Chapter 9.

Arithmetic and Geometric Progression

Exercise 9.1

1.) For the following A.P.s, write the first term a and the common difference d:

i) 3, 1, -1, -3

→ Given A.P. is 3, 1, -1, -3, ...

First term = a = 3

$$\text{Common difference } (d) = \left.\begin{aligned} &1-3=-2\\ &=-1-1=-2\\ &=-3+1=-2 \end{aligned}\right\} \boxed{d=-2}$$

ii) 1/3, 5/3, 9/3, 13/3, ...

→ Given A.P. is 1/3, 5/3, 9/3, 13/3, ...

First term (a) = 1/3

$$\text{Common difference } (d) = \left.\begin{aligned} &\frac{5}{3}-\frac{1}{3}=\frac{4}{3}\\ &=\frac{9}{3}-\frac{5}{3}=\frac{4}{3}\\ &=\frac{13}{3}-\frac{9}{3}=\frac{4}{3} \end{aligned}\right\} \boxed{d=4/3}$$

iii) $-3.2, -3, -2.8, -2.6, \cdots$

→ Given A.P. is $-3.2, -3, -2.8, -2.6, \cdots$

first term $(a) = -3.2$

Common difference $(d) = -3 + 3.2 = 0.2$

$= -2.8 + 3 = 0.2$

$= -2.6 + 2.8 = 0.2$

$\boxed{d = 0.2}$

2) Write first four terms of the A.P., when the first term a and the common difference d are given as follows:

i) $a = 10, d = 10$

→ first term $(a) = 10$

Common difference $(d) = 10$

2^{nd} term $= a + d = 10 + 10 = 20$

3^{rd} term $= a + 2d = 10 + 20 = 30$

4^{th} term $= a + 3d = 10 + 30 = 40$

5^{th} term $= a + 4d = 10 + 40 = 50$

Hence, first four terms are

$10, 20, 30, 40, 50, \cdots$

ii) $a = -2, d = 0$

→ first term $(a) = -2$

Common difference $(d) = 0$

2^{nd} term $= a + d = -2 + 0 = -2$

3^{rd} term $= a + 2d = -2 + 0 = -2$

4^{th} term $= a + 3d = -2 + 0 = -2$

Thus, the required first four terms are $-2, -2, -2, -2$.

iii) $a = 4, d = -3$

→ first term $(a) = 4$

Common difference $(d) = -3$

2^{nd} term $= a + d = 4 - 3 = 1$

3^{rd} term $= a + 2d = 4 - 6 = -2$

4^{th} term $= a + 3d = 4 - 9 = -5$

Thus, first four terms are found to be $4, 1, -2, -5$.

iv) $a = 1/2, d = -1/6$

→ first term $(a) = 1/2$

Common difference $(d) = -1/6$

2^{nd} term $= a + d = \frac{1}{2} - \frac{1}{6} = \frac{3-1}{6} = \frac{1}{3}$

3^{rd} term $= a + 2d = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

4^{th} term $= a + 3d = \frac{1}{2} - \frac{1}{2} = 0$

Thus, the required first four terms are $1/2, 1/3, 1/6, 0$.

3) Which of the following lists of numbers forms an A.P.? If they form an A.P., find the common difference 'd' and write the next three terms:

i) 4, 10, 16, 22, ...

→ Given numbers are

4, 10, 16, 22 ... ...

For Common difference:

$$\left.\begin{aligned} 10-4 &= 6 \\ 16-10 &= 6 \\ 22-16 &= 6 \end{aligned}\right\} \boxed{d=6}$$

Here, common difference is same.

Thus, given sequence is A.P.

Then, next three terms are:

$(22+6) = 28$

$(28+6) = 34$

$(34+6) = 40$.

ii) −2, 2, −2, 2, ...

→ Given numbers are

−2, 2, −2, 2, ...

For common difference:

$$\left.\begin{aligned} 2+2 &= 4 \\ -2-2 &= -4 \\ 2+2 &= 4 \end{aligned}\right\} d \neq \text{same}$$

Here, the common difference is not same for given numbers.

Hence, the given sequence is not an Arithmetic progression.

iii) 2, 4, 8, 16, ...

→ Given numbers are

2, 4, 8, 16, ...

For Common difference:

$$\left.\begin{aligned} 4-2 &= 2 \\ 8-4 &= 4 \\ 16-8 &= 8 \end{aligned}\right\} d \neq \text{same}$$

Here, the common difference is not same for given numbers.

Hence, the given sequence is not an arithmetic progression.

iv) −10, −6, −2, 2, ...

→ Given numbers are

−10, −6, −2, 2, ...

for Common difference:

$$\left.\begin{aligned} -6+10 &= 4 \\ -2+6 &= 4 \\ 2+2 &= 4 \end{aligned}\right\} \boxed{d=4}$$

Here, the common difference is same for given numbers.

Hence, the given sequence is an arithmetic progression.

Exercise 9.2

1.) Find the A.P. whose n^{th} term is $(7-3n)$, Also find the 20^{th} term.

→ Given that, n^{th} term of an A.P. ⇒ $\boxed{a_n = (7-3n)}$

Now, to find the terms of an A.P. put $n = 1, 2, 3, \cdots$

$n = 1 \Rightarrow a_1 = (7-3) = 4$

$n = 2 \Rightarrow a_2 = (7-6) = 1$

$n = 3 \Rightarrow a_3 = (7-9) = -2$

Thus, the required A.P. is $4, 1, -2, \cdots$

20th term ⇒ $\boxed{a_{20} = (7-60) = -53}$

2.) Find the indicated term in each of the following A.P.

i) $1, 6, 11, 16, \cdots, a_{20}$

→ Here first term $(a) = 1$

Common difference $= 5$

Then,

$\boxed{a_n = a + (n-1)d}$

$a_{20} = a + (20-1)d$

$a_{20} = 1 + 19(5)$

$a_{20} = 1 + 95$

$\boxed{a_{20} = 96}$

ii) $-4, -7, -10, -13, \cdots a_{25}, a_n$.

→ Here, first term $(a) = -4$

Common difference $(d) = -3$

Then, $\boxed{a_n = a + (n-1)d}$

Now, $a_{25} = -4 + (25-1)(-3)$

$= -4 + 24(-3)$

$= -4 - 72 = -76$

$\boxed{a_{25} = -76}$

And $a_n = -4 - (n-1)3$

$= -4 - 3n + 3$

$a_n = -3n - 1$

$\boxed{a_n = -1 - 3n}$

3) Find the nth term and the 12th term of the list of the numbers: $5, 2, -1, -4, \cdots$

→ Given sequence is $5, 2, -1, -4, \cdots$

Here, $\boxed{a=5}$ Common difference ⇒ $\boxed{d=-3}$

Then nth term: $a_n = a + (n-1)d$

$a_n = 5 + (n-1)(-3) = 5 - 3n + 3 = 8 - 3n$

$\boxed{a_n = 8 - 3n}$ is the required nth term of an given A.P.

Now, 12th term: $a_{12} = 8 - 3(12)$

$= 8 - 36$

$\boxed{a_{12} = -28}$

4) i) If the common difference of an A.P. is -3 and 18th term is (-5), then find the first term.

