Chapter 12.
Equation of Straight Line

Exercise 12.1

1) Find the slope of the line whose inclination is

i) 45°

→ Given that, $\theta = 45^\circ$

Then,

Slope $= \tan 45^\circ = 1$

ii) 30°

→ Given that, $\theta = 30^\circ$

Then,

Slope $= \tan 30^\circ = 1/\sqrt{3}$.

2.) Find the inclination of a line whose gradient is

i) 1

→ Given that,

$\tan\theta = 1$

$\Rightarrow \theta = \tan^{-1}(1)$

$\boxed{\theta = 45^\circ}$

ii) $\sqrt{3}$

→ Given that,

$\tan\theta = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3})$

$\boxed{\theta = 60^\circ}$

iii) $1/\sqrt{3}$

→ Given that,

$\tan\theta = 1/\sqrt{3}$

$\theta = \tan^{-1}(1/\sqrt{3})$

$\boxed{\theta = 30^\circ}$

3.) Find the equation of a straight line parallel to X-axis which is at a distance

i) 2 units above it

→ We have,

Equation of a line parallel to X-axis $y = a$

$\Rightarrow y = 2$

$\boxed{y - 2 = 0}$

is the required equation of a straight line.

ii) 3 units below it

→ We have,

Equation of a line parallel to X-axis is $y = a$

$\Rightarrow y = -3$

$\boxed{y + 3 = 0}$

is the required equation of a straight line.

4) find the equation of a straight line parallel to Y-axis which is at a distance of

i) 3 units to the right

→ We have,

The equation of a line parallel to Y-axis is $x = a$

$\Rightarrow \quad x = 3$

$\Rightarrow \boxed{x - 3 = 0}$

is the required equation of a straight line.

ii) 2 units to the left

→ We have,

The equation of a line parallel to Y-axis is $x = a$

$\Rightarrow \quad x = -2$

$\Rightarrow \boxed{x + 2 = 0}$

is the required equation of a straight line.

5) find the equation of a straight line parallel to Y-axis and passing through the point (-3,5).

→ We have,

The equation of a straight line parallel to Y-axis is $x = a$.

The line is passing through the point (-3,5).

But, here the line is passing through the point (-3,5).

Hence, the required equation of a straight line parallel to Y-axis and passing through the point (-3,5) is found to be

$x = -3 \Rightarrow \boxed{x + 3 = 0}$

6) find the equation of a line whose

i) slope = 3, y-intercept = -5

→ Given that, slope (m) = 3 and y-intercept (c) = -5

We have, equation of a line in slope-intercept form is

$y = mx + c$ m - slope

y-intercept → c

$y = 3x - 5$

$\boxed{3x - y - 5 = 0}$ is the required equation of a line.

ii) slope $= -\frac{2}{7}$, y-intercept $= 3$

→ Given that,

slope $(m) = -2/7$ and

y-intercept$(c) = 3$

We have, equation of a straight line in slope-intercept form is

$$y = mx + c$$

$$y = -\frac{2}{7}x + 3$$

$$7y = -2x + 21$$

$$\boxed{2x + 7y - 21 = 0}$$

is the required equation of a line.

iii) gradiant $= \sqrt{3}$,

y-intercept $= 3$

→ Given that,

slope = gradient $(m) = \sqrt{3}$

y-intercept $(c) = 3$

We have,

The equation of a straight line in slope-intercept form is

$$y = mx + c$$

$$y = \sqrt{3}x + 3$$

$$\boxed{\sqrt{3}x - y + 3 = 0}$$

is the required equation of a line.

7) find the slope & y-intercept of the following lines:

i) $x - 2y - 1 = 0$

→ Given equation of line is

$$x - 2y - 1 = 0$$

$$x = 2y + 1$$

$$2y = x - 1$$

$$y = \frac{1}{2}x - \frac{1}{2} \quad \text{——} ①$$

On compairing equⁿ① with $y = mx + c$

$\Rightarrow \boxed{m = 1/2}$ and

$\boxed{c = -1/2}$

Thus, here

slope $(m) = 1/2$ &

y-intercept $(c) = -1/2$

ii) $4x - 5y - 9 = 0$

→ Given equation of a line is

$$4x - 5y - 9 = 0$$

$$4x = 5y + 9$$

$$5y = 4x - 9$$

$$y = \frac{4}{5}x - 9/5 \quad \text{——} ①$$

On compairing equⁿ① with $y = mx + c$

$\Rightarrow \boxed{m = 4/5}$ and $\boxed{c = -9/5}$

Thus, here

slope $(m) = 4/5$ &

y-intercept $(c) = -9/5$

iii) $3x+5y+7=0$

$\rightarrow$ Given equation of line is

$3x+5y+7=0$

$3x=-5y-7$

$5y=-3x-7$

$y=-\frac{3}{5}x-\frac{7}{5}$ —— ①

On compairing equⁿ① with

$y=mx+c$

$\Rightarrow \boxed{m=-3/5}$ & $\boxed{c=-7/5}$

Thus, here

slope $(m)=-3/5$

Y intercept $(c)=-7/5$

iv) $\frac{x}{3}+\frac{y}{4}=1$

$\rightarrow$ Given equation of a line is

$\frac{x}{3}+\frac{y}{4}=1$

$\frac{4x+3y}{12}=1$

$4x+3y=12$

$3y=12-4x$

$y=-\frac{4}{3}x+4$ —— ①

On comparing equⁿ① with

$y=mx+c$

$\Rightarrow \boxed{m=-4/3}$ & $\boxed{c=4}$

Thus, here

slope $(m)=-4/3$ &

Y-intercept $(c)=4$

8.) The equation of the line PQ is $3y-3x+7=0$

i) Writedown the slope of the line PQ.

ii) Calculate the angle that the line PQ makes with the positive direction of X-axis.

