Chapter 5. Quadratic Equations

Exercise 5.1

1.) In each of the following, determine whether the given numbers are roots of the given equations or not.

i) $x^2-5x+6=0$; $2, -3$

→ Given quadratic equation is $x^2-5x+6=0$ ——— ①

To check given numbers are roots of equⁿ ① or not

we put $x=2$ in equⁿ ①

$$\Rightarrow (x)^2-5x+6 = (2)^2-5(2)+6$$
$$= 4-10+6$$
$$= 10-10$$
$$= 0$$
$$= \text{R.H.S.}$$

Thus, equⁿ ① satisfies for the value of $x=2$.

Hence, $x=2$ is the root of quadratic equation ①.

Now, put $x=-3$ in equⁿ ①

$$\Rightarrow x^2-5x+6 = (-3)^2-5(-3)+6$$
$$= 9+15+6$$
$$= 30$$
$$\neq 0$$

Thus, equⁿ ① not satisfies for the value of $x=-3$

Hence, $x=-3$ is not the root of quadratic equⁿ ①.

ii) $3x^2-13x-10=0$, $5, -\frac{2}{3}$

→ Given quadratic equation is $3x^2-13x-10=0$ —— ①

To check given numbers are roots of eqⁿ ①.

Initially, we put $x=5$ in eqⁿ ①

$$\Rightarrow 3x^2-13x-10 = 3(5)^2-13(5)-10$$
$$= 3(25)-65-10$$
$$= 75-75$$
$$= 0$$

Thus, eqⁿ ① satisfies for the value of $x=5$.

Hence, $x=5$ is the root of equation ①.

Now, we will put $x=-2/3$ in eqⁿ ①

$$3x^2-13x-10 = 3(-2/3)^2-13(-2/3)-10$$
$$= 3(4/9)+26/3-10$$
$$= 12/9+26/3-10$$
$$= 4/3+26/3-10$$
$$= 30/3-10$$
$$= 10-10$$
$$= 0$$

Thus, eqⁿ ① satisfies for the value of $x=-2/3$.

Hence, $x=-2/3$ is the root of eqⁿ ①.

2.) In each of the following, determine whether the given numbers are solutions of given equation or not.

i) $x^2-3\sqrt{3}x+6=0$; $\sqrt{3}, -2\sqrt{3}$

ii) $x^2-\sqrt{2}x-4=0$, $x=-\sqrt{2}, 2\sqrt{2}$

→ i) Given quadratic equation is

$$x^2-3\sqrt{3}x+6=0 \text{ —— ①}$$

To check given numbers are roots of equation ①,
we put $x=\sqrt{3}$ in equⁿ①.

$$\Rightarrow x^2-3\sqrt{3}x+6 = (\sqrt{3})^2 - 3\sqrt{3}(\sqrt{3})+6$$
$$= 3-9+6$$
$$= 9-9$$
$$= 0$$

Thus, equⁿ① satisfies for the value of $x=\sqrt{3}$.
Hence, $x=\sqrt{3}$ is the root of equation①.

Now, put $x=-2\sqrt{3}$ in equⁿ①.

$$\Rightarrow x^2-3\sqrt{3}x+6 = (-2\sqrt{3})^2 - 3\sqrt{3}(-2\sqrt{3})+6$$
$$= 12+18+6$$
$$= 36$$
$$\neq 0$$

Thus, equⁿ① not satisfies for the value of $x=-2\sqrt{3}$.
Hence, $x=-2\sqrt{3}$ is not the root of equⁿ①.

ii) Given quadratic equⁿ is

$$x^2-\sqrt{2}x-4=0, \quad x=-\sqrt{2}, 2\sqrt{2} \qquad ①$$

To check given numbers are roots of equⁿ①
we put $x=-\sqrt{2}$ in equⁿ①.

$$\Rightarrow x^2-\sqrt{2}x-4 = (-\sqrt{2})^2 - \sqrt{2}(-\sqrt{2})-4$$
$$= 2+2-4$$
$$= 4-4$$
$$= 0$$

Thus, equⁿ① satisfies for the value of $x=-\sqrt{2}$.
Hence, $x=-\sqrt{2}$ is the root of equⁿ①.

Now, put $x = 2\sqrt{2}$ in equⁿ ①

$\Rightarrow x^2 - \sqrt{2}x - 4 = (2\sqrt{2})^2 - \sqrt{2}(2\sqrt{2}) - 4$

$= 8 - 4 - 4$

$= 8 - 8$

$= 0$

Thus, equⁿ ① satisfies the value of $x = 2\sqrt{2}$.

Hence, $x = 2\sqrt{2}$ is the root of equⁿ ①.

3.) i) If $-1/2$ is a solution of the equation $3x^2 + 2kx - 3 = 0$, find the value of k.

→ Given quadratic equation is $3x^2 + 2kx - 3 = 0$ —— ①

Also, $x = -1/2$ is the solution of equⁿ ①.

To find k, we put $x = -1/2$ in equⁿ ①.

$\Rightarrow 3x^2 + 2kx - 3 = 0$

$3(-1/2)^2 + 2k(-1/2) - 3 = 0$

$3/4 - k - 3 = 0$

$\frac{3}{4} = k + 3$

$\frac{3}{4} - 3 = k$

$\boxed{k = -9/4}$ is the required value of k.

ii) If $2/3$ is a solution of the equation $7x^2 + kx - 3 = 0$, find the value of k.

→ Given quadratic equation is

$7x^2 + kx - 3 = 0$ —— ①

Also, $x = 2/3$ is the solution of equⁿ ①.

To find k, we put $x = 2/3$ in equⁿ ①

$\Rightarrow 7x^2 + kx - 3 = 0$

$$7x^2+kx-3=0$$
$$7(2/3)^2+k(2/3)-3=0$$
$$\frac{28}{9}+\frac{2}{3}k-3=0$$
$$\frac{2}{3}k=3-\frac{28}{9}$$
$$\frac{2}{3}k=-1/9$$
$$2k=-1/3$$
$\boxed{k=-1/6}$ is the required value of k.

4) i) If $\sqrt{2}$ is a root of the equation $kx^2+\sqrt{2}x-4=0$, find the value of k.