→ Given that, Common difference $(d) = -3$

18th term ⇒ $a_{18} = -5$

We have, $a_n = a + (n-1)d$

$a_{18} = a + (18-1)d = a + 17d$

$-5 = a + 17(-3)$

$-5 = a - 51$

$a = -5 + 51 = 46$ $\boxed{a=46}$ → first term

ii) If the first term of an A.P. is -18 and it's 10th term is zero, then find its common difference.

→ Given that, first term $(a) = -18$, $a_{10} = 0$

We have, $a_n = a + (n-1)d$

$a_{10} = -18 + (10-1)d$

$0 = -18 + 9d$

$18 = 9d$

$\boxed{d=2}$ → Common difference

5.) Which term of the A.P.

i) 3, 8, 13, 18, ... is 78?

→ Given that, first term (a) = 3

Common difference (d) = 5

Now, $\boxed{a_n = a + (n-1)d}$

$78 = 3 + (n-1)5$

$75 = 5n - 5$

$80 = 5n$

$\boxed{n = 16}$

Thus, 78 is the 16^{th} term of given A.P.

ii) 18, $15\frac{1}{2}$, 13, ... is −47?

→ Given that, first term (a) = 18

Common difference (d) = $-5/2$

Now, $\boxed{a_n = a + (n-1)d}$

$-47 = 18 + (n-1)(-5/2)$

$-65 = \frac{-5n}{2} + \frac{5}{2}$

$-130 = -5n + 5$

$-135 = -5n$

$\boxed{n = 27}$

Thus, −47 is the 27^{th} term of given A.P.

6.) i) Check whether (−150) is a term of A.P. 11, 8, 5, 2, ...

→ Given A.P. is 11, 8, 5, 2 ...

first term (a) = 11 and Common difference (d) = −3

We have, $\boxed{a_n = a + (n-1)d}$

$-150 = 11 + (n-1)(-3)$

$-161 = -3n + 3$

$-158 = -3n \Rightarrow \boxed{n = 52.6}$

But, here n = 52.6 is not a natural number.

Hence, −150 is not the term of given A.P.

ii) Find whether 55 is a term of the A.P. 7, 10, 13, ... or not. If yes, find which term it is.

→ Given A.P. is 7, 10, 13, ...

first term (a) = 7, Common difference (d) = 3

We have, $\boxed{a_n = a + (n-1)d}$

$55 = 7 + (n-1)3$
$48 = 3n - 3$
$51 = 3n$
$\boxed{n = 17}$

Here, $n=17$ is the natural numbers. Hence, 55 is the term of given A.P.

Also, 55 is the 17^{th} term of given A.P.

7) Find the 20th term from the last term of the A.P.
3, 8, 13, ..., 253.

→ Given A.P. is 3, 8, 13, ..., 253

First term → $(a) = 3$ Common difference → $d = 5$

We have, $\boxed{a_n = a + (n-1)d}$

$253 = 3 + (n-1)5$
$250 = 5n - 5$
$255 = 5n$
$\boxed{n = 51}$

253 is the 51^{st} term of given A.P.

Let us consider, 'P' is the 20th term from the last term.

Then, $P = L - (n-1)d$
$P = 253 - (20-1)5$
$= 253 - 19 \times 5$
$= 253 - 95$
$\boxed{P = 158}$

Thus, 158 is the 20th term from the last term.

8) Find the sum of the two middle most terms of the A.P.
$-4/3, -1, -2/3, \ldots, 4\frac{1}{3}$

→ Given A.P. is $-4/3, -1, -2/3, \ldots, 4\frac{1}{3}$.

Here, first term $(a) = -4/3$

Common difference $(d) = -1 - (-4/3) = -1 + 4/3 = 1/3$

$\boxed{d = 1/3}$

We have, $\boxed{a_n = a+(n-1)d}$

$$\frac{13}{3} = \frac{-4}{3} + (n-1)\frac{1}{3}$$

$$\frac{13}{3} + \frac{4}{3} = \frac{n}{3} - \frac{1}{3}$$

$$\frac{17}{3} = \frac{(n-1)}{3}$$

$$17 = n-1$$

$$\boxed{n = 18}$$

Thus, middle term is $18/2$ & $(18/2)+1 \longrightarrow 9^{th}$ & 10^{th} term.

Then, $a_9 + a_{10} = a+8d + a+9d$

$$a_9 + a_{10} = 2a + 17d$$

$$= 2(-4/3) + 17(1/3)$$

$$= -8/3 + 17/3$$

$$= 9/3$$

$\boxed{a_9 + a_{10} = 3}$, is the required answer.

9.) Which term of the A.P. 53, 48, 43, ... is the first negative term?

$\longrightarrow$ Given A.P. is 53, 48, 43, ...

Here, first term $\rightarrow a = 53$

Common difference $\rightarrow d = -5$

We have, $a_n = a+(n-1)d$

$$= 53 + (n-1)(-5)$$

$$= 53 - 5n + 5$$

$$= 58 - 5n \quad \rightarrow \quad 58 = 5n$$

$$n = 11.6 = 12^{th}$$

Thus, 12^{th} term is the first negative term of the A.P.

10.) Determine the A.P. whose third term is 16 & the 7th term exceeds the 5th term by 12.

→ Given that, $a_3 = 16$

$a_7 - a_5 = 12$

Let us consider, a be the first term & d be the common difference.

Then, $a_n = a + (n-1)d$ and $\boxed{a_5 = a + 4d}$

$a_3 = a + 2d$

$\boxed{16 = a + 2d}$ — ①

$\boxed{a_7 = a + 6d}$

$a_7 - a_5 = a + 6d - a - 4d$

$12 = 2d$

$\boxed{d = 6}$ put in ①

$16 = a + 12$

$\boxed{4 = a}$

Here, first term is $a = 4$ and Common difference $(d) = 6$

Then, required A.P. is $a, a+d, a+2d, a+3d, \ldots$

i.e. $4, 10, 16, 22, \ldots$

11.) find the 20th term of the A.P. whose 7th term is 24 less than the 11th term, first term being 12.

→ Given that, first term $(a) = 12$ and $a_7 = a_{11} - 24$

Then, $a_{11} - a_7 = 24$ $\quad \because a_n = a + (n-1)d$

$(a + 10d) - (a + 6d) = 24$

$4d = 24$

$\boxed{d = 6}$ $\boxed{a = 12}$

Then 20th term of A.P. ⇒ $a_{20} = 12 + 19(6)$

$= 12 + 114$

$\boxed{a_{20} = 126}$

Thus, 20th term of an A.P. is found to be 126.

12) Find the 31^{st} term of an A.P. whose 11^{th} term is 38 and 6th term is 73.

→ Given that, $a_{11} = 38$ and $a_6 = 73$

$a_{11} = a + 10d$ and $a_6 = a + 5d$

$38 = a + 10d$ —— ① and $73 = a + 5d$ —— ②

① - ② ⇒ $(73 - 38) = (5d - 10d)$

$+35 = -5d$

$\boxed{d = -7}$ put in ①

$38 = a - 70$

$70 + 38 = a$

$\boxed{a = 108}$

Thus, 31^{st} term of an A.P. is given by

$a_{31} = a + 30d$

$a_{31} = 108 - 210$

$\boxed{a_{31} = -178}$

Thus, 31^{st} term of given A.P. is found to be (-178).