$\rightarrow$ Given that, the equation of the line PQ is $3y-3x+7=0$

i.e. $3y=3x-7$

$y=x-7/3$ —— ①

On compairing equⁿ① with $y=mx+c$

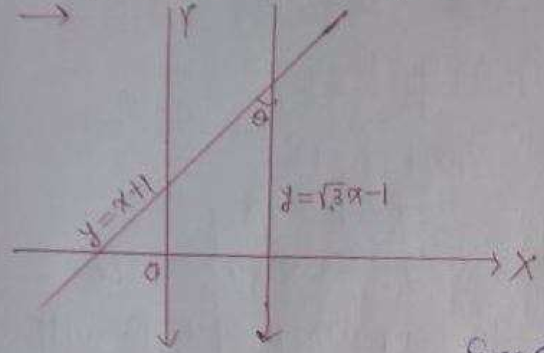
i) we get slope of the line PQ $(m)=1$

ii) And $\tan\theta=1$

$\theta=\tan^{-1}(1)$

$\boxed{\theta=45^\circ}$

9) The given fig. represents the line $y = x+1$ and $y = \sqrt{3}x - 1$. Write down the angles which the lines make with the positive direction of the X-axis. Hence, determine θ.



→ Given equations of lines are

$y = x+1$ —— ① and $y = \sqrt{3}x - 1$ —— ②

On compairing equ^n ① & ② with

$y = mx + c$

we get,

for ①: slope $(m) = 1$, y-intercept $(c) = 1$

for ②: slope $(m) = \sqrt{3}$, y-intercept $(c) = -1$

Here, $\tan\theta = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3}) = 60°$ $\boxed{\theta = 60°}$

In fig, the triangle is formed by given two lines & X-axis.

Thus, exterior angle = sum of interior opposite angles

$60° = \theta + 45°$

$\theta = 60° - 45°$

$\boxed{\theta = 15°}$ is the required angle.

10) Find the value of p, given that line $y/2 = x - p$ passes through the point $(-4, 4)$.

→ Given that, the line $\frac{y}{2} = x - p$ passes through the point $(-4, 4)$.

i.e. $\frac{y}{2} = x - p$ —— ①

put $x = -4$ and $y = 4$ in equ^n ① →

$\frac{4}{2} = -4 - p$

$2 = -4 - p$

$p = -4 - 2$

$\boxed{p = -6}$ is the required value.

11) Given that $(a, 2a)$ lies on the line $\frac{y}{2} = 3x - 6$ find a.

→ Given that,

The point $(a, 2a)$ lies on the line $\frac{y}{2} = 3x - 6$ —— ①

Then, put $x = a$ and $y = 2a$ in eqn ① ⇒

$$\frac{2a}{2} = 3a - 6$$

$$a = 3a - 6$$

$$6 = 2a \Rightarrow a = 6/2 = 6/2 \Rightarrow \boxed{a = 3}$$ is the required value of a

12) The graph of the equation $y = mx + c$ passes through the points $(1, 4)$ and $(-2, -5)$. Determine the values of m and c.

→ Given equation of line $y = mx + c$ passes through the points $(1, 4)$ and $(-2, -5)$.

put ~~(1,~~ $x = 1$ & $y = 4$ in $y = mx + c$ ⇒

$$\boxed{4 = m + c} —— ①$$

put $x = -2$ and $y = -5$ in $y = mx + c$ ⇒

$$\boxed{-5 = -2m + c} —— ②$$

① × 2 ⇒ $8 = 2m + 2c$

② ⇒ $-5 = -2m + c$

$3 = 3c$

$\boxed{c = 1}$ put in ① ⇒

$4 = m + c$

$4 = m + 1$

$\boxed{m = 3}$

Thus, the values of m and c are found to be $m = 3$ & $c = 1$.

13) Find the equation of the line passing through the point $(2, -5)$ and making an intercept of -3 on the y-axis.

→ Given that,

A line passes through the point $(2, -5)$ and makes an intercept of -3 on the y-axis.

We have, the equation of line in slope-intercept form is given by, $y = mx + c \Rightarrow y = mx - 3$ — ①

and put $x = 2$ & $y = -5$ in $y = mx + c$ $\quad \because \boxed{c = -3}$

$$-5 = 2m + c$$

$$2m = -5 - c = -5 + 3 \qquad \because \boxed{c = -3}$$

$$2m = -2$$

$$\boxed{m = -1}$$

Thus, the required equation of line is found to be

$y = -x - 3$ i.e. $x + y + 3 = 0$

14) Find the equation of a straight line passing through $(-1, 2)$ and whose slope is $2/5$.

→ Given that,

A line passes through the point $(-1, 2)$ & having slope $2/5$.

$\Rightarrow x_1 = -1$, $y_1 = 2$ and $m = 2/5$

We have, the equation of a line in slope-point form as

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 2/5(x + 1)$$

$$5y - 10 = 2x + 2$$

$2x - 5y + 12 = 0$ is the required equation of line.

15) Find the equation of a straight line whose inclination is $60°$ and which passes through the point $(0, -3)$.

→ Given that, a line is having inclination of $60°$ & passes through the point $(0, -3)$.

$\Rightarrow x_1 = 0$, $y_1 = -3$ and $\theta = 60°$

$\Rightarrow$ slope$(m) = \tan\theta = \tan 60° = \sqrt{3}$

We have, equation of a line in slope-point form is found to be

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \sqrt{3}(x - 0)$$

$\sqrt{3}x - y - 3 = 0$ is the required equation of line.

16) Find the gradient of a line passing through the following pairs of points.

i) $(0,-2), (3,4)$

→ Here $(x_1, y_1) \equiv (0,-2)$

and $(x_2, y_2) \equiv (3,4)$

Then slope/gradient of a line is given by

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{4+2}{3-0} = \frac{6}{3}$$

$\boxed{m=2}$

ii) $(3,-7), (-1,8)$

→ Here, $(x_1,y_1) \equiv (3,-7)$

and $(x_2,y_2) \equiv (-1,8)$

Then slope/gradient of a line is given by

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{8+7}{-1-3} = \frac{15}{-4}$$

$\boxed{m=-15/4}$

17) The coordinates of two points E and F are $(0,4)$ and $(3,7)$ respectively. Find i) the gradient of EF

ii) the equation of EF

iii) the coordinates of the point where the line EF intersects the X-axis.