→ Given quadratic equation is $kx^2+\sqrt{2}x-4=0$ —— ①

Given that, $x=\sqrt{2}$ is the solution of ①.

we put $x=\sqrt{2}$ in equⁿ ①.

$$\Rightarrow kx^2+\sqrt{2}x-4=0$$
$$k(\sqrt{2})^2+\sqrt{2}(\sqrt{2})-4=0$$
$$2k+2-4=0$$
$$2k-2=0$$
$\boxed{k=1}$ is the required value of k.

ii) If 'a' is a root of the equation $x^2-(a+b)x+k=0$, find the value of k.

→ Given quadratic equation is

$$x^2-(a+b)x+k=0 \quad \text{——①}$$

$x=a$ is the solution of equation ①.

To find k, put $x=a$ in equⁿ ①.

$$x^2-(a+b)x+k=0$$
$$a^2-(a+b)a+k=0$$
$$k=-a^2+(a+b)a$$

$K = -a^2 + (a+b)a$

$K = -a^2 + a^2 + ab$

$\boxed{K = ab}$ is the required value of K.

5) If $2/3$ and -3 are the roots of the equation $px^2 + 7x + q = 0$, find the values of p and q.

→ Given quadratic equation is

$$px^2 + 7x + q = 0 \quad \text{——} \; (1)$$

$x = 2/3$ & $x = -3$ are the solutions of equⁿ (1).

put $x = 2/3$ in equⁿ (1)

$\Rightarrow px^2 + 7x + q = 0$

$p(2/3)^2 + 7(2/3) + q = 0$

$p(4/9) + (14/3) + q = 0$

$\frac{4p}{9} + \frac{14}{3} + q = 0$

$4p + 42 + 9q = 0$

$4p + 9q = -42 \quad \text{——} \; (2)$

put $x = -3$ in equⁿ (1)

$px^2 + 7x + q = 0$

$p(-3)^2 + 7(-3) + q = 0$

$p(9) - 21 + q = 0$

$9p + q = 21 \quad \text{——} \; (3)$

equⁿ (3) × 9

$\Rightarrow 81p + 9q = 189 \quad \text{——} \; (4)$

equⁿ (2) − (4) $\Rightarrow$

$$\begin{array}{r} 81p + 9q = 189 \\ 4p + 9q = -42 \\ - \quad - \quad + \\ \hline 77p = 231 \end{array}$$

$p = 231/77$

$\boxed{p = 3}$

put $p = 3$ in equⁿ (2)

$\Rightarrow 12 + 9q = -42$

$9q = -54$

$\boxed{q = -6}$

Thus, the required values of p & q are 3 & −6 respectively

Exercise 5.2

1.) i) $x^2-3x-10=0$ Solve by the method of factorization.

$\rightarrow$ Given quadratic equation is $x^2-3x-10=0$

$$x^2-3x-10=0$$
$$x^2-5x+2x-10=0$$
$$x(x-5)+2(x-5)=0$$
$$(x+2)(x-5)=0$$
$$\boxed{x=-2} \text{ or } \boxed{x=5}$$

ii) $x(2x+5)=3$

$\rightarrow$ Given equation is

$$x(2x+5)=3$$
$$2x^2+5x-3=0$$
$$2x^2+6x-x-3=0$$
$$2x(x+3)-1(x+3)=0$$
$$(x+3)(2x-1)=0$$
$$\boxed{x=-3} \text{ or } \boxed{x=1/2}$$

2) i) $3x^2-5x-12=0$

$\rightarrow$ Given equation is

$$3x^2-5x-12=0$$
$$3x^2-9x+4x-12=0$$
$$3x(x-3)+4(x-3)=0$$
$$(x-3)(3x+4)=0$$
$$x-3=0 \text{ or } 3x+4=0$$
$$\boxed{x=3} \text{ or } \boxed{x=-4/3}$$

ii) $21x^2-8x-4=0$

$\rightarrow$ Given quadratic equ^n is

$$21x^2-8x-4=0$$
$$21x^2-14x+6x-4=0$$
$$7x(3x-2)+2(3x-2)=0$$
$$(3x-2)(7x+2)=0$$
$$3x-2=0 \text{ or } 7x+2=0$$
$$\boxed{x=2/3} \text{ or } \boxed{x=-2/7}$$

3.) i) $3x^2=x+4$

$\rightarrow$ Given equ^n is $3x^2-x-4=0$

$$3x^2-4x+3x-4=0$$
$$x(3x-4)+1(3x-4)=0$$
$$(3x-4)(x+1)=0$$
$$3x-4=0 \text{ or } x+1=0$$
$$\boxed{x=4/3} \text{ or } \boxed{x=-1}$$

ii) $x(6x-1)=35$

$\rightarrow$ Given equ^n is $6x^2-x-35=0$

$$6x^2-x-35=0$$
$$6x^2-15x+14x-35=0$$
$$3x(2x-5)+7(2x-5)=0$$
$$(2x-5)(3x+7)=0$$
$$\boxed{x=5/2} \text{ or } \boxed{x=-7/3}$$

