\ -\cos A \sin A & \cos^2 A \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2 A + \cos^2 A & -\sin A \cdot \cos A + \sin A \cdot \cos A \\ +\sin A \cdot \cos A - \cos A \cdot \sin A & \sin^2 A + \cos^2 A \end{bmatrix}$$

$$\because \sin^2 A + \cos^2 A = 1$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

4) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$ find $(A+2B-3C)$.

→ Here, $A+2B-3C = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + 2\begin{bmatrix} -2 & -1 \\ 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-4-0 & 2-2-9 \\ -2+2-6 & 3+4+3 \end{bmatrix}$$

$$(A+2B-3C) = \begin{bmatrix} -3 & -9 \\ -6 & 10 \end{bmatrix}$$

5) If $A=\begin{bmatrix}0 & -1\\ 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}$ find matrix X, if

i) $3A+X=B$ ii) $X-3B=2A$

→ Given that, $A=\begin{bmatrix}0 & -1\\ 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}$

i) $3A+X=B$

$$X=B-3A=\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}-3\begin{bmatrix}0 & -1\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}-\begin{bmatrix}0 & -3\\ 3 & 6\end{bmatrix}=\begin{bmatrix}1-0 & 2+3\\ -1-3 & 1-6\end{bmatrix}$$

$$\left\{X=\begin{bmatrix}1 & 5\\ -4 & -5\end{bmatrix}\right\}$$ is the required matrix

ii) $X-3B=2A$

$$X=2A+3B=2\begin{bmatrix}0 & -1\\ 1 & 2\end{bmatrix}+3\begin{bmatrix}1 & 2\\ -1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}0 & -2\\ 2 & 4\end{bmatrix}+\begin{bmatrix}3 & 6\\ -3 & 3\end{bmatrix}=\begin{bmatrix}0+3 & -2+6\\ 2-3 & 4+3\end{bmatrix}$$

$$\left\{X=\begin{bmatrix}3 & 4\\ -1 & 7\end{bmatrix}\right\}$$ is the required matrix

6) Solve the matrix equation $\begin{bmatrix}2 & 1\\ 5 & 0\end{bmatrix}-3X=\begin{bmatrix}-7 & 4\\ 2 & 6\end{bmatrix}$

→ Given that, $\begin{bmatrix}2 & 1\\ 5 & 0\end{bmatrix}-3X=\begin{bmatrix}-7 & 4\\ 2 & 6\end{bmatrix}$

$$\begin{bmatrix}2 & 1\\ 5 & 0\end{bmatrix}-\begin{bmatrix}-7 & 4\\ 2 & 6\end{bmatrix}=3X$$

$$3X=\begin{bmatrix}2+7 & 1-4\\ 5-2 & 0-6\end{bmatrix}=\begin{bmatrix}9 & -3\\ 3 & -6\end{bmatrix}$$

$$X=\begin{bmatrix}3 & -1\\ 1 & -2\end{bmatrix}$$ is the required matrix.

7) If $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$, find the matrix M.

→ Given that, $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix}$$

$$\Rightarrow 2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 9-1 & 6-4 \\ 0+2 & -9-3 \end{bmatrix}$$

$$2M = \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix}$$

$M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix}$ is the required matrix.

8) Find the values of x, y, a and b if $\begin{bmatrix} x-2 & y \\ a+2b & 3a-b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$

→ Given that, $\begin{bmatrix} x-2 & y \\ a+2b & 3a-b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix}$

On comparing corresponding elements of the matrix,

$x-2=3$ $\quad \boxed{y=1}$ $\quad a+2b=5$ —— ①

$\boxed{x=5}$ $\quad 3a-b=1$ —— ②

① ⇒ $a+2b=5$

②×2 ⇒ $6a-2b=2$

$7a=7$

$\boxed{a=1}$ put in ①

$1+2b=5$

$2b=4$

$\boxed{b=2}$

Thus, the required values are $x=5, y=1, a=1, b=2$.