→ Given that, the co-ordinates of two points E & F are $(0,4)$ & $(3,7)$ respectively.

i) Gradient of EF is given by $m = \frac{y_2-y_1}{x_2-x_1} = \frac{7-4}{3-0} = \frac{3}{3} = 1$

$\boxed{m=1}$

ii) We have, equation of a line in slope-point form is given by

$$y-y_1 = m(x-x_1)$$

$\Rightarrow y-7 = 1(x-3)$

$y-7 = x-3$ $\Rightarrow$ $\boxed{x-y+4=0}$ is the required equation of a line

iii) If the line $x-y+4=0$ intersects X-axis then $\boxed{y=0}$

put $y=0$ in $x-y+4=0$

$x+4=0$

$\boxed{x=-4}$

Thus, the coordinates of the point where the line EF intersects X-axis is found to be $x=-4$ & $y=0$

18.) Find the intercept made by the line $2x-3y+12=0$ on the coordinate axis.

→ To get intercept made on X-axis put $y=0$ in $2x-3y+12=0$

$\Rightarrow 2x+12=0$

$2x=-12$

$\boxed{x=-6}$

To get intercept made on Y-axis put $x=0$ in $2x-3y+12=0$

$\Rightarrow 0-3y+12=0$

$3y=12$

$\boxed{y=4}$

19.) Find the equation of the line passing through the points $P(5,1)$ and $Q(1,-1)$. Hence, show that the points P, Q & $R(11,4)$ are collinear.

→ Given points are $P(5,1)$, $Q(1,-1)$, $R(11,4)$

To find equation of a line passing through points P & Q:

We have, $\text{Slope}(m) = \frac{y_2-y_1}{x_2-x_1} = \frac{-1-1}{1-5} = \frac{-2}{-4} = 1/2$

$\boxed{m=1/2}$

Now, equation of a line passing through points P & Q and having slope 1/2 is given by,

$y-y_1 = m(x-x_1)$

$y-1 = 1/2(x-5)$

$2y-2 = x-5$

$\boxed{x-2y-3=0}$ is the required equation.

To show points P, Q & R are collinear:

put $x=11$ & $y=4$ in $x-2y-3=0$

$11-8-3=0$

$11-11=0$

$\boxed{0=0}$

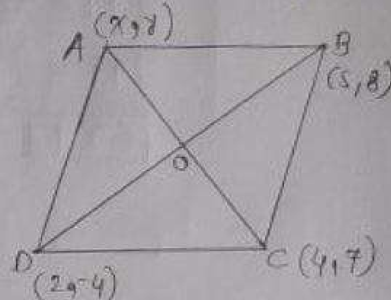
Thus, the points $P(5,1)$, $Q(1,-1)$ & $R(11,4)$ are collinear.

Hence proved.

22) ABCD is a parallelogram where $A(x,y)$, $B(5,8)$, $C(4,7)$ and $D(2,-4)$. find i) co-ordinates of A
ii) the equation of the diagonal BD.

→ Given that,
ABCD is a parallelogram where $A(x,y)$, $B(5,8)$, $C(4,7)$ & $D(2,-4)$.

i) let us consider 'O' is the point of intersection of diagonals of the parallelogram ABCD.



$$\Rightarrow \quad (x,y) = \left(\frac{5+2}{2}, \frac{8-4}{2}\right) = (3\cdot5, 2)$$

For line AC:

$$3\cdot5 = \frac{(x+4)}{2} \quad \& \quad 2 = \frac{(y+7)}{2}$$

$$7 = x+4 \quad \& \quad 4 = y+7$$

$$\boxed{x=3} \quad \& \quad \boxed{y=-3}$$

Thus, the co-ordinates of point A are found to be $A(3,-3)$.

ii) The slope of a line BD is given by

$$\text{slope}(m) = \frac{-4-8}{2-5} = \frac{-12}{-3} = 4 \quad \Rightarrow \boxed{m=4}$$

We have, equation of a line BD in slope-point form is

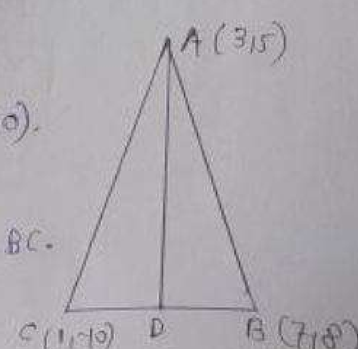
$$y - y_1 = m(x - x_1)$$

$$y - 8 = 4(x-5)$$

$$4x - y - 20 + 8 = 0$$

$\boxed{4x - y - 12 = 0}$ is the required equation of the diagonal BD.

23) In ΔABC, $A(3,5)$, $B(7,8)$ and $C(1,-10)$. Find the equation of the median through A.



→ Given that,
In ΔABC, $A(3,5)$, $B(7,8)$ and $C(1,-10)$.

In fig. AD is the median.

⇒ point D is the mid-point of side BC.

Then, By mid-point formula,

$D(x, y) \equiv \left(\frac{7+1}{2}, \frac{8-10}{2}\right) \equiv (4, -1)$

Now, slope of the median AD is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5+1}{3-4} = \frac{6}{-1} = -6 \Rightarrow \boxed{m = -6}$$

Now, equation of a line AD in slope-point form is given by,

$$y - y_1 = m(x - x_1)$$

$$y + 1 = -6(x - 4)$$

$$y + 1 = -6x + 24$$

$\boxed{6x + y - 23 = 0}$ is the required equation of median through point A.