4) i) $6p^2+11p-10=0$

$\rightarrow$ Given equ^n is $6p^2+11p-10=0$

$6p^2+15p-4p-10=0$

$3p(2p+5)-2(2p+5)=0$

$(2p+5)(3p-2)=0$

$2p+5=0$ or $3p-2=0$

$2p=-5$ or $3p=2$

$\boxed{p=-5/2}$ or $\boxed{p=2/3}$

ii) $\frac{2}{3}x^2-\frac{1}{3}x-1=0$

$\rightarrow$ Given quadratic equation is

$\frac{2}{3}x^2-\frac{1}{3}x-1=0$

$2x^2-x-3=0$

$2x^2-3x+2x-3=0$

$x(2x-3)+1(2x-3)=0$

$(2x-3)(x+1)=0$

$2x-3=0$ or $x+1=0$

$\boxed{x=3/2}$ or $\boxed{x=-1}$

5) i) $3(x-2)^2=147$

$\rightarrow$ Given equation is

$3(x^2-4x+4)=147$

$3x^2-12x+12=147$

$3x^2-12x+12-147=0$

$3x^2-12x-135=0$

$x^2-4x-45=0$

$x^2-9x+5x-45=0$

$x(x-9)+5(x-9)=0$

$(x-9)=0$ or $(x+5)=0$

$\boxed{x=9}$ or $\boxed{x=-5}$

$\because (a+b)^2=a^2+2ab+b^2$

$(a-b)^2=a^2-2ab+b^2$

ii) $\frac{4}{7}(3x-5)^2=28$

$\rightarrow$ Given equation is

$\frac{4}{7}(3x-5)^2=28$

$(3x-5)^2=4$

$9x^2-30x+25=4$

$9x^2-30x+25-4=0$

$9x^2-30x+21=0$

$3x^2-10x+7=0$

$3x^2-7x-3x+7=0$

$x(3x-7)-1(3x-7)=0$

$(3x-7)(x-1)=0$

$3x-7=0$ or $x-1=0$

$\boxed{x=7/3}$ or $\boxed{x=1}$

6) $x^2-4x-12=0,\ x\in N$

→ Given quadratic equⁿ is

$$x^2-4x-12=0 \quad \text{—— (1)}$$
$$x^2-6x+2x-12=0$$
$$x(x-6)+2(x-6)=0$$
$$(x-6)(x+2)=0$$
$$\boxed{x=6} \text{ or } \boxed{x=-2}$$

But $x\in N$

Hence, $x=6$ is the only solution of equⁿ (1).

8) i) $a^2x^2+2ax+1=0,\ a\neq 0$

→ Given quadratic equⁿ is

$$a^2x^2+2ax+1=0 \quad \text{—— (1)}$$
$$a^2x^2+ax+ax+1=0$$
$$ax(ax+1)+1(ax+1)=0$$
$$(ax+1)(ax+1)=0$$
$$(ax+1)^2=0$$
$$ax=-1$$
$$\boxed{x=-1/a}$$

Thus, $x=-1/a, -1/a$ is the required solⁿ.

7) $2x^2-9x+10=0$

i) $x\in N$ ii) $x\in Q$

→ Given quadratic equⁿ is

$$2x^2-9x+10=0 \quad \text{—— (1)}$$
$$2x^2-5x-4x+10=0$$
$$x(2x-5)-2(2x-5)=0$$
$$(2x-5)(x-2)=0$$
$$\boxed{x=5/2} \text{ or } \boxed{x=2}$$

i) when $x\in N$

$x=2$ is the only one solution of equation (1).

ii) when $x\in Q$ then $x=2, 5/2$ is the solution of equⁿ (1).

ii) $x^2-(p+q)x+pq=0$

→ Given quadratic equⁿ is

$$x^2-(p+q)x+pq=0 \quad \text{—— (1)}$$
$$x^2-px-qx+pq=0$$
$$x(x-p)-q(x-p)=0$$
$$(x-p)(x-q)=0$$
$$(x-p)=0 \text{ or } (x-q)=0$$
$$\boxed{x=p} \text{ or } \boxed{x=q}$$

is the required solution of equⁿ (1).

9.) $a^2x^2 + (a^2+b^2)x + b^2 = 0, a \neq 0$

→ Given quadratic equation is

$$a^2x^2 + (a^2+b^2)x + b^2 = 0, \quad a \neq 0$$
$$a^2x^2 + a^2x + b^2x + b^2 = 0$$
$$a^2x(x+1) + b^2(x+1) = 0$$
$$(x+1)(a^2x+b^2) = 0$$
$$(x+1) = 0 \quad \text{or} \quad a^2x + b^2 = 0$$
$$\boxed{x = -1} \quad \text{or} \quad a^2x = -b^2$$
$$\boxed{x = -b^2/a^2}$$

This is the required solution of given quadratic eqn

10.) i) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

→ Given quadratic eqn is

$$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0 \text{ — } ①$$
$$\sqrt{3}x^2 + 7x + 3x + 7\sqrt{3} = 0$$
$$\sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$$
$$\sqrt{3}x(x+\sqrt{3}) + 7(x+\sqrt{3}) = 0$$
$$(x+\sqrt{3})(\sqrt{3}x+7) = 0$$
$$x+\sqrt{3} = 0 \quad \text{or} \quad \sqrt{3}x + 7 = 0$$
$$\boxed{x = -\sqrt{3}} \quad \text{or} \quad \boxed{x = -7/\sqrt{3}}$$

This is the required solution of given equation.

ii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

→ Given quadratic eqn is

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0 \text{ — } ①$$
$$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$
$$4x(\sqrt{3}x+2) - \sqrt{3}(\sqrt{3}x+2) = 0$$
$$(\sqrt{3}x+2)(4x-\sqrt{3}) = 0$$
$$\sqrt{3}x + 2 = 0 \quad \text{or} \quad 4x - \sqrt{3} = 0$$
$$\boxed{x = -2/\sqrt{3}} \quad \text{or} \quad \boxed{x = \sqrt{3}/4}$$

This is the required solution of given equation.

11.) i) $x^2-(1+\sqrt{2})x+\sqrt{2}=0$

→ Given equation is

$$x^2-(1+\sqrt{2})x+\sqrt{2}=0$$
$$x^2-x-\sqrt{2}x+\sqrt{2}=0$$
$$x(x-1)-\sqrt{2}(x-1)=0$$
$$(x-1)(x-\sqrt{2})=0$$
$$x-1=0 \text{ or } x-\sqrt{2}=0$$
$$\boxed{x=1} \text{ or } \boxed{x=\sqrt{2}}$$

This is the required solution of given equation.

ii) $x+\frac{1}{x}=2\frac{1}{20}$

→ Given equation is

$$x+1/x=2\,1/20$$
$$\frac{x^2+1}{x}=\frac{41}{20}$$
$$20x^2+20=41x$$
$$20(x^2+1)-41x=0$$
$$20x^2-41x+20=0$$
$$20x^2-25x-16x+20=0$$
$$5x(4x-5)-4(4x-5)=0$$
$$(4x-5)(5x-4)=0$$
$$\boxed{x=5/4} \text{ or } \boxed{x=4/5}$$

This is the required solution of given equation.

12.) i) $\frac{2}{x^2}-\frac{5}{x}+2=0,\ x\neq 0$

→ Given quadratic equn is $\frac{2}{x^2}-\frac{5}{x}+2=0,\ x\neq 0$

$$2-5x+2x^2=0 \quad \because \text{multiplying by } x^2$$
$$2x^2-5x+2=0$$
$$2x^2-4x-x+2=0$$
$$2x(x-2)-1(x-2)=0$$
$$(x-2)(2x-1)=0$$
$$x-2=0 \text{ or } 2x-1=0$$
$$\boxed{x=2} \text{ or } \boxed{x=1/2}$$

This is the required solution of given equn.