9) Find the values of a, b, c and d if $\begin{bmatrix} a+b & 3 \\ 5+c & ab \end{bmatrix} = \begin{bmatrix} 6 & d \\ -1 & 8 \end{bmatrix}$

→ Given that, $\begin{bmatrix} a+b & 3 \\ 5+c & ab \end{bmatrix} = \begin{bmatrix} 6 & d \\ -1 & 8 \end{bmatrix}$

On comparing corresponding elements of the matrix,

$a+b=6$, $\quad \boxed{d=3}$ $\quad 5+c=-1$

$ab=8$ $\quad \boxed{c=-6}$

We have, $(a-b)^2 = (a+b)^2 - 4ab$

$(a-b)^2 = 36 - 4(8) = 36-32 = 4$

$a-b=\pm 2$

If $(a-b)=-2$ the $a+b=6$

$(a-b)+(a+b)=4$ $\qquad$ $a-b=-2$

$2a=4$ $\qquad$ $2-b=-2$

$\boxed{a=2}$ $\qquad$ $\boxed{b=4}$ are the required values.

10.) Given $A=\begin{bmatrix}2 & -6\\ 2 & 0\end{bmatrix}$ and $B=\begin{bmatrix}-3 & 2\\ 4 & 0\end{bmatrix}$, $C=\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}$

Find the matrix X such that $A+2X=2B+C$.

→ Given that, $A=\begin{bmatrix}2 & -6\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}-3 & 2\\ 4 & 0\end{bmatrix}$, $C=\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}$

Then, $\quad A+2X=2B+C$

$$2X=2B+C-A=\begin{bmatrix}-6 & 4\\ 8 & 0\end{bmatrix}+\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}-\begin{bmatrix}2 & -6\\ 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-6+4-2 & 4+0+6\\ 8+0-2 & 0+2-0\end{bmatrix}$$

$$2X=\begin{bmatrix}-4 & 10\\ 6 & 2\end{bmatrix}$$

$\boxed{X=\begin{bmatrix}-2 & 5\\ 3 & 1\end{bmatrix}}$ is the required matrix.

11.) Find X and Y if $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$.

→ Given that, $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

$$(X+Y)+(X-Y)=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}=\begin{bmatrix}7+3 & 0+0\\ 2+0 & 5+3\end{bmatrix}$$

$$2X=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$$

$\boxed{X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}}$ is the required matrix.

Then, $\quad Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-X=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$

$\boxed{Y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}}$ is the required matrix.

12.) If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find the values of x & y.

→ Given that, $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$$\begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 6+1 & 8+y \\ 10+0 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

On comparing corresponding elements of the matrix,

$8+y=0$ $\quad\quad$ $2x+1=5$

$\boxed{y=-8}$ $\quad\quad$ $2x=4$

$\boxed{x=2}$

The required values of x & y are $x=2$, $y=-8$.

13.) If $\begin{bmatrix} 5 & 2 \\ -1 & y+1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2x-1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$ find the values of x & y.

→ Given that, $\begin{bmatrix} 5 & 2 \\ -1 & y+1 \end{bmatrix} - 2\begin{bmatrix} 1 & (2x-1) \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$

$$\begin{bmatrix} 5 & 2 \\ -1 & y+1 \end{bmatrix} - \begin{bmatrix} 2 & (4x-2) \\ 6 & -4 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5-2 & 2-4x+2 \\ -1-6 & y+1+4 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 4-4x \\ -7 & y+5 \end{bmatrix} = \begin{bmatrix} 3 & -8 \\ -7 & 2 \end{bmatrix}$$

On comparing corresponding elements of the matrix,

$4-4x=-8$

$-4x=-12$ $\quad\quad$ $y+5=2$

$\boxed{x=3}$ $\quad\quad$ $\boxed{y=-3}$

Thus, the required values of x & y are $x=3$, $y=-3$

14.) If $\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$ find a, b & c.