24) Find the equation of a line passing through the point $(-2, 3)$ and having x-intercept 4 units.

→ Given that,

A line passes through the point $(-2, 3)$ & having x-intercept 4 units.

⇒ The coordinates of the point be $(4, 0)$.

Now, slope of the line passing through two points $(-2, 3)$ and $(4, 0)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{4+2} = \frac{-3}{6} = \frac{-1}{2} \Rightarrow \boxed{m = -1/2}$$

Now, equation of a line in slope-point form is given by

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -1/2\,(x - 4)$$

$$2y = -x + 4$$

$\boxed{x + 2y = 4}$ is the required equation of a line.

25) Find the equation of the line whose x-intercept is 6 & y-intercept is -4.

$\longrightarrow$ Given that,

The x-intercept of a line = 6 and

The y-intercept of a line = -4. $\Rightarrow \boxed{c=-4}$

Thus, the required line passes through the points (6,0) & (0,-4).

$$\text{slope } (m) = \frac{y_2-y_1}{x_2-x_1} = \frac{-4-0}{0-6} = \frac{-4}{-6} = \frac{2}{3} \Rightarrow \boxed{m = 2/3}$$

Now, equation of a line in slope-intercept form is given by

$$y = mx + c$$

$$y = \frac{2}{3}x - 4$$

$$3y = 2x - 12$$

$\boxed{2x-3y-12=0}$ is the required equation of a line.

27) Find the equation of the line passing through point (1,4) & intersecting the line $x-2y-11=0$ on the y-axis.

$\longrightarrow$ Given that,

A line passes through the point (1,4) & intersects the line $x-2y-11=0$ on the y-axis.

$\Rightarrow \boxed{x=0}$ put in $x-2y-11=0$

$\Rightarrow \boxed{y=-11/2}$

Now, slope of the line passing through points (1,4) & (0,-11/2) is

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{-11/2-4}{0-1} = 19/2 \Rightarrow \boxed{m = 19/2}$$

Thus, the equation of a line in slope-point form is given by,

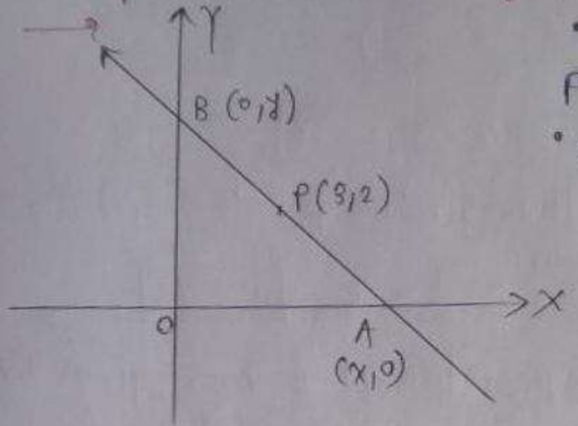
$$y - y_1 = m(x - x_1)$$

$$y + 11/2 = 19/2\,(x-0)$$

$$2y + 11 = 19x$$

$\boxed{19x-2y-11=0}$ is the required equation of a line.

28) Find the equation of the straight line containing the point (3,2) & making positive equal intercepts on axes.



- Let us consider the required line passes through the point P(3,2).
- Also, it passes through X-axis at point A(x,0) and Y-axis at point B(0,y).

From given condition & fig.,

$OA = OB$

$\Rightarrow \boxed{x = y}$

The slope of the line is given by,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - y}{x - 0} = \frac{-x}{x} = -1 \Rightarrow \boxed{m = -1}$$

Thus, equation of a line in slope-point form is given by,

$y - y_1 = m(x - x_1)$

$y - 2 = -1(x - 3)$

$y - 2 = -x + 3$

$\boxed{x + y - 5 = 0}$ is the required equation of line.

30) A and B are two points on the X-axis and Y-axis respectively. P(2,-3) is the mid-point of AB. Find

i) the coordinates of A and B

ii) the slope of line AB

iii) the equation of line AB

→ Given that,

A and B are two points on the X-axis & Y-axis respectively.

& P(2,-3) is the mid-point of AB.

i) Let us consider the coordinates be A(x,0) & B(0,y).

$\Rightarrow \quad 2 = \frac{(x+0)}{2} \quad \& \quad -3 = \frac{(0+y)}{2}$

$\boxed{x = 4} \quad \& \quad \boxed{y = -6}$

ii) The slope of line AB is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-0}{0-4} = \frac{-6}{-4} = \frac{3}{2} \Rightarrow \boxed{m = 3/2}$$

iii) Now, equation of line AB in slope-point form is given by,

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = \frac{3}{2}(x-2)$$
$$y + 3 = \frac{3}{2}(x-2)$$
$$2y + 6 = 3x - 6$$
$$\boxed{3x - 2y - 12 = 0}$$ is the required equation of line.

32) The line through point $P(5,3)$ intersects y-axis at Q.

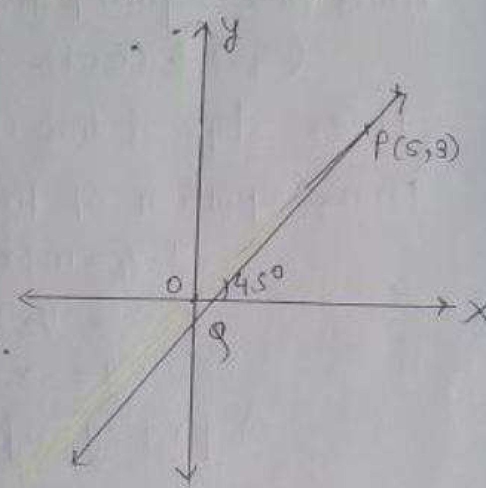
i) Write the slope of the line.

ii) Write the equation of the line.

iii) Find the co-ordinates of Q.