13) i) $3x - \frac{8}{x} = 2$

→ Given equation is

$$3x - \frac{8}{x} = 2$$

$$(3x^2 - 8)/x = 2$$

$$3x^2 - 8 = 2x$$

$$3x^2 - 2x - 8 = 0$$

$$3x^2 - 6x + 4x - 8 = 0$$

$$3x(x-2) + 4(x-2) = 0$$

$$(x-2)(3x+4) = 0$$

$$x - 2 = 0 \text{ or } 3x + 4 = 0$$

$$\boxed{x = 2} \text{ or } \boxed{x = -4/3}$$

This is the required solution of given equation.

ii) $\frac{x+2}{x+3} = \frac{2x-3}{3x-7}$

→ Given equation is

$$\frac{x+2}{x+3} = \frac{2x-3}{3x-7}$$

$$(x+2)(3x-7) = (2x-3)(x+3)$$

$$(3x^2 - 7x + 6x - 14) = (2x^2 + 6x - 3x - 9)$$

$$3x^2 - 2x^2 - x - 3x - 14 + 9 = 0$$

$$x^2 - 4x - 5 = 0$$

$$x^2 - 5x + x - 5 = 0$$

$$x(x-5) + 1(x-5) = 0$$

$$(x-5)(x+1) = 0$$

$$x - 5 = 0 \text{ or } x + 1 = 0$$

$$\boxed{x = 5} \text{ or } \boxed{x = -1}$$

This is the required solution of given equation.

14) i) $\frac{8}{(x+3)} - \frac{3}{(2-x)} = 2$

→ Given equation is

$$\frac{8}{(x+3)} - \frac{3}{(2-x)} = 2$$

$$\frac{8(2-x) - 3(x+3)}{(x+3)(2-x)} = 2$$

$$(16 - 8x) - 3x - 9 = 2(x+3)(2-x)$$

$$7 - 11x = 2(2x - x^2 + 6 - 3x)$$

$$7 - 11x = (4x - 2x^2 + 12 - 6x)$$

$$7 - 11x = -2x - 2x^2 + 12$$

$$2x^2 - 9x - 5 = 0$$

$$2x^2 - 10x + x - 5 = 0$$

$$2x(x-5) + 1(x-5) = 0$$

$$(x-5)(2x+1) = 0$$

$$x - 5 = 0 \text{ or } 2x + 1 = 0$$

$$\boxed{x = 5} \text{ or } \boxed{x = -1/2}$$

ii) $\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}$

→ Given equation is

$$\frac{x}{x-1} + \frac{x-1}{x} = \frac{5}{2}$$

$$\frac{x^2 + (x-1)^2}{x(x-1)} = \frac{5}{2}$$

$$2(x^2 + x^2 - 2x + 1) = 5(x^2 - x)$$

$$2(2x^2 - 2x + 1) = 5x^2 - 5x$$

$$4x^2 - 4x + 2 = 5x^2 - 5x$$

$$x^2 - x - 2 = 0$$

$$x^2 - 2x + x - 2 = 0$$

$$x(x-2) + 1(x-2) = 0$$

$$(x-2)(x+1) = 0$$

$$x - 2 = 0 \text{ or } x + 1 = 0$$

$$\boxed{x = 2} \text{ or } \boxed{x = -1}$$

15) i) $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 3$

$\rightarrow$ Given equation is

$$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 3$$

$$\frac{(x+1)(x+2)+(x-2)(x-1)}{(x-1)\ (x+2)} = 3$$

$$\frac{(x^2+2x+x+2)+(x^2-x-2x+2)}{(x^2+2x-x-2)} = 3$$

$$\frac{(x^2+3x+2+x^2-3x+2)}{(x^2+x-2)} = 3$$

$$2x^2+4 = 3x^2+3x-6$$

$$x^2+3x-10 = 0$$

$$x^2+5x-2x-10 = 0$$

$$x(x+5)-2(x+5) = 0$$

$$(x+5)(x-2) = 0$$

$$x+5 = 0 \text{ or } x-2 = 0$$

$$\boxed{x=-5} \text{ or } \boxed{x=2}$$

ii) $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$

$\rightarrow$ Given equation is

$$\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$$

$$\frac{(x+5)-(x-3)}{(x-3)(x+5)} = \frac{1}{6}$$

$$\frac{(x+5-x+3)}{(x^2+5x-3x-15)} = \frac{1}{6}$$

$$\frac{(8)}{x^2+2x-15} = \frac{1}{6}$$

$$48 = x^2+2x-15$$

$$x^2+2x-63 = 0$$

$$x^2+9x-7x-63 = 0$$

$$x(x+9)-7(x+9) = 0$$

$$(x+9)(x-7) = 0$$

$$x+9 = 0 \text{ or } x-7 = 0$$

$$\boxed{x=-9} \text{ or } \boxed{x=7}$$

16) i) $\frac{a}{ax-1} + \frac{b}{(bx-1)} = (a+b)$, $a+b \neq 0$, $ab \neq 0$

$\rightarrow$ Given equation is $\frac{a}{(ax-1)} + \frac{b}{(bx-1)} = (a+b)$

$$\frac{a(bx-1)+b(ax-1)}{(ax-1)(bx-1)} = a+b$$

$$\frac{abx-a+abx-b}{(abx^2-ax-bx+1)} = (a+b)$$

$$2abx-a-b = a^2bx^2-a^2x-abx+a+ab^2x^2-abx-b^2x+b$$

$$a^2bx^2+ab^2x^2-a^2x-abx-abx-b^2x+a+b+a+b-2abx = 0$$

$$(a^2b+ab^2)x^2-(a^2+2ab+b^2)x+2a+2b-2abx = 0$$

$$(a^2b+ab^2)x^2-(a^2+4ab+b^2)x+2(a+b) = 0$$

$$\frac{a}{(ax-1)} + \frac{b}{(bx-1)} = a+b$$

$$\left\{\frac{a}{(ax-1)} - b\right\} + \left\{\frac{b}{(bx-1)} - a\right\} = 0$$