→ Given that, $\begin{bmatrix} a & 3 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$

$$\begin{bmatrix} a+2 & 3+b \\ 4+1 & 2-2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2+a & 3+b \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2+a-1 & 3+b-1 \\ 5+2 & 0-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1+a & 2+b \\ 7 & -c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

On compairing corresponding elements of the matrix,

$1+a=5$ and $-c=3$ $\quad 2+b=0$

$\boxed{a=4}$ $\quad \boxed{c=-3}$ $\quad \boxed{b=-2}$

Thus, the required values of a, b, c are $a=4$, $b=-2$, $c=-3$.

15.) If $A = \begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix}$, $C = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$. and $5A+2B=C$, find the values of a, b, c.

→ $5A+2B=C \Rightarrow 5\begin{bmatrix} 2 & a \\ -3 & 5 \end{bmatrix} + 2\begin{bmatrix} -2 & 3 \\ 7 & b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$

$$\begin{bmatrix} 10 & 5a \\ -15 & 25 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ 14 & 2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 10-4 & 5a+6 \\ -15+14 & 25+2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 5a+6 \\ -1 & 25+2b \end{bmatrix} = \begin{bmatrix} c & 9 \\ -1 & -11 \end{bmatrix}$$

On compairing corresponding elements of the matrix,

$\boxed{c=6}$ $\quad 5a+6=9$ $\quad 25+2b=-11$

$5a=3$ $\quad 2b=-36$

$\boxed{a=3/5}$ $\quad \boxed{b=-18}$

Exercise 8.3

1.) If $A=\begin{bmatrix}3 & 5\\ 4 & -2\end{bmatrix}$ and $B=\begin{bmatrix}2\\ 4\end{bmatrix}$, is the product AB possible? Give a reason, if yes find AB.

→ Here, matrix A is of order (2×2)
and matrix B is of order (2×1)

Thus, AB: no. of columns of A = no. of rows of B

Hence, product AB is possible.

Because, $(2\times2)\times(2\times1)\to(2\times1)$

Thus, $AB=\begin{bmatrix}3 & 5\\ 4 & -2\end{bmatrix}\begin{bmatrix}2\\ 4\end{bmatrix}=\begin{bmatrix}3(2)+5(4)\\ 4(2)+(-2)(4)\end{bmatrix}=\begin{bmatrix}6+20\\ 8-8\end{bmatrix}=\begin{bmatrix}26\\ 0\end{bmatrix}$

$\therefore AB=\begin{bmatrix}26\\ 0\end{bmatrix}_{2\times1}$ is the required matrix.

2.) If $A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$, $B=\begin{bmatrix}1 & -1\\ -3 & 2\end{bmatrix}$, find AB and BA, Is AB=BA?

→ Given that, $A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$, $B=\begin{bmatrix}1 & -1\\ -3 & 2\end{bmatrix}$

$AB=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}\begin{bmatrix}1 & -1\\ -3 & 2\end{bmatrix}=\begin{bmatrix}2-15 & -2+10\\ 1-9 & -1+6\end{bmatrix}=\begin{bmatrix}-13 & 8\\ -8 & 5\end{bmatrix}$

And $BA=\begin{bmatrix}1 & -1\\ -3 & 2\end{bmatrix}\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}=\begin{bmatrix}2-1 & 5-3\\ -6+2 & -15+6\end{bmatrix}=\begin{bmatrix}1 & 2\\ -4 & -9\end{bmatrix}$

Here, $AB\neq BA$

3.) If $A=\begin{bmatrix}3 & 7\\ 2 & 4\end{bmatrix}$, $B=\begin{bmatrix}0 & 2\\ 5 & 3\end{bmatrix}$ and $C=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

find AB−5C.