13) If the seventh term of an A.P. is 1/9 and its ninth term is 1/7, find its 63^{rd} term.

→ Given that, $a_7 = 1/9$ and $a_9 = 1/7$

$a_7 = a + 6d$ —— ① and $a_9 = a + 8d$ —— ②

① - ② ⇒ $(1/9 - 1/7) = (a + 6d) - (a + 8d) = -2d$

$\left(\frac{-2}{63}\right) = -2d$

$\boxed{d = 1/63}$ put in ①

$\frac{1}{9} = a + \frac{6}{63}$

$7 = 63a + 6$

$1 = 63a$

$\boxed{a = 1/63}$

Thus, 63^{rd} term of an A.P. is given by $a_{63} = a + (62)d$

$= \frac{1}{63} + \frac{62}{63}$

$a_{63} = \frac{63}{63} = 1$

Thus, 63^{rd} term of given A.P. is found to be 1.

15.) If 8th term of an A.P. is zero, prove that its 38th term is triple of its 18th term.

→ Given that, $a_8 = 0$

Let us consider a is the first term & d is common difference of given A.P.

Then, $a_8 = a + 7d$

$\Rightarrow a + 7d = 0$

$\boxed{a = -7d}$

Now, $a_{18} = a + 17d$ and $a_{38} = a + 37d$

$a_{18} = -7d + 17d$ $\quad a_{38} = -d(7) + 37d$

$\boxed{a_{18} = 10d}$ —— ① $\quad \boxed{a_{38} = 30d}$ —— ②

from ① & ② $\Rightarrow$ $a_{38} = 30d = 3(10d)$

$\boxed{a_{38} = 3(a_{18})}$ Hence proved.

16.) Which term of the A.P. 3, 10, 17, ... will be 84 more than its 13th term?

→ Given that, A.P. is 3, 10, 17, ...

first term $\Rightarrow a = 3$ common difference $(d) = 7$

Now, $a_{13} = a + 12d = 3 + 84 = 87$

$\boxed{a_{13} = 87}$

Given that, $a_n = a_{13} + 84$

$= 87 + 84$

$a_n = 171$

$a + (n-1)d = 171$

$3 + (n-1)7 = 171$

$7n - 7 = 168$

$7n = 175$

$\boxed{n = 25}$

Thus, 25th term of given A.P. is 84 more than its 13th term.

18.) If the numbers $(n-2)$, $(4n-1)$ and $(5n+2)$ are in A.P. find the value of n.

→ Given that, $(n-2)$, $(4n-1)$ and $(5n+2)$ are the numbers which are in A.P.

That means common difference should be same.

Thus, $(4n-1)-(n-2) = (5n+2)-(4n-1)$

$3n+1 = n+3$

$2n = 2$

$\boxed{n=1}$ is the required value

19.) The sum of three numbers in A.P. is 3 and their product is -35. Find the numbers.

→ Given that, the sum of three numbers in A.P. is 3.

Also, their product $= -35$

Let us consider the numbers $(a-d)$, a and $(a+d)$ are in A.P.

$\Rightarrow \quad a-d+a+a+d = 3$

$3a = 3$

$\boxed{a=1}$

And $(a-d)a(a+d) = -35$

$a(a^2-d^2) = -35$

$(a^3-ad^2) = -35$

$(1-d^2) = -35$

$1+35 = d^2 \Rightarrow d^2 = 36 \Rightarrow \boxed{d=\pm 6}$

If $\boxed{d=-6}$ then the required three numbers are

$a-d = 1+6 = +7$

$a = 1$

$a+d = 1-6 = -5$.

Hence, the three numbers 7, 1, -5, ... are in A.P.

20.) The sum of three numbers in A.P. is 30 & the ratio of first number to the third number is 3:7. Find the numbers.

→ Given that, the sum of three numbers in A.P. is 30.
And the ratio of first number to third number is 3:7.
Let us consider the three numbers be $(a-d)$, a & $(a+d)$.

$$\Rightarrow \quad (a-d)+a+(a+d)=30$$

$$3a=30$$

$$\boxed{a=10}$$

Again, $3:7=(a-d):(a+d)$

$$\frac{3}{7}=\frac{(a-d)}{(a+d)}$$

$$\Rightarrow 3(a+d)=7(a-d)$$

$$3a+3d=7a-7d$$

$$\Rightarrow 7d+3d=7a-3a$$

$$10d=4a \quad \Rightarrow$$

$$40=10d$$

$$\boxed{d=4}$$

Thus, the required numbers are: $a=10$

$a+d=10+4=14$, $a-d=10-4=6$

Thus, the required three numbers are $6, 10, 14, \ldots$ in A.P.

21.) The sum of the first three terms of an A.P. is 33. If the product of the first and the third terms exceeds the second term by 29, find the A.P.

→ Given that, the sum of the first three terms of an A.P. is 33.
Let us consider the three numbers be $(a-d)$, a, $(a+d)$.

$$\Rightarrow \quad a-d+a+a+d=33$$

$$3a=33 \quad \Rightarrow \quad \boxed{a=11}$$

Also, given that product of first & third term exceeds the second term by 29.

$$\Rightarrow \quad (a-d)(a+d)=a+29$$

$$a^2-d^2=11+29$$

$$11^2-d^2=40$$

$$121-40=d^2$$
$$d^2=81 \Rightarrow$$
$$\boxed{d=\pm 9}$$

If $d=9$, then the required numbers are
$a-d=11-9=2$, $a=11$, $a+d=11+9=20$

If $d=-9$, then the required numbers are
$(a-d)=11-(-9)=11+9=20$; $a=11$, $a+d=11-9=2$.

Hence, the required three numbers are $2, 11, 20, \ldots$ and $20, 11, 2, \ldots$ which are in A.P.

Exercise 9.3

1) Find the sum of the following A.P:

i) $2, 7, 12, \ldots$ to 10 terms

→ first term $(a)=2$

Common difference $(d)=5$

$$S_{10}=\frac{n}{2}[2a+(n-1)d]$$
$$=\frac{10}{2}[4+9\times5]$$
$$=5(4+45)$$
$$S_{10}=5(49)$$
$$\boxed{S_{10}=245}$$

ii) $1/15, 1/12, 1/10, \ldots$ to 11 terms

→ first term $(a)=1/15$

Common difference $(d)=\frac{1}{12}-\frac{1}{15}$
$$=\frac{(15-12)}{15\times12}$$
$$\boxed{d=1/60}$$

$$S_{11}=\frac{11}{2}[2a+(n-1)d]$$
$$=\frac{11}{2}\left[2\times\frac{1}{15}+10\times\frac{1}{60}\right]$$
$$=\frac{11}{2}\left(\frac{2}{15}+\frac{1}{6}\right)$$
$$=\frac{11}{2}(4+5)/30$$
$$=\frac{11}{2}(9/30)$$
$$=11/2\,(3/10)$$
$$\boxed{S_{11}=33/20}$$