$$\left\{\frac{a-b(ax-1)}{(ax-1)}\right\} + \left\{\frac{b-a(bx-1)}{(bx-1)}\right\} = 0$$

$$\frac{(a-abx+b)}{(ax-1)} + \frac{(b-abx+a)}{(bx-1)} = 0$$

$$(a-abx+b)\left[\frac{1}{(ax-1)} + \frac{1}{(bx-1)}\right] = 0$$

$$(a-abx+b)\left[\frac{(bx-1)+(ax-1)}{(ax-1)(bx-1)}\right] = 0$$

$$(a-abx+b)\left[\frac{(ax+bx-2)}{(ax-1)(bx-1)}\right] = 0$$

$a-abx+b=0$ or $\frac{(ax+bx-2)}{(ax-1)(bx-1)} = 0$

If $a-abx+b=0$

$\Rightarrow a+b = abx$

$$\boxed{x = \frac{(a+b)}{ab}}$$

If $\frac{(ax+bx-2)}{(ax-1)(bx-1)} = 0$

$\Rightarrow ax+bx-2=0$

$(a+b)x-2=0$

$$\boxed{x = \frac{2}{(a+b)}}$$

Thus, the required values of x is found to be

$$x = \frac{(a+b)}{ab}, \quad x = \frac{2}{(a+b)}.$$

17) $\frac{1}{x+6} + \frac{1}{x-10} = \frac{3}{x-4}$

→ Given equation is $\frac{1}{x+6} + \frac{1}{x-10} = \frac{3}{x-4}$

$$\frac{x-10+x+6}{(x+6)(x-10)} = \frac{3}{x-4}$$

$$\frac{2x-4}{(x^2-10x+6x-60)} = \frac{3}{x-4}$$

$(2x-4)(x-4) = 3(x^2-4x-60)$

$(2x^2-8x-4x+16) = 3x^2-12x-180$

$x^2-180-16=0$

$x^2-196=0$

$x^2=196$

$x=\sqrt{196}$

$\boxed{x=\pm 14}$ This is the required value of x.

18.) i) $\sqrt{3x+4}=x$

→ Given equation is

$\sqrt{3x+4}=x$

squaring on both sides,

$(3x+4)=x^2$

$x^2-3x-4=0$

$x^2-4x+x-4=0$

$x(x-4)+1(x-4)=0$

$(x-4)(x+1)=0$

$x-4=0$ or $x+1=0$

$\boxed{x=4}$ or $\boxed{x=-1}$

ii) $\sqrt{x(x-7)}=3\sqrt{2}$

→ Given equation is

$\sqrt{x(x-7)}=3\sqrt{2}$

squaring on both sides,

$x(x-7)=18$

$x^2-7x=18$

$x^2-7x-18=0$

$x^2-9x+2x-18=0$

$x(x-9)+2(x-9)=0$

$(x-9)(x+2)=0$

$x-9=0$ or $x+2=0$

$\boxed{x=9}$ or $\boxed{x=-2}$

19.) Use the substitution $y=3x+1$ to solve for x:

$5(3x+1)^2+6(3x+1)-8=0$

→ Given that, $5(3x+1)^2+6(3x+1)-8=0$ —— ①

put $y=3x+1$ in ①

① ⇒ $5y^2+6y-8=0$

$5y^2+10y-4y-8=0$

$5y(y+2)-4(y+2)=0$

$(y+2)(5y-4)=0$

$y+2=0$ or $5y-4=0$

$\boxed{y=-2}$ or $\boxed{y=4/5}$ But $y=3x+1$

$\Rightarrow 3x+1=-2$ or $3x+1=4/5$

$3x=-3$ or $3x=4/5-1=-1/5$

$\boxed{x=-1}$ or $\boxed{x=-1/15}$

This are the required values of x for equation ①.

20.) Find the values of x, if $p+1=0$ and $x^2+px-6=0$

→ Given that, $p+1=0$

$\boxed{p=-1}$ put in given equⁿ.

$\Rightarrow x^2+px-6=0$

$x^2-x-6=0$

$x^2-3x+2x-6=0$

$x(x-3)+2(x-3)=0$

$(x-3)(x+2)=0$

$x-3=0$ or $x+2=0$ This are the required values

$\boxed{x=3}$ or $\boxed{x=-2}$ of x for given equation.

21.) Find the values of x if $p+7=0$, $q=12$ & $x^2+px+q=0$

→ Given equation is $x^2+px+q=0$ —— ①

Also $p+7=0$ and $\boxed{q=12}$

$\Rightarrow \boxed{p=-7}$

By putting the values of p & q in equⁿ①.

$\Rightarrow x^2-7x+12=0$

$x^2-4x-3x+12=0$

$x(x-4)-3(x-4)=0$

$(x-4)(x-3)=0$

$x-4=0$ or $x-3=0$ This are the required values

$\boxed{x=4}$ or $\boxed{x=3}$ of x for given equⁿ.

22.) If $x=3$ is a solution of equn $(k+2)x^2-kx+6=0$ find the value of k & find the other root of equation also.

→ Given equation is $(k+2)x^2-kx+6=0$ —— ①

$x=3$ is the solution of equn ①.

put $x=3$ in equn ① ⇒ $(k+2)(3)^2-k(3)+6=0$

$$9(k+2)-3k+6=0$$
$$9k+18-3k+6=0$$
$$6k+24=0$$
$$6k=-24$$
$$\boxed{k=-4}$$

Then, put $k=-4$ in equn ① ⇒ $(-4+2)x^2+4x+6=0$

$$-2x^2+4x+6=0$$
$$2x^2-4x+6=0$$
$$x^2-2x-3=0$$
$$x^2-3x+x-3=0$$
$$x(x-3)+1(x-3)=0$$
$$(x-3)(x+1)=0$$

$x-3=0$ or $x+1=0$

$\boxed{x=3}$ or $\boxed{x=-1}$

Thus, the another root of equn ① is found to be $x=-1$.

23.) If $x=p$ is a solution of equn $x(2x+5)=3$, then find the value of p.

→ Given equn is $x(2x+5)=3$

$$2x^2+5x-3=0 \quad \text{——} ①$$
$$2x^2+6x-x-3=0$$
$$2x(x+3)-1(x+3)=0$$
$$(x+3)(2x-1)=0$$

$x+3=0$ or $2x-1=0$

$\boxed{x=-3}$ or $\boxed{x=1/2}$

But, given that $x=p$

Thus, the required values of p for given equation ① are found to be $p=-3$ and $p=1/2$.