→ Given that, $A=\begin{bmatrix}3 & 7\\ 2 & 4\end{bmatrix}$, $B=\begin{bmatrix}0 & 2\\ 5 & 3\end{bmatrix}$ and

$C=\begin{bmatrix}1 & -5\\ -4 & 6\end{bmatrix}$

Now, $AB - 5C = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

$= \begin{bmatrix} 0+35 & 6+21 \\ 0+20 & 4+12 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$

$= \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$

$= \begin{bmatrix} 35-5 & 27+25 \\ 20+20 & 16-30 \end{bmatrix}$

$(AB - 5C) = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$ is the required matrix.

4.) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find $A(BA)$.

→ Given that, $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Now, $BA = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

Now, $A(BA) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

$A(BA) = \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix} = \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$ is the required matrix

5.) Given the matrices:

$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$

find the products of i) ABC ii) ACB and state whether they are equal.

→ Given that, $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$

$$ABC = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6-1 & 8-2 \\ 12-2 & 16-4 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 6 \\ 10 & 12 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -15+0 & 5-12 \\ -30+0 & 10-24 \end{bmatrix} = \begin{bmatrix} -15 & -7 \\ -30 & -14 \end{bmatrix}$$

$\boxed{ABC = \begin{bmatrix} -15 & -7 \\ -30 & -14 \end{bmatrix}}$ is the required matrix.

Now, $ACB = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$

$$= \begin{bmatrix} -6+0 & 2-2 \\ -12+0 & 4-4 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$$

$$ACB = \begin{bmatrix} -6 & 0 \\ -12 & 0 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -18 & -24 \\ -36 & -48 \end{bmatrix}$$

$\boxed{ACB = \begin{bmatrix} -18 & -24 \\ -36 & -48 \end{bmatrix}}$ is the required matrix.

And here, $ABC \neq ACB$

6.) Evaluate: $\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$.

→ Given that, $\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 4\times 1/2 & 2\times 1/2 \\ 1 & 2\times 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 8+5 & 10+4 \\ 4+10 & 5+8 \end{bmatrix} = \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

Thus, $\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$.

7) If $A=\begin{bmatrix}-1 & 3\\ 2 & 4\end{bmatrix}$, $B=\begin{bmatrix}2 & -3\\ -4 & -6\end{bmatrix}$, find the matrix $AB+BA$.

$\rightarrow$ Given that, $A=\begin{bmatrix}-1 & 3\\ 2 & 4\end{bmatrix}$, $B=\begin{bmatrix}2 & -3\\ -4 & -6\end{bmatrix}$

Then, $AB=\begin{bmatrix}-1 & 3\\ 2 & 4\end{bmatrix}\begin{bmatrix}2 & -3\\ -4 & -6\end{bmatrix}=\begin{bmatrix}-2-12 & 3-18\\ 4-16 & -6-24\end{bmatrix}$

$$AB=\begin{bmatrix}-14 & -15\\ -12 & -30\end{bmatrix}$$

Now, $BA=\begin{bmatrix}2 & -3\\ -4 & -6\end{bmatrix}\begin{bmatrix}-1 & [illegible]\\ 2 & 4\end{bmatrix}=\begin{bmatrix}-2-6 & [illegible]-12\\ 4-[illegible]2 & [illegible]-24\end{bmatrix}$

$$BA=\begin{bmatrix}-8 & -[illegible]\\ -[illegible] & -3[illegible]\end{bmatrix}$$

Now, $AB+BA=\begin{bmatrix}-14 & -15\\ -12 & -30\end{bmatrix}+\begin{bmatrix}-8 & -6\\ -8 & -36\end{bmatrix}$

$(AB+BA)=\begin{bmatrix}-14-8 & -15-6\\ -12-8 & -30-36\end{bmatrix}=\begin{bmatrix}-22 & -21\\ -20 & -66\end{bmatrix}$

is the required matrix.

8.) If $A=\begin{bmatrix}1 & -2\\ 2 & -1\end{bmatrix}$ and $B=\begin{bmatrix}3 & 2\\ -2 & +1\end{bmatrix}$, find $2B-A^2$.