→ Given that,

The line passes through point $P(5,3)$ & intersects y-axis at point Q as shown.



i) slope of line $(m) = \tan 45° = 1$

$$\boxed{m = 1}$$

ii) Equation of line PQ in slope-point form is given by

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 1(x-5)$$
$$y = x - 2 \quad \text{or} \quad \boxed{x - y - 2 = 0}$$ is the required equation of line.

iii) put $x = 0$ in equation $x - y - 2 = 0$

$$\Rightarrow \boxed{y = -2}$$

Thus, co-ordinates of point Q are found to be $Q(0,-2)$.

35) find the equation of a straight line passing through the origin & through the point of intersection of the lines $5x+7y=3$ and $2x-3y=7$.

→ Given that,

A line passes through the origin & through the point of intersection of the lines $5x+7y=3$ & $2x-3y=7$.

$$5x+7y=3 \quad \text{——} ①$$
$$2x-3y=7 \quad \text{——} ②$$

① × 3 ⇒ $15x+21y=9$

② × 7 ⇒ $14x-21y=49$

$$29x=58$$

$\boxed{x=2}$ put in ①

⇒ $5(2)+7y=3$

$10+7y=3$

$7y=-7$

$\boxed{y=-1}$

Thus, the required line passes through the points $(0,0)$ & $(2,-1)$.

Then, slope of line $(m) = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{(0+1)}{(0-2)} = -1/2 \Rightarrow \boxed{m=-1/2}$

Thus, equation of line is given by,

$$y-y_1=m(x-x_1)$$
$$y-0=-1/2\,(x-0)$$
$$2y=-x$$

⇒ $\boxed{x+2y=0}$ is the required equation of line.

Exercise 12.2

1.) State which one of the following is true:

The straight lines $y = 3x - 5$ and $2y = 4x + 7$ are

i) parallel ii) perpendicular iii) neither parallel nor perpendicular

→ Given equation of lines are $y = 3x - 5$ — ① and

$2y = 4x + 7$ — ②

Slope of line ① ⇒ $\boxed{m_1 = 3}$

slope of line ② ⇒ $\boxed{m_2 = 2}$

Here, $m_1 \neq m_2$ and $m_1 m_2 \neq -1$

Hence, the required lines are neither parallel nor perpendicular.

2.) If $6x + 5y - 7 = 0$ and $2px + 5y + 1 = 0$ are parallel lines, find the value of p.

→ Given lines are $6x + 5y - 7 = 0$ — ①

i.e. $5y = -6x + 7$

$y = -6/5\,x + 7/5$ i.e. $y = mx + c$

Here, the slope of line ① is found to be $\boxed{m_1 = -6/5}$

Now, Given equation of line are $2px + 5y + 1 = 0$ — ②

i.e. $5y = -2px - 1$

$y = -\frac{2}{5}px - \frac{1}{5}$ i.e. $y = mx + c$

Here, the slope of line ② is found to be $\boxed{m_2 = -\frac{2}{5}p}$

But, given lines ① & ② are parallel.

⇒ $m_1 = m_2$

$-\frac{6}{5} = -\frac{2}{5}p$

⇒ $\frac{6}{2} = p$

$\boxed{p = 3}$ is the required value of p.

3) Lines $2x-by+5=0$ and $ax+3y=2$ are parallel. find the relation connecting a and b.

$\longrightarrow$ Given equation of lines are $2x-by+5=0$ —— ①

and $ax+3y-2=0$ —— ②

① $\Rightarrow$ $-by=-2x-5$

$y=\frac{2}{b}x+\frac{5}{b}$ i.e. $y=mx+c$

Here, slope of line ① is found to be $\boxed{m_1=2/b}$

② $\Rightarrow$ $3y=-ax+2$

$y=-\frac{a}{3}x+\frac{2}{3}$ i.e. $y=mx+c$

Here, slope of line ② is found to be $\boxed{m_2=-a/3}$

But, given that, the lines ① & ② are parallel.

$\Rightarrow m_1=m_2$

$\frac{2}{b}=-a/3$

$\Rightarrow 6=-ab$

$\boxed{ab=-6}$ is the required relation between a and b.

4) If the straight lines $3x-5y=7$ and $4x+ay+9=0$ are perpendicular to one another, find the value of a.

$\longrightarrow$ Given equations of line are

$3x-5y=7$ —— ①

and $4x+ay+9=0$ —— ②

① $\Rightarrow$ $5y=3x-7$

$y=\frac{3}{5}x-\frac{7}{5}$ i.e. $y=mx+c$

Here, the slope of line ① is found to be $\boxed{m_1=3/5}$

② $\Rightarrow$ $4x+ay+9=0$

$ay=-4x-9$

$y=-\frac{4}{a}x-\frac{9}{a}$ i.e. $y=mx+c$

Here, the slope of line ② is found to be $\boxed{m_2=-4/a}$

Given that, lines ① & ② are perpendicular.

$m_1 m_2 = -1$

$(3/5)(-4/a) = -1$

$\frac{12}{5a} = 1$

$5a = 12 \Rightarrow \boxed{a = 12/5}$ is the required value of a.

5) If the lines $3x+by+5=0$ and $ax-5y+7=0$ are perpendicular to each other, find the relation betn a and b.

→ Given equation of lines are $3x+by+5=0$ —— ①

and $ax-5y+7=0$ —— ②

① ⇒ $by = -3x-5$

$y = -3/b\,x - 5/b$ i.e. $y = mx+c$

Here, slope of line ① is found to be $\boxed{m_1 = -3/b}$

② ⇒ $5y = ax+7$

$y = \frac{a}{5}x + 7/5$ i.e $y = mx+c$

Here, slope of line ② is found to be $\boxed{m_2 = a/5}$

Given that, the lines ① & ② are perpendicular to each other.

⇒ $m_1 m_2 = -1$

$\left(\frac{-3}{b}\right)\left(\frac{a}{5}\right) = -1$

$\frac{-3a}{5b} = -1$

⇒ $\boxed{3a = 5b}$ is the required relation betn a and b.