$\boxed{\text{Exercise 5.3}}$

1.) i) $2x^2-7x+6=0$

→ Given equation is

$$2x^2-7x+6=0 \quad \text{--- ①}$$

$$2x^2-3x-4x+6=0$$

$$2x^2-4x-3x+6=0$$

$$2x(x-2)-3(x-2)=0$$

$$(x-2)(2x-3)=0$$

$$x-2=0 \text{ or } 2x-3=0$$

$$\boxed{x=2} \text{ or } \boxed{x=3/2}$$

This are the required values of x for given equation ①.

ii) $2x^2-6x+3=0$

→ Given equation is

$$2x^2-6x+3=0 \quad \text{--- ①}$$

Comparing equⁿ ① with $ax^2+bx+c=0$

we get, $a=2$, $b=-6$, $c=3$

$$x=\frac{-(-6)\pm\sqrt{36-4(6)}}{4}$$

$$x=\frac{6\pm\sqrt{36-24}}{4}$$

$$x=\frac{6\pm\sqrt{12}}{4}=\frac{6\pm\sqrt{3\times4}}{4}$$

$$x=\frac{6\pm2\sqrt{3}}{4}=\frac{3\pm\sqrt{3}}{2}$$

$$\boxed{x=\frac{3+\sqrt{3}}{2}} \text{ or } \boxed{x=\frac{3-\sqrt{3}}{2}}$$

This are the required values of x for given equation ①.

2.) i) $25x^2+30x+7=0$

→ Given equⁿ is $25x^2+30x+7=0$ --- ①

on comparing equⁿ ① with $ax^2+bx+c=0$

we get, $a=25$, $b=+30$, $c=7$

Then, $$x=\frac{-(+30)\pm\sqrt{900-700}}{2\times25}$$

$$x=\frac{-30\pm\sqrt{200}}{50}=\frac{-30\pm10\sqrt{2}}{50}=\frac{-3\pm\sqrt{2}}{5}$$

$$\boxed{x=\frac{-3+\sqrt{2}}{5}} \text{ or } \boxed{x=\frac{-3-\sqrt{2}}{5}}$$

This are the required values of x for given equation ①

3) i) $2x^2+\sqrt{5}x-5=0$

→ Given equation is

$2x^2+\sqrt{5}x-5=$ ——①

on compairing equⁿ① with

$ax^2+bx+c=0$

$\Rightarrow a=2,\ b=\sqrt{5},\ c=-5$

$b^2-4ac = 5-4(2)(-5)$

$= 5+40$

$\boxed{b^2-4ac = 45}$

Now,

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-\sqrt{5}\pm\sqrt{45}}{4}$

$x = \frac{-\sqrt{5}\pm 3\sqrt{5}}{4}$

$x = \frac{-\sqrt{5}+3\sqrt{5}}{4}$ or $x = \frac{-\sqrt{5}-3\sqrt{5}}{4}$

$x = \frac{2\sqrt{5}}{4}$ or $x = \frac{-4\sqrt{5}}{4}$

$\boxed{x = \frac{\sqrt{5}}{2}}$ or $\boxed{x = -\sqrt{5}}$

This are the required roots of given equation ①.

ii) $\sqrt{3}x^2+10x-8\sqrt{3}=0$

→ Given equation is

$\sqrt{3}x^2+10x-8\sqrt{3}=0$ ——①

On compairing equⁿ① with

$ax^2+bx+c=0$

we get $a=\sqrt{3},\ b=10,\ c=-8\sqrt{3}$

$b^2-4ac = 100-4(\sqrt{3})(-8\sqrt{3})$

$=100+32\times 3$

$=100+96$

$\boxed{b^2-4ac = 196}$

Now,

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-10\pm\sqrt{196}}{2\sqrt{3}} = \frac{-10\pm 14}{2\sqrt{3}}$

$x = \frac{-10+14}{2\sqrt{3}}$ or $x = \frac{-10-14}{2\sqrt{3}}$

$x = \frac{4}{2\sqrt{3}}$ or $x = \frac{-24}{2\sqrt{3}}$

$\boxed{x = \frac{2}{\sqrt{3}}}$ or $x = \frac{-12}{\sqrt{3}} = \frac{-3\times 4}{\sqrt{3}}$

$\boxed{x = -4\sqrt{3}}$

This are the required roots of given equation ①.

5) i) $4x^2-4ax+(a^2-b^2)=0$

→ Given equation is $4x^2-4ax+(a^2-b^2)=0$ ——①

On compairing equⁿ① with $ax^2+bx+c=0$

we get $a=4,\ b=-4a,\ c=a^2-b^2$

Now, $b^2-4ac = 16a^2-4(4)(a^2-b^2)$

$=16a^2-16a^2+16b^2$

$b^2-4ac = 16b^2$

$\sqrt{b^2-4ac} = 4b$

Then, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{4a \pm 4b}{8} = \frac{a \pm b}{2}$

$\boxed{x = \frac{a+b}{2}}$ or $\boxed{x = \frac{a-b}{2}}$

This are the required values of x for given equation ①.

6.) i) $x - \frac{1}{x} = 3$, $x \neq 0$

→ Given equation is

$x - \frac{1}{x} = 3$ —— ①

$\frac{x^2-1}{x} = 3$

$x^2 - 1 = 3x$

$x^2 - 3x - 1 = 0$

Here, $a=1$, $b=-3$, $c=-1$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{3 \pm \sqrt{9+4}}{2}$

$x = \frac{3 \pm \sqrt{13}}{2}$

$\boxed{x = \frac{3+\sqrt{13}}{2}}$ or $\boxed{x = \frac{3-\sqrt{13}}{2}}$

This are the required values of x for given eqn ①.

ii) $\frac{1}{x} + \frac{1}{x-2} = 3$, $x \neq 0, 2$

→ Given equation is

$\frac{1}{x} + \frac{1}{x-2} = 3$ —— ①

$\frac{(x-2)+x}{x(x-2)} = 3$

$\frac{2x-2}{x^2-2x} = 3$

$2x - 2 = 3x^2 - 6x$

$3x^2 - 6x - 2x + 2 = 0$

$3x^2 - 8x + 2 = 0$

Here, $a=3$, $b=-8$, $c=+2$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{8 \pm \sqrt{64 - 12(+2)}}{6}$

$x = \frac{8 \pm \sqrt{64-24}}{6} = \frac{8 \pm \sqrt{}}{6}$

$x = \frac{8 \pm \sqrt{4 \times 10}}{6} = \frac{4 \pm \sqrt{10}}{3}$

$\boxed{x = \frac{4+\sqrt{10}}{3}}$ or $\boxed{x = \frac{4-\sqrt{10}}{3}}$

This are the required values of x for given eqn ①.