2) Find the sum given below:

i) $34+32+30+\cdots+10$

→ Here, first term $(a)=34$

Common difference $(d)=-2$

Last term $(l)=10$

Then, $a_n=a+(n-1)d$

$10=34+(n-1)(-2)$

$-24=-2(n-1)$

$-24=-2n+2$

$2n=24+2$

$2n=26$

$\boxed{n=13}$

Thus, $S_n=\frac{n}{2}(a+l)$

$=\frac{13}{2}(34+10)$

$=\frac{13}{2}(44)$

$=13(22)$

$\boxed{S_n=286}$

ii) $-5+(-8)+(-11)+\cdots+(-230)$

→ first term $(a)=-5$

Common difference $(d)=-3$

Last term $(l)=-230$

We have, $a_n=a+(n-1)d$

$-230=-5+(n-1)(-3)$

$-230=-5-3n+3$

$-230=-2-3n$

$3n=230-2$

$3n=228$

$\boxed{n=76}$

Thus, $S_n=\frac{n}{2}(a+l)$

$=\frac{76}{2}(-5-230)$

$=38(-5-230)$

$=38(-235)$

$\boxed{S_n=-8930}$

3) In an A.P. (with usual notations):

In an A.P. given $a=5$, $d=3$, $a_n=50$ find n and S_n.

i) given $a=5$, $d=3$, $a_n=50$

→ $a_n=a+(n-1)d$

$50=5+(n-1)3$

$50=5+3n-3$

$50=2+3n$

$3n=50-2$

$3n=48$

$n=48/3$

$\boxed{n=16}$

Thus,

$S_n=\frac{n}{2}[2a+(n-1)d]$

$=\frac{16}{2}[2\times5+15\times3]$

$=8(10+45)$

$S_n=8\times55$

$\boxed{S_n=440}$

ii) given $a=7$, $a_{13}=35$, find d and S_{13}.

→ We have, $a_n = a+(n-1)d$

$35 = 7+(13-1)d$

$35 = 7+13d-d$

$35 = 7+12d$

$12d = 28$

$\boxed{d=7/3}$

Thus, $S_{13} = \frac{n}{2}[2a+(n-1)d]$

$= \frac{13}{2}[2\times7+12\times\frac{7}{3}]$

$= \frac{13}{2}[14+28] = \frac{13}{2}\times42 = 13\times21$

$\boxed{S_{13}=273}$

4.) i) The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

→ Given that, first term $(a)=5$, Last term $(l)=45$

Sum $=400$,

$a_n = a+(n-1)d$

$45 = 5+(n-1)d$

$\Rightarrow (n-1)d = 45-5 = 40$

$\Rightarrow (n-1)d = 40$ —— ①

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$400 = \frac{n}{2}[2\times5+40]$ from ①

$800 = n(10+40)$

$800 = 50n$

$\boxed{n=16}$ put in ① $\Rightarrow$ $(16-1)d=40$

$d = 40/15 = 8/3$

$\boxed{d=8/3}$

Thus, the number terms are 16.

And the common differences is 8/3.

5) The first and last terms of an A.P. are 17 and 350 resp. If the common difference is 9, how many terms are there and what is their sum?

→ Given that, first term $(a) = 17$, Last term $(l) = 350$.

common difference $(d) = 9$,

$a_n = a + (n-1)d$

$350 = 17 + (n-1) \times 9$

$350 - 17 = 9n - 9$

$333 + 9 = 9n$

$9n = 342$

$n = 342/9$

$\boxed{n = 38}$

Thus, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{38}{2}[2 \times 17 + 37d]$

$= 19[34 + 37 \times 9]$

$= 19(34 + 333)$

$= 19 \times 367$

$\boxed{S_n = 6973}$

6) Solve for x: $1 + 4 + 7 + 10 + \cdots + x = 287$.

→ Given that, first term $(a) = 1$, Common difference $(d) = 3$

$a_n = x$

$a_n = a + (n-1)d$

$x = 1 + (n-1)d$

$(x-1) = (n-1)d$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$287 = \frac{n}{2}[2 \times 1 + (n-1)3]$

$574 = n(2 + 3n - 3)$

$574 = 2n + 3n^2 - 3n$

$574 = -n + 3n^2$

$3n^2 - n - 574 = 0$

$3n^2 - 42n + 41n - 574 = 0$

$3n(n-14) + 41(n-14) = 0$

$(n-14)(3n+41) = 0$

If $(n-14) = 0 \Rightarrow \boxed{n = 14}$

or $3n + 41 = 0 \Rightarrow n = -41/3$

Hence, we will take $\boxed{n = 14}$

Thus, $a_n = a + (n-1)d$

$x = 1 + 13 \times 3$

$x = 1 + 39$

$\boxed{x = 40}$ is the required value.

8.) Find the sum of first 22 terms of an A.P. in which $d=7$ and a_{22} is 149.

→ Given that, the sum of first 22 terms of A.P. whose $d=7$. and $a_{22}=149$

$a_{22} = (n-1)d$

$149 = a+21d$

$149 = a+21\times7$

$149 = a+147$

$\boxed{a=2}$

Thus, $S_{22} = \frac{22}{2}[2a+(n-1)d]$

$= 11[2\times2+21\times7]$

$= 11[4+21\times7]$

$= 11(4+147)$

$S_{22} = 11\times151 = 1661$

Thus, the sum of first 22 terms is found to be 1661.

9.) In an A.P. the fourth & sixth term are 8 & 14 respectively find i) first term ii) common difference iii) sum of first 20 term

→ Given that, $a_4=8$ and $a_6=14$.

⇒ $a+3d=8$ —① and $a+5d=14$ —②

⇒ $3d-5d=8-14$

$-2d=-6$

$\boxed{d=3}$ put in ① ⇒ $a+9=8$

$\boxed{a=-1}$

Thus, i) first term $(a)=-1$

ii) Common difference $(d)=3$

iii) Then the sum of first 20 terms is given by

$S_{20} = \frac{20}{2}[2a+19\times d]$

$= 10[2\times(-1)+19\times3]$

$= 10[-2+57]$

$= 10\times55$

$\boxed{S_{20}=550}$ is the required sum.

10) find the sum of first 51 terms of the A.P. whose second and third terms are 14 and 18 respectively.

→ Given that, the sum of first 51 term of an A.P, where

$a_2 = 14, \quad a_3 = 18$

Then, common difference $= a_3 - a_2 = 18 - 14 = 4$

$\boxed{d=4}$

Now, first term $= 14 - 4 = 10$ and $\boxed{n = 51}$

$\boxed{a=10}$

We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{51}{2}[2 \times 10 + 50 \times 4]$

$= \frac{51}{2}[20 + 200]$

$S_{51} = \frac{51}{2} \times 220 = 5610$

$\boxed{S_{51} = 5610}$ is the required sum of first 51 terms.