6) Is the line through (-2,3) and (4,1) perpendicular to the line $3x = y+1$? Does the line $3x = y+1$ bisect the join of (-2,3) & (4,1).

→ Given line is passing through the points (-2,3) & (4,1).

Then slope of a line $(m_1) = \frac{y_2-y_1}{x_2-x_1} = \frac{1-3}{4+2} = \frac{-2}{6} = -1/3$ $\boxed{m_1 = -1/3}$

And slope of the line $3x = y+1$

i.e. $y = 3x-1$

slope $(m_2) = 3$ $\boxed{m_2 = 3}$

Here, $\boxed{m_1 m_2 = -1}$

Thus, the given lines are perpendicular to each other.

Now, co-ordinates of the mid-point of the line joining the points $(-2,3)$ and $(4,1)$ is found to be

$$\left(\frac{-2+4}{2}, \frac{3+1}{2}\right) \equiv (1,2)$$

Put $x=1$ & $y=2$ in equation $3x = y+1$

$$\Rightarrow \quad 3(1) = 2+1$$
$$3 = 3$$

Thus, the line $3x = y+1$ bisects the line which joins the points $(-2,3)$ & $(4,1)$.

8) The line through $A(-2,3)$ and $B(4,b)$ is perpendicular to the line $2x-4y=5$. Find the value of b.

→ Given line is passing through two points $A(-2,3)$ & $B(4,b)$.

Then slope of a line $(m_1) = \frac{y_2-y_1}{x_2-x_1} = \frac{b-3}{4+2} = \frac{b-3}{6}$

And the line with equation $2x-4y=5$ is perpendicular to above line through points A and B.

Here, $4y = 2x-5$

$y = \frac{1}{2}x - 5/4$ i.e. $y = mx + c$

$$\Rightarrow \boxed{m_2 = 1/2}$$

Since, given two lines are perpendicular to each other.

$$\Rightarrow \quad m_1 m_2 = -1$$
$$\left(\frac{b-3}{6}\right)\left(\frac{1}{2}\right) = -1$$
$$b-3 = -12$$
$$b = -12+3$$

$\boxed{b=-9}$ is the required value of b.

9) Find the equation of a line, which has the y-intercept 4, and is parallel to the line $2x-3y-7=0$. Find the coordinates of the point where it cuts the x-axis.

→ Given that,

The y-intercept of required line is $\boxed{c=4}$

which is parallel to the line $2x-3y-7=0$ —— ①

$$3y = 2x-7$$

$$y = \frac{2}{3}x - 7/3 \quad \text{i.e. } y = mx+c$$

$$\Rightarrow \boxed{m = 2/3} \text{ is the slope of line ①}$$

If two lines are parallel then their slopes are also equal.

Thus, the required line is having slope 2/3 and y-intercept as 4 whose equation is given by

$$y = mx + c$$

$$y = \frac{2}{3}x + 4 \quad \Rightarrow \quad 3y = 2x+12$$

i.e. $\boxed{2x-3y+12=0}$ is the required equation of line —— ②

When the line ② cuts the x-axis then $\boxed{y=0}$

put $y=0$ in ② $\Rightarrow$ $2x+12=0$

$$2x=-12$$

$$\boxed{x=-6}$$

Thus, the co-ordinates of the required point are $(-6,0)$.

10) Find the equation of a straight line perpendicular to the line $2x+5y+7=0$ and with y intercept -3.

→ Given equation of a line is $2x+5y+7=0$ —— ①

$$5y = -2x-7$$

$$y = \frac{-2}{5}x - 7/5 \quad \text{i.e. } y = mx+c$$

Here, slope of line ① is found to be $\boxed{m_1 = -2/5}$

And the line with y-intercept -3 is perpendicular to ①.

$$\Rightarrow m_1 m_2 = -1$$

$$m_2 = \frac{-1}{m_1} = \frac{-1}{(-2/5)} = \frac{5}{2} \qquad \boxed{m_2 = 5/2}$$

Now, equation of a line with slope 5/2 and y-intercept −3 is given by

$$y = mx + c$$
$$y = 5/2\, x - 3$$
$$2y = 5x - 6$$

$\boxed{5x - 2y - 6 = 0}$ is the required equation of line.

11.) Find the equation of a straight line perpendicular to the line $3x - 4y + 12 = 0$ and having same y-intercept as $2x - y + 5 = 0$.

→ Given equation of a line is $3x - 4y + 12 = 0$ —— ①

slope of line ① ⇒ $4y = 3x + 12$

$$y = \frac{3}{4}x + 3 \quad \text{i.e. } y = mx + c$$

slope of line ① ⇒ $\boxed{m_1 = 3/4}$

Line ① is perpendicular to the line having y-intercept as $2x - y + 5 = 0$.

Here $y = 2x + 5$ i.e. $y = mx + c$

i.e. y-intercept is found to be $\boxed{c = 5}$

But, given lines are perpendicular to each other.

$$m_1 m_2 = -1$$
$$\frac{3}{4}(m_2) = -1 \Rightarrow \boxed{m_2 = -4/3}$$

Thus, required equation of line with slope −4/3 & y-intercept 5 is given by,

$$y = mx + c$$
$$y = \frac{-4}{3}x + 5$$
$$3y = -4x + 15$$

i.e. $\boxed{4x + 3y - 15 = 0}$ is the required equation of line.

13) Write down the equation of the line perpendicular to $3x+8y=12$ and passing through the point $(-1,-2)$.

$\rightarrow$ Given equation of a line is $3x+8y=12$ —— ①

① $\Rightarrow$ $8y = -3x+12$

$y = \frac{-3}{8}x + \frac{6}{4} = \frac{-3}{8}x + \frac{3}{2}$ i.e. $y = mx+c$

Then, slope of line ① is found to be $\boxed{m_1 = -3/8}$

The required line is perpendicular to line ①.