$\rightarrow$ $A^2=AA=\begin{bmatrix}1 & -2\\ 2 & -1\end{bmatrix}\begin{bmatrix}1 & -2\\ 2 & -1\end{bmatrix}=\begin{bmatrix}1-4 & -2+2\\ 2-2 & -4+1\end{bmatrix}=\begin{bmatrix}-3 & 0\\ 0 & -3\end{bmatrix}$

$(2B-A^2)=2\begin{bmatrix}3 & 2\\ -2 & +1\end{bmatrix}-\begin{bmatrix}-3 & 0\\ 0 & -3\end{bmatrix}$

$=\begin{bmatrix}6 & 4\\ -4 & +2\end{bmatrix}-\begin{bmatrix}-3 & 0\\ 0 & -3\end{bmatrix}=\begin{bmatrix}6+3 & 4-0\\ -4-0 & +2+3\end{bmatrix}$

$\boxed{(2B-A^2)=\begin{bmatrix}9 & 4\\ -4 & 5\end{bmatrix}}$ is the required matrix.

10.) If $A=\begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}$ and $B=\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}$, $C=\begin{bmatrix}1 & 3\\3 & 1\end{bmatrix}$

find the matrix $C(B-A)$.

$\rightarrow$ Given that, $A=\begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}$, $B=\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}$ and $C=\begin{bmatrix}1 & 3\\3 & 1\end{bmatrix}$

$(B-A)=\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}-\begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}=\begin{bmatrix}2-1 & 1-2\\3-2 & 2-3\end{bmatrix}=\begin{bmatrix}1 & -1\\1 & -1\end{bmatrix}$

Now, $C(B-A)=\begin{bmatrix}1 & 3\\3 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\1 & -1\end{bmatrix}$

$C(BA)=\begin{bmatrix}1+3 & -1-3\\3+1 & -3-1\end{bmatrix}=\begin{bmatrix}4 & -4\\4 & -4\end{bmatrix}$

$\boxed{C(B-A)=\begin{bmatrix}4 & -4\\4 & -4\end{bmatrix}}$ is the required matrix.

11.) Let $A=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\-1 & 0\end{bmatrix}$ find (A^2+AB+B^2).

$\rightarrow$ Given that, $A=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\-1 & 0\end{bmatrix}$

$A^2=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}=\begin{bmatrix}1+0 & 0\\2+2 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\4 & 1\end{bmatrix}$

$AB=\begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\-1 & 0\end{bmatrix}=\begin{bmatrix}2 & 3\\4-1 & 6\end{bmatrix}=\begin{bmatrix}2 & 3\\3 & 6\end{bmatrix}$

$B^2=\begin{bmatrix}2 & 3\\-1 & 0\end{bmatrix}\begin{bmatrix}2 & 3\\-1 & 0\end{bmatrix}=\begin{bmatrix}4-3 & 6+0\\-2 & -3\end{bmatrix}=\begin{bmatrix}1 & 6\\-2 & -3\end{bmatrix}$

Now, $(A^2+AB+B^2)=\begin{bmatrix}1 & 0\\4 & 1\end{bmatrix}+\begin{bmatrix}2 & 3\\3 & 6\end{bmatrix}+\begin{bmatrix}1 & 6\\-2 & -3\end{bmatrix}$

$(A^2+AB+B^2)=\begin{bmatrix}1+2+1 & 0+3+6\\4+3-2 & 1+6-3\end{bmatrix}$

$(A^2+AB+B^2)=\begin{bmatrix}4 & 9\\5 & 4\end{bmatrix}$ is the requpred matrix.

13) $A=\begin{bmatrix}2&3\\5&7\end{bmatrix}$, $B=\begin{bmatrix}0&4\\-1&7\end{bmatrix}$ and $C=\begin{bmatrix}1&0\\-1&4\end{bmatrix}$, find $AC+B^2-10C$.