11.) If the sum of first 6 terms of an A.P is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

→ Given that, $S_6 = 36$ and $S_{16} = 256$

We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

Then, $S_6 = \frac{6}{2}[2a + 5d]$ and $S_{16} = \frac{16}{2}[2a + 15d]$

$36 = 3(2a + 5d)$ $\qquad 256 = 8[2a + 15d]$

$12 = 2a + 5d$ —— ① $\qquad 32 = (2a + 15d)$ —— ②

② − ① ⇒ $32 - 12 = (2a + 15d) - (2a + 5d)$

$20 = 10d$

$\boxed{d=2}$ put in ① ⇒ $12 = 2a + 10$

$2 = 2a$

Thus, the sum of first ten terms $\boxed{a=1}$ is given by,

$S_{10} = \frac{n}{2}[2a + (n-1)d] = 5[2 + 9 \times 2]$

$S_{10} = 5(2 + 18)$

$S_{10} = 5(20) = 100$

Thus, the sum of first ten terms is found to be 100.

12.) Show that: $a_1, a_2, a_3, \ldots$ form an A.P. where a_n is defined as ($a_n = 3+4n$). Also find the sum of first 15 terms.

→ Given that, $a_n = 3+4n$ —— ①

put n=1 in ① ⇒ $a_1 = 3+4 = 7$

$n=2 \Rightarrow a_2 = 3+8 = 11$

$n=3 \Rightarrow a_3 = 3+12 = 15$

$n=4 \Rightarrow a_4 = 3+16 = 19$

Thus, the required numbers are found to be 7, 11, 15, 19, ...

first term ⇒ a = 7 and Common difference (d)

$$\left.\begin{array}{l} d = 11-7 = 4 \\ d = 15-11 = 4 \\ d = 19-15 = 4 \end{array}\right\} \boxed{d=4}$$

Thus, 7, 11, 15, 19, ... (i.e. $a_1, a_2, a_3, \ldots$) forms an A.P.

Now, The sum of first 15 terms is given by

$$S_{15} = \frac{15}{2}[2\times 7 + 14\times 4]$$

$$= \frac{15}{2}[14 + 14\times 4] = \frac{15}{2}\times \overset{7}{\cancel{14}}(1+4)$$

$$= 105\times 5$$

$\boxed{S_{15} = 525}$ is the required sum of first 15 terms.

13.) Sum of first 6 terms of an arithmetic progression is 42 and the ratio of the 10th and 30th term is 1:3. Calculate first and 13th term.

→ Given that, $S_6 = 42$ and $a_{10} : a_{30} = 1:3$

We have, $S_n = \frac{n}{2}[2a+(n-1)d]$ & $a_n = a+(n-1)d$

Thus, $S_6 = 3[2a+5d]$

$42 = 3(2a+5d)$

$14 = 2a+5d$ —— ①

$a_{10} = a+9d$ & $a_{30} = a+29d$

$$\frac{a_{10}}{a_{30}} = \frac{(a+9d)}{(a+29d)} = \frac{1}{3}$$

$\Rightarrow 3(a+9d) = (a+29d)$

$3a - a + 27d - 29d = 0$

$2a - 2d = 0$

$\boxed{a=d}$

put $\boxed{a=d}$ in ① ⇒

$14 = 2a+5a$

$14 = 7a$

$\boxed{a=2}$ put in ①

$14 = 2a + 5d$
$14 = 4 + 5d$
$10 = 5d$
$\boxed{d = 2}$

Thus, first term $\Rightarrow \boxed{a = 2}$

And $a_{13} = a + (n-1)d$
$= 2 + 12 \times 2$
$= 2 + 24$
$\boxed{a_{13} = 26}$ is the required term.

14.) In an A.P. the sum of first n terms is $(6n - n^2)$. Find 25th term.

→ Given that $\boxed{S_n = 6n - n^2}$

Then, $S_{(n-1)} = 6(n-1) - (n-1)^2$
$= 6n - 6 - (n^2 + 2n + 1)$
$= 6n - 6 - n^2 + 2n + 1$
$= -n^2 + 8n - \cancel{7}$
$\boxed{S_{n-1} = -n^2 + 8n - 7}$

We know that, $a_n = S_n - S_{n-1}$
$a_{25} = S_{25} - S_{24}$
$= (6 \times 25 - 25^2) - (6 \times 24 - 24^2)$
$= 25(6 - 25) - 24(6 - 24)$
$= 25(-19) - 24(-18)$
$= -475 + 432$
$\boxed{a_{25} = -43}$ is the required 25th term.

15.) If S_n denote the sum of first n terms of an A.P. prove that $S_{30} = 3(S_{20} - S_{10})$.

→ We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

Given that, $S_{30} = 3(S_{20} - S_{10})$ — ①

$S_{20} = 10[2a + 19d] = 20a + 190d$
$S_{10} = 5(2a + 9d) = 10a + 45d$

$\Rightarrow$ L.H.S. of ① $= 3(S_{20} - S_{10})$

$S_{30} = 3(S_{20} - S_{10})$

$= 3[20a + 190d - (10a + 45d)]$

$= 3[20a + 190d - 10a - 45d]$

$= 3[10a + 145d]$

$\boxed{S_{30} = 30a + 435d}$ —— ②

Now, $S_{30} = 15(2a + 29d) = 30a + 435d$ —— ③

from ② and ③ ⇒ $\boxed{S_{30} = 3(S_{20} - S_{10})}$ Hence proved.

16.) i) Find the sum of first 1000 positive integers

→ Let us consider the sum of first 1000 positive integers is $(1 + 2 + 3 + 4 + \cdots + 1000)$

Here, $\boxed{a = 1}$ and common difference $\boxed{d = 1}$

Thus, we have $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{1000} = 500[2 + 999 \times 1]$

$= 500[2 + 999] = 500 \times 1001$

$\boxed{S_{1000} = 500500}$

This is the required sum of first 1000 positive integers.

ii) Find the sum of first 15 multiple of 8

→ The multiples of 8 are : 8, 16, 24, 40, ... & given $\boxed{n = 15}$

Here, $\boxed{a = 8}$, $\boxed{d = 8}$

Thus, $S_{15} = \frac{15}{2}[2 \times 8 + 14 \times 8]$

$S_{15} = 15 \times 4[2 + 14] = 60[16] = 960$

$\boxed{S_{15} = 960}$

This is the required sum of first 15 multiples of 8.

17) i) Find the sum of two digit natural numbers which are divisible by 4.

→ The two digit natural numbers which are divisible by 4 starts from 12, 16, 20, ... is an arithmetic progression.

Here, $\boxed{a=12}$ and $\boxed{d=4}$

Thus, the last two digit number which is divisible by 4 is 96

Hence, $\boxed{\text{Last term} = 96}$

Hence, $a_n = a+(n-1)d$

$96 = 12+(n-1)4 = 12+4n-4 = 8+4n$

$4n = 88$

$\boxed{n=22}$ Hence, 96 is the 22^{nd} term here.

So, we have to find the sum of 22 terms here.

Thus, $S_{22} = \frac{22}{2}[2\times12+(22-1)\times4]$

$= 11[24+21\times4]$

$= 11[24+84] = 11(108)$

$\boxed{S_{22} = 1188}$ is the required sum of two digit natural numbers which are divisible by 4.

ii) Find the sum of all natural numbers less than 100 which are divisible by 6.

→ The natural numbers divisible by 6 starts from 6, 12, 18, 24, ... and the last number less than 100 which is divisible by 6 is found to be 98.