8.) Solve the following equation by using quadratic equation for x and give your answer. i) $x^2-5x-10=0$

$\rightarrow$ Given equation is $x^2-5x-10=0$ — ①

On comparing equⁿ ① with $ax^2+bx+c=0$

we get, $a=1$, $b=-5$, $c=-10$

Now, $b^2-4ac=(-5)^2-4(1)(-10)=25+40=65$

Now, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{5\pm\sqrt{65}}{2}$

$x=\dfrac{5+\sqrt{65}}{2}$ or $x=\dfrac{5-\sqrt{65}}{2}$

$x=\dfrac{5+8.06}{2}$ or $x=\dfrac{5-8.06}{2}$

$\boxed{x=6.53}$ or $\boxed{x=-1.53}$

These are the required values of x for given equⁿ ①.

9.) i) $4x^2-5x-3=0$

$\rightarrow$ Given equation is $4x^2-5x-3=0$ — ①

On comparing equⁿ ① with $ax^2+bx+c=0$

we get, $a=4$, $b=-5$, $c=-3$

Then, $b^2-4ac=(-5)^2-4(4)(-3)=25+48=73$

Now, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{5\pm\sqrt{73}}{8}$

$x=\dfrac{5+\sqrt{73}}{8}$ or $x=\dfrac{5-\sqrt{73}}{8}$

$x=\dfrac{5+8.54}{8}$ or $x=\dfrac{5-8.54}{8}$

$\boxed{x=1.69}$ or $\boxed{x=-0.44}$

These are the required values of x for given equⁿ ①.

10) i) $x^2-4x-8=0$

$\rightarrow$ Given equn is $x^2-4x-8=0$ —①

On comparing equn① with

$ax^2+bx+c=0$

$a=1,\ b=-4,\ c=-8$

$b^2-4ac=(-4)^2-4(1)(-8)$

$=16+32$

$=48$

Now,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{4\pm\sqrt{16+32}}{2}=\frac{4\pm\sqrt{48}}{2}$$

$$x=\frac{4\pm4\sqrt{3}}{2}=2\pm2\sqrt{3}$$

$\boxed{x=5.465}$ or $\boxed{x=-1.465}$

ii) $x-18/x=6$

$\rightarrow$ Given equn is

$x-\frac{18}{x}=6$ —①

On compairing equn① with $ax^2+bx+c=0$

$a=1,\ b=-1$

$\frac{x^2-18}{x}=6$

$x^2-18=6x$

$x^2-6x-18=0$ —①

$a=1,\ b=-6,\ c=-18$

$b^2-4ac=36-4(1)(-18)$

$=36+72$

$\boxed{b^2-4ac=108}$

Now, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{6\pm\sqrt{108}}{2}$$

$$x=\frac{6\pm3\sqrt{12}}{2}$$

$\boxed{x=8.19}$ or $\boxed{x=-2.19}$

11.) Solve the equn $5x^2-3x-4=0$ & give your answer correct to 3 significant figures.

$\rightarrow$ Given equn is $5x^2-3x-4=0$ —①

On comparing equn① with $ax^2+bx+c=0$

$a=5,\ b=-3,\ c=-4$ $\quad$ $b^2-4ac=9-4(5)(-4)$

$=9+80=89$

Now, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{3\pm\sqrt{89}}{10}$

$x=\frac{3+\sqrt{89}}{10}$ or $x=\frac{3-\sqrt{89}}{10}$

$\boxed{x=1.24}$ or $\boxed{x=-0.643}$

These are the required values of x for given equn①.

Exercise 5.4

1.) Find the discriminant of the following equations & hence find the nature of the roots.

i) $3x^2-5x-2=0$

Here, Given equⁿ is

$3x^2-5x-2=0$ —— ①

On comparing equⁿ① with

$ax^2+bx+c=0$

$\Rightarrow a=3, b=-5, c=-2$

$D=b^2-4ac$

$=(-5)^2-4(3)(-2)$

$=25+24$

$\boxed{D=49}$

$D>0$

$\Rightarrow$ The roots are real & distinct.

ii) $2x^2-3x+5=0$

Here, Given equation is

$2x^2-3x+5=0$ —— ①

On comparing equⁿ① with

$ax^2+bx+c=0$

$\Rightarrow a=2, b=-3, c=5$

$D=b^2-4ac$

$=(-3)^2-4(2)(5)$

$=9-40$

$\boxed{D=-31}$

$D<0$

$\Rightarrow$ The roots are imaginary or not real.

2.) Discuss the nature of the roots of the following quadratic equations:

i) $3x^2-4\sqrt{3}x+4=0$

Given equⁿ is

$3x^2-4\sqrt{3}x+4=0$ —— ①

On comparing equⁿ① with

$ax^2+bx+c=0$

$\Rightarrow a=3, b=-4\sqrt{3}, c=4$

$D=b^2-4ac=(-4\sqrt{3})^2-4(3)(4)$

$=48-48$

$\boxed{D=0}$

Thus, the roots are real & equal.

ii) $x^2-\frac{1}{2}x+4=0$

Given equation is

$x^2-\frac{1}{2}x+4=0$ —— ①

On comparing equⁿ① with

$ax^2+bx+c=0$

$\Rightarrow a=1, b=-1/2, c=4$

$D=b^2-4ac$

$=\frac{1}{4}-4(1)(4)=\frac{1}{4}-16$

$D=(1-64)/4=-63/4$

$\boxed{D=-63/4}$

$D<0$

Thus, the roots are not real or imaginary.