→ $AC=\begin{bmatrix}2&3\\5&7\end{bmatrix}\begin{bmatrix}1&0\\-1&4\end{bmatrix}=\begin{bmatrix}2-3&0+12\\5-7&0+28\end{bmatrix}=\begin{bmatrix}-1&12\\-2&28\end{bmatrix}$

$B^2=\begin{bmatrix}0&4\\-1&7\end{bmatrix}\begin{bmatrix}0&4\\-1&7\end{bmatrix}=\begin{bmatrix}-4&28\\-7&-4+49\end{bmatrix}=\begin{bmatrix}-4&28\\-7&+45\end{bmatrix}$

$(AC+B^2-10C)=\begin{bmatrix}-1&12\\-2&28\end{bmatrix}+\begin{bmatrix}-4&28\\-7&45\end{bmatrix}-\begin{bmatrix}10&0\\-10&40\end{bmatrix}$

$=\begin{bmatrix}-1-4-10&12+28-0\\-2-7+10&28+45-40\end{bmatrix}$

$(AC+B^2-10C)=\begin{bmatrix}-15&40\\1&33\end{bmatrix}$ is the required matrix

15.) If $X=\begin{bmatrix}4&1\\-1&2\end{bmatrix}$, show that $6X-X^2=9I$, where I is the unit matrix.

→ Given that, $X=\begin{bmatrix}4&1\\-1&2\end{bmatrix}$ and $I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

Then $X^2=\begin{bmatrix}4&1\\-1&2\end{bmatrix}\begin{bmatrix}4&1\\-1&2\end{bmatrix}=\begin{bmatrix}16-1&4+2\\-4-2&-1+4\end{bmatrix}=\begin{bmatrix}15&6\\-6&3\end{bmatrix}$

Now, $(6X-X^2)=\begin{bmatrix}24&6\\-6&12\end{bmatrix}-\begin{bmatrix}15&6\\-6&3\end{bmatrix}$

$=\begin{bmatrix}24-15&6-6\\-6+6&12-3\end{bmatrix}$

$=\begin{bmatrix}9&0\\0&9\end{bmatrix}=9\begin{bmatrix}1&0\\0&1\end{bmatrix}$

$\boxed{(6X-X^2)=9I}$ Hence proved.

16.) Show that $\begin{bmatrix}1&2\\2&1\end{bmatrix}$ is a solution of the matrix $X^2-2X-3I=0$, where I is the unit matrix.

→ Given matrix equation is $X^2-2X-3I=0$ — ①

We put $X=\begin{bmatrix}1&2\\2&1\end{bmatrix}$ to check solution.

$$X^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix}$$

$$X^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

Now, $(X^2 - 2X - 3I) = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$$

$$(X^2-2X-3I) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Thus, $\boxed{(X^2-2X-3I)=0}$ Hence proved

18.) If $A = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}$, find the value of x, so that $A^2 = 0$.

→ Given that, $A = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 1 \\ x & x \end{bmatrix} = \begin{bmatrix} 1+x & 1+x \\ x+x^2 & x+x^2 \end{bmatrix}$$

Given that, $A^2 = 0 \Rightarrow \begin{bmatrix} 1+x & 1+x \\ x+x^2 & x+x^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

On comparing corresponding elements of above matrix,

we get $1+x=0$

$\boxed{x=-1}$

19.) Find x and y if $\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$.

→ Given that, $\begin{bmatrix} -3 & 2 \\ 0 & -5 \end{bmatrix}\begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$

$$\begin{bmatrix} -3x+4 \\ 0-10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} -3x+4 \\ -10 \end{bmatrix} = \begin{bmatrix} -5 \\ y \end{bmatrix}$$

On comparing elements of the matrix,

$-3x+4=-5$ $\boxed{y=-10}$

$4+5=3x$

$9=3x$ Thus, the values of x & y are

$\boxed{x=3}$ found to be $\boxed{x=3}$ $\boxed{y=-10}$

21.) If $\begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$, find the values of x & y.