$\Rightarrow$ $m_1 m_2 = -1$

$(-3/8)m_2 = -1$

$\boxed{m_2 = 8/3}$

Now, equation of a line with slope $8/3$ & passing through point $(-1,-2)$ is given by

$y - y_1 = m(x - x_1)$

$y+2 = 8/3(x+1)$

$3y+6 = 8x+8$

$8x-3y+2=0$ is the required equation of line.

15) Find the equation of the line that is parallel to $2x+5y-7=0$ and passes through the mid-point of the line segment joining the points $(2,7)$ and $(-4,1)$.

$\rightarrow$ Given equation of the line is $2x+5y-7=0$ —— ①

① $\Rightarrow$ $5y = -2x+7$

$y = -2/5x + 7/5$ i.e. $y = mx+c$

Here, slope of line ① is found to be $\boxed{m_1 = -2/5}$

The required line is parallel to line ① $\Rightarrow$ $\boxed{m_2 = -2/5}$

Also, the required line passes through the mid-point of the line segment joining the points $(2,7)$ & $(-4,1)$.

Then, $(x,y) \equiv \left(\frac{2-4}{2}, \frac{7+1}{2}\right) \equiv \left(\frac{-2}{2}, \frac{8}{2}\right) \equiv (-1,4)$ is the mid-point

Now, equation of line passing through $(-1,4)$ & with slope $-2/5$ is given by,

$y - y_1 = m(x - x_1)$

$y - 4 = \left(\frac{-2}{5}\right)(x+1)$

$5y - 20 = -2x - 2$

$\boxed{2x + 5y - 18 = 0}$ is the required equation.

17) Find the equation of a straight line passing through the intersection of $2x+5y-4=0$ with X-axis and parallel to the line $3x-7y+8=0$

→ Given equation of line is $3x - 7y + 8 = 0$ —— ①

① ⇒ $7y = 3x + 8$

$y = 3/7\, x + 8/7$ i.e. $y = mx + c$

Then, slope of a line ① is found to be $\boxed{m_1 = 3/7}$

The required line is parallel to line ① ⇒ $\boxed{m_2 = 3/7}$

Also, the required line passes through the intersection of line $2x+5y-4=0$ & X-axis.

put $y=0$ in $2x + 5y - 4 = 0$

$2x - 4 = 0$

$2x = 4$

$\boxed{x = 2}$

Thus, required line is passing through the point $(2,0)$ & having slope $3/7$.

Thus, equation of a line in slope-point form is given by,

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{3}{7}(x - 2)$

$7y = 3x - 6$

$\boxed{3x - 7y - 6 = 0}$ is the required equation of line.

19) Find the equation of the line perpendicular from the point $(1,-2)$ on the line $4x-3y-5=0$. Also, find the coordinates of the foot of perpendicular.

→ Given equation of line is $4x-3y-5=0$ —— ①

$\Rightarrow 3y = 4x-5$

$y = \frac{4}{3}x - 5/3$ i.e. $y = mx+c$

Hence, slope of line ① is found to be $\boxed{m_1 = 4/3}$

Then, the slope of line perpendicular to ① is $\boxed{m_2 = -3/4}$

Now, equation of a line passing through point $(1,-2)$ and having slope $-3/4$ is given by,

$y - y_1 = m(x - x_1)$

$y+2 = \frac{-3}{4}(x-1)$

$4y+8 = -3x+3$

$3x+4y+5 = 0$ —— ②

①×4 ⇒ $16x - 12y = 20$

②×3 ⇒ $9x+12y = -15$

$25x = 5$

$\boxed{x = 1/5}$ put in ① ⇒

$4\left(\frac{1}{5}\right) - 3y = 5$

$\frac{4}{5} - 3y = 5$

$4 - 15y = 25$

$4 - 25 = 15y$

$-21 = 15y$

$\boxed{y = -21/15}$

Thus, the coordinates of foot of perpendicular is found to be $(1/5, -21/15)$.

20) Prove that the line through $(0,0)$ and $(2,3)$ is parallel to the line through $(2,-2)$ and $(6,4)$.

→ Given line is passing through points $(0,0)$ & $(2,3)$.

Then slope of line ⇒ $m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{3-0}{2-0} = \frac{2}{3}$ $\boxed{m_1 = 2/3}$

Again line which is passing through points $(2,-2)$ & $(6,4)$.

Then slope of line ⇒ $m_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{4+2}{6-2} = \frac{6}{4} = \frac{3}{2}$

Here, $\boxed{m_1 = m_2}$

Thus, given two lines are parallel to each other.

Hence proved.

21) Prove that the line through (-2,6) and (4,8) is perpendicular to the line through (8,12) and (4,24).

→ Given line is passing through points (-2,6) and (4,8).

Then slope of line is found to be

$$m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{8-6}{4+2} = \frac{2}{6} = \frac{1}{3} \quad \boxed{m_1 = 1/3}$$

And, the line is passing through points (8,12) & (4,24).

Then slope of line is found to be

$$m_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{24-12}{4-8} = \frac{12}{-4} = -3 \quad \Rightarrow \boxed{m_2 = -3}$$

Here, $\boxed{m_1 \cdot m_2 = -1}$

Thus, given two lines are perpendicular to each other.

Hence proved

22) Show that the triangle formed by the points A(1,3), B(3,-1) and C(-5,-5) is a right angled triangle by using slopes.

→ Given that,

The three vertices of a triangle are A(1,3), B(3,-1) and C(-5,-5).

Then, slope of line AB $\Rightarrow m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{-1-3}{3-1} = \frac{-4}{2} = -2$

$\boxed{m_1 = -2}$

and slope of line BC $\Rightarrow m_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{-5+1}{-5-3} = \frac{-4}{-8} = \frac{1}{2}$

$\boxed{m_2 = 1/2}$

Thus, $\boxed{m_1 m_2 = -1}$

Hence, required triangle is a right angled triangle.

Since line AB is perpendicular to line BC.