Hence, here $\boxed{a=6}$ $\boxed{d=6}$ and $\boxed{\text{Last term} = 98}$

Then, $a_n = 6+(n-1)6$

$98 = 6+6n-6$

$98 = 6n$

$\boxed{n=13}$

98 is the 13^{th} term.

Hence, we have to find the sum of first 13^{th} terms which are divisible by 6.

$S_{13} = \frac{13}{2}[2\times6+12\times6]$

$= 13\times3(2+12)$

$= 39\times14$

$\boxed{S_{13} = 546}$

This is the required sum.

Exercise 9.4

1.) i) Find the next term of the list of numbers $\frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \ldots$

$\rightarrow$ Given list numbers are $\frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \ldots$

Here, $a = 1/6$, common ratio $r \rightarrow \frac{1/3}{1/6} = \frac{6}{3} = 2$

Here, common ratio $r = 2$ is same. $\frac{2/3}{1/3} = 2$ $\boxed{r=2}$

Hence, the list of numbers given

$\frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \ldots$ is a geometric progression.

Hence, the next term will be $\rightarrow \frac{2}{3} \times 2 = \frac{4}{3}$.

ii) Find the next term of the list of numbers $\frac{3}{16}, -\frac{3}{8}, \frac{3}{4}, \frac{-3}{2}, \ldots$

$\rightarrow$ Given list of numbers is $\frac{3}{16}, \frac{-3}{8}, \frac{3}{4}, \frac{-3}{2}, \ldots$

Here, $\boxed{a = 3/16}$ and common ratio $\rightarrow$

And the common ratio $\boxed{r = -2}$ is same.

$\frac{-3/8}{3/16} = \frac{-3}{8} \times \frac{16^2}{3} = -2$

$\frac{3/4}{-3/8} = \frac{3}{4} \times \left(\frac{-8}{3}\right) = -2$

$\boxed{r=-2}$

$\rightarrow$ Thus, the list of numbers $\frac{3}{16}, \frac{-3}{8}, \frac{3}{4}, \frac{-3}{2}, \ldots$ is G.P.

- The next term will be $\frac{-3}{2} \times (-2) = 3$.

iii) Find the 10th and nth term of the list of numbers $5, 25, 125, \ldots$

$\rightarrow$ Given list of numbers is $5, 25, 125, \ldots$

Here, , $\boxed{r = 5}$

Then, $a_n = ar^{n-1}$

$a_{10} = 5(5)^{10-1} = 5 \times 5^9 = 5^{10}$

$\boxed{a_{10} = 5^{10}}$ is the 10th term

And nth term is given by

$a_n = 5(5)^{n-1} = 5^n$

$\boxed{a_n = 5^n}$

2. i) Which term of the G.P. i) $2, 2\sqrt{2}, 4, \cdots$ is 128?

→ Given G.P. is $2, 2\sqrt{2}, 4, \cdots$

Here, $\boxed{a=2}$ and common ratio ⇒

$$\left.\begin{aligned}\frac{2\sqrt{2}}{2} &= \sqrt{2}\\ \frac{4}{2\sqrt{2}} &= \frac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}} = \sqrt{2}\end{aligned}\right\} \boxed{r=\sqrt{2}}$$

We have, $a_n = ar^{n-1}$

$$128 = 2(\sqrt{2})^{n-1}$$

$$(\sqrt{2})^{n-1} = 64$$

$$(\sqrt{2})^{n-1} = 2^6$$

$$\Rightarrow (\sqrt{2})^{n-1} = (\sqrt{2})^{12}$$

Comparing exponential on both sides,

⇒ we get $n-1=12$

$\boxed{n=13}$

Hence, 128 is the 13th term of given geometric progression.

ii) Determine the 12th term of G.P, Whose 8th term is 192 and common ratio is 2.

→ Given that, 8th term of G.P. ⇒ $a_8 = 192$

and common ratio $(r) = 2$.

We have, $a_n = ar^{n-1}$

$$a_8 = a(2)^{8-1} = a(2)^7 \Rightarrow 192 = a(128)$$

$$\Rightarrow \boxed{a = \frac{192}{128}} \quad \text{——} ①$$

Now, $a_{12} = a(2)^{12-1} = a\,2^{11}$

$$\Rightarrow \boxed{a = \frac{a_{12}}{2^{11}}} \quad \text{——} ②$$

from ① & ②

$$\frac{a_{12}}{2^{11}} = \frac{192}{2^7}$$

$$a_{12} = \frac{192}{2^7} \times 2^{11} = 192 \times 2^{11} \times 2^{-7}$$

$$= 192 \times 2^{11-7}$$

$$= 192 \times 2^4$$

$$= 192 \times 16$$

$$\boxed{a_{12} = 3072}$$

Thus, the required 12th term of G.P. is found to be 3072.

4) In a G.P. the third term is 24 and 6th term is 192. find the 10th term.

→ Given that, 3rd term of G.P. is 24. ⇒ $a_3 = 24$

6th term of G.P. is 192 ⇒ $a_6 = 192$

We have, $a_n = ar^{n-1}$

$a_3 = ar^2 \Rightarrow 24 = ar^2 \Rightarrow a = 24/r^2$ —— ①

and $a_6 = ar^5 \Rightarrow 192 = ar^5 \Rightarrow a = 192/r^5$ —— ②

from ① & ② ⇒ $\frac{24}{r^2} = \frac{192}{r^5}$

$\frac{192}{24} = \frac{r^5}{r^2} = r^3 \Rightarrow 8 = r^3$

$\boxed{r = 2} \Rightarrow \boxed{a = 6}$

Thus, 10th term of G.P. is given by, $a_{10} = 6(2)^9$

$\boxed{a_{10} = 3072}$

is the required 10th term of G.P.

5.) find the number of terms of a G.P. whose first term is 3/4, Common ratio is 2 and last term is 384.

→ Given that, first term $(a) = 3/4$, Common ratio $(r) = 2$

and $\boxed{\text{Last term} = 384}$

We have, $a_n = ar^{n-1}$

$384 = \left(\frac{3}{4}\right) 2^{n-1}$

$\frac{384 \times 4}{3} = 2^{n-1}$

$512 = 2^{n-1}$

$\Rightarrow 2^9 = 2^{n-1}$

$9 = n-1$

$\boxed{n = 10}$

Thus, the required total no. of terms are found to be 10.

6.) find the value of 'x' such that

i) $(x+9)$, $(x-6)$ and 4 are three consecutive terms of a G.P.

→ Given that, $(x+9)$, $(x-6)$ and 4 are three consecutive terms of G.P.

$$\Rightarrow \frac{x-6}{x+9} = \frac{4}{x-6} = \text{constant} = r$$

$\Rightarrow (x-6)^2 = 4x+36$

$x^2-12x+36 = 4x+36$

$x^2-16x=0$

$x(x-16)=0$

$\boxed{x=0}$ or $\boxed{x=16}$ are the required values of x.

ii) $x, (x+3), (x+9)$ are first three terms of G.P. find x.

→ Given that, $x, (x+3), (x+9)$ are first three terms of G.P.