→ Given that, $\begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$

$$\begin{bmatrix} x+0 & 0+2y \\ 3x+0 & 0+3y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x & 2y \\ 3x & 3y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$$

On compairing corresponding elements of the matrix,

$3x = 9$ $\qquad$ $2y = 0$

$\boxed{x = 3}$ $\qquad$ $\boxed{y = 0}$

Thus, the values of x & y are found to be $x = 3$, $y = 0$.

22.) If $\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, write down the values of a, b, c and d.

→ Given that, $\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} a+0 & b \\ c & d \end{bmatrix}$$

On comparing corresponding elements of the matrix,

$\boxed{a = 3}$, $\boxed{b = 4}$, $\boxed{c = 2}$, $\boxed{d = 5}$

24.) If $A = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$, find the value of x, given that $(A^2 = B)$.

→ $A^2 = \begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2x+x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix}$

Given that, $A^2 = B \Rightarrow \begin{bmatrix} 4 & 3x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix}$

On compairing corresponding elements of the matrix,

$3x = 36$

$\boxed{x = 12}$ is the required value.

25) If $A=\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$, find x & y when $A^2=B$.

→ $A^2=\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 9 & 3x+x \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix}$

Given that $A^2=B$

$\Rightarrow \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$

On compairing corresponding matrix elements,

$4x=16$ $\boxed{y=-1}$

$\boxed{x=4}$

Thus, the required values of x & y are 4 & -1 respectively.

26) find x, y if $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix}+3\begin{bmatrix} -2 \\ 1 \end{bmatrix}=2\begin{bmatrix} y \\ 3 \end{bmatrix}$

→ Given that, $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 2x \end{bmatrix}+3\begin{bmatrix} -2 \\ 1 \end{bmatrix}=2\begin{bmatrix} y \\ 3 \end{bmatrix}$

$\begin{bmatrix} -2+0 \\ -3+2x \end{bmatrix}+\begin{bmatrix} -6 \\ 3 \end{bmatrix}=\begin{bmatrix} 2y \\ 6 \end{bmatrix}$

$\begin{bmatrix} -2 \\ -3+2x \end{bmatrix}+\begin{bmatrix} -6 \\ 3 \end{bmatrix}=\begin{bmatrix} 2y \\ 6 \end{bmatrix}$

$\begin{bmatrix} -8 \\ 2x \end{bmatrix}=\begin{bmatrix} 2y \\ 6 \end{bmatrix}$

On compairing corresponding elements of the matrix,

$2y=-8$ and $2x=6$

$\boxed{y=-4}$ $\boxed{x=3}$

Thus, the requlred values of x & y are 3 & -4 respectively

27) If $\begin{bmatrix} a & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}=\begin{bmatrix} b & 11 \\ 4 & c \end{bmatrix}$, find a, b and c

→ Given that, $\begin{bmatrix} a & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}=\begin{bmatrix} b & 11 \\ 4 & c \end{bmatrix}$

$\begin{bmatrix} 4a-3 & 3a+2 \\ 4 & 3 \end{bmatrix}=\begin{bmatrix} b & 11 \\ 4 & c \end{bmatrix}$

$$\begin{bmatrix} 4a-3 & 3a+2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} b & 11 \\ 4 & c \end{bmatrix}$$

On compairing corresponding elements of the matrix.

$4a-3=b$

$4a=3+b$

$b=12-3$

$\boxed{b=9}$

$\boxed{c=3}$

$3a+2=11$

$3a=9$

$\boxed{a=3}$

Thus, the required values are found to be $a=3$, $b=9$, $c=3$.

28.) If $A=\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, find x & y so that $A^2-xA+yI=0$

→

$$A^2=\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}=\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$(A^2-xA+yI)=\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}-x\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}+\begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix}$$

$$(A^2-xA+yI)=\begin{bmatrix} 7-2x+y & 12-3x \\ 4-x & 7-2x+y \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

on compairing corresponding elements of the matrix,

$12-3x=0$

$3x=12$

$\boxed{x=4}$

$7-2x+y=0$

$7-8+y=0$

$y-1=0$

$\boxed{y=1}$

Thus, the required values of x & y are found to be 4 & 1 respectively.