Hence proved

23) Find the equation of line through the point $(-1, 3)$ and parallel to the line joining the points $(0, -2)$ and $(4, 5)$.

→ Given line is passing through the points $(0, -2)$ and $(4, 5)$

Then slope of line $(m_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5+2}{4-0} = \frac{7}{4}$

But the required line is parallel to above line.

Thus, slope of required line is found to be

$$\boxed{m_2 = 7/4}$$

Then equation of line passing through point $(-1, 3)$ and having slope $7/4$ is given by,

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{7}{4}(x+1)$$

$$4y - 12 = 7x + 7$$

$\boxed{7x - 4y + 19 = 0}$ is the required equation of line.

25) Find the equation of the line through $(0, -3)$ and perpendicular to the line joining the points $(-3, 2)$ and $(9, 1)$.

→ Given line is passing through the points $(-3, 2)$ & $(9, 1)$.

Then, slope of line ⇒ $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-2}{9+3} = \frac{-1}{12}$

$$\boxed{m_1 = -1/12}$$

But, required line is perpendicular to above line.

$$\Rightarrow m_2 = \frac{-1}{m_1} = 12 \Rightarrow \boxed{m_2 = 12}$$

Now, the required line is passing through point $(0, -3)$ & having slope 12 whose equation is given by

$$y - y_1 = m(x - x_1)$$

$$y + 3 = 12(x - 0)$$

$$y + 3 = 12x$$

$\boxed{12x - y - 3 = 0}$ is the required equation of line.

27) The vertices of a triangle are $A(10,4)$, $B(4,-9)$ and $C(-2,-1)$. Find the equation of the altitude through A.
(The perpendicular drawn from a vertex of a triangle to the opposite side is called altitude).

→ Given that,
The vertices of a triangle are $A(10,4)$, $B(4,-9)$ and $C(-2,-1)$.
Then slope of line BC ⇒ $m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{-1+9}{-2-4} = \frac{8}{-6} = \frac{-4}{3}$

Let us consider, the slope of the altitude from the point $A(10,4)$ to BC be m_2.

Then, $m_1 \times m_2 = -1$

$$-\frac{4}{3}(m_2) = -1$$

$$\boxed{m_2 = 3/4}$$

Thus, equation of line passing through point $(10,4)$ & having slope $3/4$ is given by

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 3/4\,(x - 10)$$

$$4y - 16 = 3x - 30$$

$\boxed{3x - 4y - 14 = 0}$ is the required equation of line.

29) Find the equation of the right bisector of the line segment joining the points $(1,2)$ and $(5,-6)$.

→ Given line is passing through the points $(1,2)$ & $(5,-6)$.
Slope of the line ⇒ $m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{-6-2}{5-1} = \frac{-8}{4} = -2$

$$\boxed{m_1 = -2}$$

Let us consider m_2 be the slope of right bisector of the line segment.

$$m_1 m_2 = -1$$

$$m_2 = \frac{-1}{m_1} = \frac{-1}{-2} = 1/2$$

$$\Rightarrow \boxed{m_2 = 1/2}$$

Then, the midpoint of the line joining points (1,2) and (5,-6) is found to be

$$\left(\frac{1+5}{2}, \frac{2-6}{2}\right) \equiv \left(\frac{6}{2}, \frac{-4}{2}\right) \equiv (3,-2)$$

Thus, equation of line passing through point (3,-2) and with slope 1/2.

Then, $y - y_1 = m(x - x_1)$

$y + 2 = 1/2\,(x - 3)$

$2y + 4 = x - 3$

$\boxed{x - 2y - 7 = 0}$ is the required equation of line.

31) The points B(1,3) and D(6,8) are two opposite vertices of a square ABCD. find the equation of the diagonal AC.

→ Given that,

The points B(1,3) and D(6,8) are two opposite vertices of a square ABCD.

Then slope of line BD ⇒ $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-3}{6-1} = \frac{5}{5} = 1$

$\Rightarrow \boxed{m_1 = 1}$

We have, the diagonal AC is a perpendicular bisector of diagonal BD.

$\Rightarrow \quad m_1 \times m_2 = -1$

$m_2 = -1/m_1 = \frac{-1}{1} = -1 \quad \Rightarrow \boxed{m_2 = -1}$

Then, co-ordinates of mid-point of BD and AC is found to be

$$\left(\frac{1+6}{2}, \frac{3+8}{2}\right) \equiv \left(\frac{7}{2}, \frac{11}{2}\right)$$

Then, equation of line passing through point (7/2, 11/2) and having slope -1 is given by

$y - y_1 = m(x - x_1)$

$y - 11/2 = -1(x - 7/2)$

$2y - 11 = -2x + 7$

$2x + 2y - 18 = 0$ i.e. $\boxed{x + y - 9 = 0}$ is the required equation.

32) ABCD is a rhombus. The coordinates of A and C are (3,6) and (-1,2) respectively. Write down the equation of BD.

→ Given that, ABCD is a rhombus with co-ordinates of vertices A(3,6) & C(-1,2) respectively.

Then, slope of AC ⇒ $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-6}{-1-3} = \frac{-4}{-4} = 1$,

$\boxed{m_1 = 1}$

Already we know that,

The diagonals of a rhombus bisect each other at right angle.

⇒ diagonals BD and AC are perpendicular to each other.

If m_2 is the slope of BD ⇒ $\boxed{m_2 = -1}$

Now, co-ordinates of mid-point of AC is found to be

$\left(\frac{3-1}{2}, \frac{6+2}{2}\right) \equiv \left(\frac{2}{2}, \frac{8}{2}\right) \equiv (1,4)$

Thus, equation of line passing through point (1,4) and having slope -1 is given by

$y - y_1 = m(x - x_1)$

$y - 4 = -1(x - 1)$

$\Rightarrow y - 4 = -x + 1$

$\boxed{x + y - 5 = 0}$ is the required equation of line.