$\Rightarrow$ common ratio $= \frac{x+3}{x} = \frac{x+9}{x+3}$

$\Rightarrow (x+3)^2 = x(x+9)$

$x^2+6x+9 = x^2+9x$

$-3x+9=0 \Rightarrow -3x=-9$

$x = 9/3 = 3$ $\boxed{x=3}$ is the required value of x.

7) If the fourth, seventh and tenth terms of a G.P. are x, y, z respectively then prove that x, y, z are in G.P.

→ Given that, x, y, z are 4th, 7th & 10th terms of G.P. respectively.

Thus, $a_n = ar^{n-1}$

$\Rightarrow a_4 = ar^3$, $a_7 = ar^6$, $a_{10} = ar^9$

We know that, the Geometric mean of G.P. is

$b^2 = ac \Rightarrow b = \sqrt{ac}$

$\Rightarrow ar^6 = \sqrt{ar^3 \times ar^9}$

$ar^6 = \sqrt{a^2r^{12}} = \sqrt{(ar^6)^2} = ar^6$

$\boxed{y^2 = xz}$

Thus, x, y, z are in G.P.

Hence proved.

8.) The 5^{th}, 8^{th} & 11^{th} terms of a G.P. are p, q and s respectively, Show that $q^2 = ps$

→ Given that, 5^{th}, 8^{th} and 11^{th} terms of a G.P. are p, q and s respectively.

We have, $a_n = ar^{n-1}$

$\Rightarrow$ $a_5 = a_1 r^4$. $\quad a_8 = a_1 r^7 \quad a_{11} = a_1 r^{10}$

$p = a_1 r^4 \quad q = a_1 r^7 \quad s = a_1 r^{10}$

Now, $q^2 = a_1^2 r^{14}$ —— ①

and $ps = (a_1 r^4)(a_1 r^{10}) = a_1^2 r^{14}$ —— ②

from ① & ② $\Rightarrow$ $\boxed{q^2 = ps}$ Hence proved.

9.) If a, (a^2+2) and (a^3+10) are in G.P. then find the values of a

→ Given that, a, (a^2+2) and (a^3+10) are in G.P.

$\Rightarrow$ common ratio $\Rightarrow \dfrac{(a^2+2)}{a} = \dfrac{(a^3+10)}{(a^2+2)}$

$\Rightarrow (a^2+2)^2 = a(a^3+10)$

$\Rightarrow a^4 + 4a^2 + 4 = a^4 + 10a$

$4a^2 + 4 - 10a = 0$

$4a^2 - 8a - 2a + 4 = 0$

$4a(a-2) - 2(a-2) = 0$

$(4a-2)(a-2) = 0$

$\boxed{a=2}$ or $\boxed{a = 1/2}$ are the required values of a.

11.) The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

→ Given that, the sum of first three terms of G.P. is $\frac{39}{10}$.

Their product $= 1$

Let us consider, $\frac{a}{r}$, a and ar be the first three terms which are in G.P.

$$\Rightarrow \frac{a}{r} + a + ar = \frac{39}{10} \quad \text{—①}$$

and $\left(\frac{a}{r}\right)(a)(ar) = 1$ ∵ from given condition.

$$a^3 = 1$$

$$\Rightarrow \boxed{a=1} \text{ put in ①}$$

$$\frac{1}{r} + 1 + r = \frac{39}{10} \Rightarrow 1 + r + r^2 = \frac{39}{10} r$$

$$\Rightarrow 10 + 10r + 10r^2 - 39r = 0$$

$$10r^2 - 29r + 10 = 0$$

$$10r^2 - 25r - 4r + 10 = 0$$

$$5r(2r-5) - 2(2r-5) = 0$$

$$(5r-2)(2r-5) = 0 \Rightarrow \boxed{r = 2/5 \text{ or } 5/2}$$

Thus, when common ratio $r = 2/5$ then required terms are

$$\frac{a}{r}, a, ar \Rightarrow \frac{5}{2}, 1, 2/5$$

And, when common ratio $r = 5/2$ then required terms are

$$\frac{a}{r}, a, ar \Rightarrow \frac{2}{5}, 1, 5/2.$$

Exercise 9.5

1) find the sum of i) 20 terms of the series $2+6+18+, \ldots$

→ Given series is $2+6+18+, \ldots$

Here, first term ⇒ $a=2$

Common ratio $r=\frac{6}{2}=3$, $r=\frac{18}{6}=3$ ⇒ common ratio $\boxed{r=3}$

As $r>1$ ⇒ $S_n=\frac{a(r^n-1)}{(r-1)}$

$$S_n=\frac{2(3^{20}-1)}{3-1}=3^{20}-1$$

$\boxed{S_{20}=3^{20}-1}$ is the required sum of 20 terms for given GP

ii) 10 terms of series $1+\sqrt{3}+3+, \ldots$

→ Given series is $1+\sqrt{3}+3+, \ldots$

Here, first term $(a)=1$, common ratio $r=\sqrt{3}$

$\boxed{n=10}$ $r>1$ ⇒ $S_n=\frac{a(r^n-1)}{(r-1)}$

$$S_{10}=\frac{1(\sqrt{3}^{10}-1)}{(\sqrt{3}-1)}$$

⇒ $S_{10}=\frac{((\sqrt{3})^2-1^2)}{(\sqrt{3}-1)}$ is the required sum of 10 terms of given G.P.

2) Find the sum of the series $81-27+9 \ldots -\frac{1}{27}$.

→ Given series is $81-27+9 - \ldots -\frac{1}{27}$.

first term $\boxed{(a)=81}$, Common ratio ⇒ $r=-27/81$ $\boxed{r=-1/3}$

And Last term $=-1/27$

Thus, $S_n=\frac{(a-lr)}{(1-r)}$

$$=\frac{[81+(1/27)\times(-1/3)]}{(-1/27+1/3)]}$$

$$= \frac{[(81-1/81)]}{4/3}$$

$$= \frac{(6561-1)}{81\times 4/3} = \frac{(6560\times 3)}{81\times 4}$$

$\boxed{S_n = \frac{1640}{27}}$ is the required sum.

3) The nth term of a G.P. is 128 and the sum of its n terms is 255. If its common ratio is 2, then find its first term.

→ Given that, nth term of a G.P. is 128 ⇒ $\boxed{a_n=128}$

and $\boxed{S_n = 255}$ and $\boxed{r=2}$

We have, $a_n = ar^{n-1}$

$128 = a2^{n-1}$

$\boxed{a = 128/2^{n-1}}$ —— ①

Also, $S_n = \frac{a(r^n-1)}{(r-1)}$

$255 = \frac{a(2^n-1)}{(2-1)}$

⇒ $255\times 1 = a(2^n-1)$

$\boxed{a = 255/(2^n-1)}$ —— ②

from ① and ② ⇒

$$\frac{255}{(2^n-1)} = \frac{128}{(2^{n-1})}$$

$255\times 2^{n-1} = 128(2^n-1)$

$255\times 2^{n-1} = 128\times 2^n - 128$

$(255\times 2^n)/2 = 128\times 2^n - 128$

$255\times 2^n = 256\times 2^n - 256$

$256\times 2^n - 255\times 2^n = 256$

After simplification, $2^n = 256$

$\boxed{2^n = 2^8}$ $\boxed{n=8}$

∴ $128 = a2^7$

$128 = a\times 128$

$\boxed{a=1}$

Thus, the first term is found to be $\boxed{a=1}$

4) i) How many terms of the G.P. $3, 3^2, 3^3, \ldots$ are needed to give the sum 120?