29.) If $P=\begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$, $Q=\begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix}$, find x & y such that $PQ=0$.

→

$$PQ=\begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}\begin{bmatrix} 3 & x \\ y & 2 \end{bmatrix}=\begin{bmatrix} 6+6y & 2x+12 \\ 9+9y & 3x+18 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

on compairing corresponding elements,

$2x+12=0$

$2x=-12$

$\boxed{x=-6}$

$6+6y=0$

$6y=-6$

$\boxed{y=-1}$

31.) Given, $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ i) the order of the matrix ii) the matrix X.

→ Given $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

Let, $AX = B$

$A_{(2\times2)} X = B_{(2\times1)}$

i) Thus, the order of matrix should be (2×1).

ii) Let us consider matrix 'X' be $X = \begin{bmatrix} x \\ y \end{bmatrix}$.

Then, $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$

$\begin{bmatrix} 2x+y \\ -3x+4y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ On compairing corresponding elements of the matrix,

$2x+y=7$ —— ①

$-3x+4y=6$ —— ②

① × 4 ⇒ $8x+4y=28$

② ⇒ $-3x+4y=6$

$\quad\quad + \quad - \quad -$

$11x = 22$

$\boxed{x=2}$ put in ①

⇒ $2(2)+y=7$

$4+y=7$

$\boxed{y=3}$

Thus, the required matrix $X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

32.) Solve the matrix equation, $\begin{bmatrix} 4 \\ 1 \end{bmatrix} X = \begin{bmatrix} -4 & 8 \\ -1 & 2 \end{bmatrix}$.

→ Given that, $\begin{bmatrix} 4 \\ 1 \end{bmatrix} X = \begin{bmatrix} -4 & 8 \\ -1 & 2 \end{bmatrix}$

Let, $AX = B$

$A_{(2\times1)} X = B_{(2\times2)}$

Hence, the order of matrix 'X' should be (1×2).

Let 'X' be the matrix $X = [x \; y]$

Then, $\begin{bmatrix} 4 \\ 1 \end{bmatrix}[x \; y] = \begin{bmatrix} -4 & 8 \\ -1 & 2 \end{bmatrix}$

$\begin{bmatrix} 4x & 4y \\ x & y \end{bmatrix} = \begin{bmatrix} -4 & 8 \\ -1 & 2 \end{bmatrix}$

On compairing corresponding elements of the matrix,

$4y = 8$ and $4x = -4$

$\boxed{y = 2}$ $\boxed{x = -1}$

34) If $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$, find matrix B such that $BA = I$, where I is unity matrix of order 2.

→ Given that, $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$ and $BA = I$

$$B \times A_{(2\times2)} = I_{(2\times2)}$$

Hence, matrix B should be of order (2×2).

Let us consider, $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$BA = I$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3a-b & -4a+2b \\ 3c-d & -4c+2d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing corresponding elements of the matrix,

$3a - b = 1$ — ①

$-4a + 2b = 0$

$-4a = -2b$

$\boxed{2a = b}$ put in ①

$3a - 2a = 1$

$\boxed{a = 1}$ put in ①

$3 - b = 1$

$\boxed{2 = b}$

$3c - d = 0$ $-4c + 2d = 1$ — ②

$\boxed{3c = d}$ put in ②

$-4c + 6c = 1$

$+2c = 1$

$\boxed{c = 1/2}$ put in ②

$-2 + 2d = 1$

$2d = 3$

$\boxed{d = 3/2}$

Thus, the required matrix is found to be

$$B = \begin{bmatrix} 1 & 2 \\ 1/2 & 3/2 \end{bmatrix}$$