→ Given G.P. is $3, 3^2, 3^3, \ldots$

Here, first term $(a) = 3$, common ratio $(r) = 3$

Thus, we can write

$$\Rightarrow \frac{a(r^n-1)}{(r-1)} = \frac{3(3^n-1)}{(3-1)} = 120$$

$$3(3^n-1) = 240$$

$$3^n-1 = 80$$

$$\Rightarrow 3^n = 81 = 3^4$$

$$\Rightarrow \boxed{n=4}$$

Thus, the sum of first 4 terms of given G.P. is found to be 120.

ii) How many terms of the G.P. $1, 4, 16, \ldots$ must be taken to have their sum equal to 341?

→ Given G.P. is $1, 4, 16, \ldots$

Here, first term $(a) = 1$, common ratio $r = 4$.

Thus, we can write,

$$\therefore \quad S_n = \frac{a(r^n-1)}{(r-1)} = 341 \Rightarrow \frac{1(4^n-1)}{(4-1)} = 341$$

$$4^n-1 = 341 \times 3$$

$$4^n = 1024$$

$$4^n = 4^5$$

$$\Rightarrow \boxed{n=5}$$

Thus, the sum of first 5 terms of given G.P. is found to be 341.

6) The 2nd and 5th terms of G.P. are $-1/2$ and $\frac{1}{16}$ respectively. Find the sum of the series upto 8 terms.

→ Given that, $a_2 = -1/2$ and $a_5 = 1/16$

Let us consider, $a \rightarrow$ first term and

$r \rightarrow$ common ratio of G.P.

Then, $a_2 = ar^{2-1} = ar = 1/2$

$a_5 = ar^{5-1} = ar^4 = 1/16$

Now, $\frac{ar^4}{ar} = r^3 = \frac{1/16}{-1/2} = \frac{-2}{16}$

$$r^3 = \frac{-1}{8}$$

$$r^3 = \left(\frac{-1}{2}\right)^3$$

$$\Rightarrow \boxed{r = -1/2}$$

Thus, $ar = -1/2$
$a(-1/2) = -1/2$
$\boxed{a = 1}$

and $S_8 = \frac{a(1-r^n)}{(1-r)}$ $\because r<1$

$= \frac{1[1-(-1/2)^8]}{(1+1/2)}$

$= \frac{[1-1/256)}{3/2}$

$= 255/256 \times \frac{2}{3}$

$= 510/768$

$\boxed{S_8 = 85/128}$

Thus, the sum of given series upto 8 terms is found to be 85/128.

7) The first term of a G.P. is 27 and 8th term is 1/81. find the sum of its first 10 terms.

→ Given that, $a = 27$, $a_8 = 1/81$.
Let us consider 'a' be first term & r is the common ratio.

Then, $a_8 = ar^{n-1} = ar^{8-1} = ar^7 = 1/81$

$\Rightarrow 27r^7 = \frac{1}{81} \Rightarrow r^7 = \frac{1}{81 \times 27} = \frac{1}{2187} = \frac{1}{3^7}$

$\therefore \boxed{r = 1/3} < 1$

Thus, $S_{10} = \frac{a(1-r^n)}{(1-r)} = \frac{27[1-(1/3)^{10}]}{(1-1/3)}$

$= \frac{27(1-1/3^{10})}{(3-1)/3}$

$S_{10} = \frac{27 \times 3}{2}\left[1-\frac{1}{3^{10}}\right) = \frac{81}{2}\left(1-\frac{1}{3^{10}}\right)$
is the required sum of first 10 terms of given G.P.

8) Find the first term of the G.P. whose common ratio is 3, last term is 486 and the sum of whose terms is 728.

→ Given that, common ratio $(r) = 3$

Last term $(l) = 486$

Sum of the terms $= 728$

We have, $S_n = \frac{a(r^n - 1)}{(r-1)}$ $\because r > 1$

$$728 = \frac{a(3^n - 1)}{(3-1)}$$

$$\Rightarrow a(3^n - 1) = 728 \times 2$$

$$\boxed{a(3^n - 1) = 1456} \text{ — ①}$$

But, Last term $= ar^{n-1}$

$$486 = a \times 3^{n-1}$$

$$486 = a(3^n/3)$$

$$486 \times 3 = a3^n$$

$$\boxed{1458 = a3^n} \text{ — ②}$$

from ① $\Rightarrow$ $a(3^n - 1) = 1456$

$$3^n a - a = 1456$$

$$1458 - a = 1456 \quad \because \text{from ②}$$

$\boxed{a = 2}$ is the required first term of G.P.

9.) In a G.P. the first term is 7, the last term is 448 and the sum is 889. Find the common ratio.

→ Given that, first term $(a) = 7$, Last term $= 448$

Sum of terms $= 889$.

We know that, $a, ar, ar^2, ar^3, \cdots ar^n$ are the terms in G.P.

Let r be the common ratio here.

Then, $a_n = ar^{n-1} = 448$

$$S_n = \frac{a(r^n - 1)}{(r-1)} = 889$$

$$\Rightarrow S_n = \frac{ar^{n-1}r - a}{r-1} = 889$$

$$448r - 7 = 889r - 889$$

$$882 = 441r$$

$\boxed{r=2}$ is the required common ratio.

10.) Find the third term of a G.P. whose common ratio is 3 and the sum of whose first seven terms is 2186.

$\longrightarrow$ Given that, common ratio $(r) = 3$,

Sum of first seven terms $= 2186 = S_7$.

We have, $S_n = \dfrac{a(r^n - 1)}{(r-1)}$

$$S_7 = \frac{a(3^7-1)}{(3-1)} = 2186$$

$$\Rightarrow \frac{a(3^7-1)}{2} = 2186$$

$$\Rightarrow a(3^7-1) = 4372$$

$$a(2187-1) = 4372$$

$\Rightarrow \boxed{a=2}$ is the required first term of G.P.

11.) If the first term of a G.P. is 5 and the sum of first three terms is 31/5, find the common ratio.

$\longrightarrow$ Given that, $\boxed{a=5}$ $\boxed{S_3 = 31/5}$

We have, $S_n = \dfrac{a(r^n-1)}{(r-1)} \Rightarrow S_3 = \dfrac{a(r^3-1)}{(r-1)}$

$$\frac{31}{5} = \frac{5(r^3-1)}{(r-1)}$$

$$\frac{31}{25} = \frac{r^3-1}{r-1} \Rightarrow \frac{(r-1)(r^2+r+1)}{(r-1)} = 31/25$$

$$\Rightarrow r^2+r+1 = 31/25$$

$$25r^2+25r+25 = 31$$

$$25r^2+25r-6 = 0$$

$$25r^2+30r-5r-6 = 0$$

$$5r(5r+6) - 1(5r+6) = 0$$

$$(5r+6)(5r-1) = 0$$

$$5r-1 = 0 \Rightarrow \boxed{r = 1/5}$$

and $5r+6=0$

$\boxed{r = -6/5}$

are the required common ratio