3) Find the nature of the roots of the following quadratic equations.

i) $x^2 - \frac{1}{2}x - \frac{1}{2} = 0$

→ Given quadratic equⁿ is $x^2 - \frac{1}{2}x - \frac{1}{2} = 0$ —— ①

$2x^2 - x - 1 = 0$ —— ②

On compairing equⁿ② with $ax^2 + bx + c = 0$

$a = 2, \quad b = -1, \quad c = -1$

$D = b^2 - 4ac = (-1)^2 - 4(2)(-1)$

$D = 1 + 8 = 9$ $\quad \boxed{D = 9}$ $\quad D > 0$

⇒ Thus, the roots are real and distinct.

ii) $x^2 - 2\sqrt{3}x - 1 = 0$, if real roots exist, find them.

→ Given equation is $x^2 - 2\sqrt{3}x - 1 = 0$ —— ①

On compairing equⁿ① with $ax^2 + bx + c = 0$

⇒ $a = 1, \quad b = -2\sqrt{3}, \quad c = -1$

$D = b^2 - 4ac = (-2\sqrt{3})^2 - 4(1)(-1)$

$= 12 + 4$

$\boxed{D = 16}$ $\quad D > 0$

⇒ Thus, the roots are real & distinct.

4) Without solving the quadratic equation, find the value of 'p' for which the given equations have real & equal roots:

i) $px^2 - 4x + 3 = 0$

Given equation is $px^2 - 4x + 3 = 0$ —— ①

Here, $a = p, \; b = -4, \; c = 3$

Given that equⁿ① has real & equal roots ⇒ $\boxed{D = 0}$

$D = b^2 - 4ac$

$0 = (-4)^2 - 4(p)(3)$

$0 = 16 - 12p$ $\quad \boxed{p = 4/3}$

$12p = 16$

$p = 16/12 = 4/3$

ii) $x^2 + (p-3)x + p = 0$

Here, $a = 1, \; b = (p-3), \; c = p$

Given that, roots are real & equal. ⇒ $D = 0$

$D = b^2 - 4ac$

$0 = p^2 - 6p + 9 - 4p$

$p^2 - 10p + 9 = 0$

$$p^2 - 10p + 9 = 0$$
$$p^2 - 9p - p + 9 = 0$$
$$p(p-9) + 1(p-9) = 0$$
$$(p-9)(p+1) = 0$$
$$p - 9 = 0 \quad \text{or} \quad p + 1 = 0$$
$$\boxed{p = 9} \quad \text{or} \quad \boxed{p = +1}$$

5) i) Find the value of k for which following quadratic equation has equal roots : $x^2 + 4kx + (k^2 - k + 2) = 0$

→ Given quadratic equation is
$$x^2 + 4kx + (k^2 - k + 2) = 0 \quad \text{——} \ (1)$$
On comparing equ^n (1) with $ax^2 + bx + c = 0$
$$\Rightarrow a = 1, \ b = 4k, \ c = k^2 - k + 2$$
Given, that roots of equ^n (1) are equal. $\Rightarrow \boxed{D = 0}$
$$D = b^2 - 4ac = (4k)^2 - 4(1)(k^2 - k + 2)$$
$$= 16k^2 - 4(k^2 - k + 2)$$
$$= 16k^2 - 4k^2 + 4k - 8$$
$$0 = 12k^2 + 4k - 8$$
$$\Rightarrow 3k^2 + k - 2 = 0$$
$$3k^2 + 3k - 2k - 2 = 0$$
$$3k(k+1) - 2(k+1) = 0$$
$$(k+1)(3k-2) = 0$$
$$k + 1 = 0 \quad \text{or} \quad 3k - 2 = 0$$
$$\boxed{k = -1} \quad \text{or} \quad \boxed{k = 2/9}$$
These are the required values of k for given equation.

6) Find the values of m for which the following quadratic equation has real & equal roots.

$(3m+1)x^2 + 2(m+1)x + m = 0$

$\rightarrow$ Given quadratic equation is $(3m+1)x^2+2(m+1)x+m=0$ —— ①

On comparing equⁿ① with $ax^2+bx+c=0$

$\Rightarrow a = (3m+1)$, $b = 2(m+1)$, $c = m$

Then, $D = b^2 - 4ac$

$= 2^2(m+1)^2 - 4(3m+1)(m)$

$= 4(m^2+2m+1) - (12m^2+4m)$

$= 4m^2 + 8m + 4 - 12m^2 - 4m$

$D = -8m^2 + 4m + 4$

But, given that equation ① has real & equal roots.

$\Rightarrow$ $D = 0 \Rightarrow -8m^2+4m+4=0$

$2m^2 - m - 1 = 0$

$2m^2 - 2m + m - 1 = 0$

$2m(m-1) + 1(m-1) = 0$

$(m-1)(2m+1) = 0$

$m-1=0$ or $2m+1=0$

$\boxed{m=1}$ or $\boxed{m=-1/2}$

These are the required values of 'm' for given equation ①.

7) Find the values of k for which following quadratic equation has equal roots: $9x^2+kx+1=0$

$\rightarrow$ Given quadratic equation is $9x^2+kx+1=0$ —— ①

On comparing equⁿ① with $ax^2+bx+c=0$

$\Rightarrow a=9$, $b=k$, $c=1$

Then, $D = b^2-4ac = k^2-36$

But, given that roots are equal. $\Rightarrow \boxed{D=0}$

$k^2-36=0$

$k^2=36$

$\boxed{k=\pm 6}$

8.) Find the values of p for which the quadratic equation $(2p+1)x^2-(7p+2)x+(7p-3)=0$ has equal roots. Also find these roots.

→ Given quadratic equation is

$$(2p+1)x^2-(7p+2)x+(7p-3)=0 \quad \text{--- (1)}$$

On comparing equ^n (1) with $ax^2+bx+c=0$

$\Rightarrow \quad a=(2p+1), \quad b=-(7p+2), \quad c=7p-3$

$$D=b^2-4ac$$
$$=(7p+2)^2-4(2p+1)(7p-3)$$
$$=49p^2+28p+4-4(14p^2-6p+7p-3)$$
$$=49p^2+28p+4-56p^2+24p-28p+12$$
$$=-7p^2+28p-4p+16$$
$$\boxed{D=-7p^2+24p+16}$$

But, given that, the roots are equal. $\Rightarrow \boxed{D=0}$

$$\Rightarrow -7p^2+24p+16=0$$
$$7p^2-24p-16=0$$
$$7p^2-28p+4p-16=0$$
$$7p(p-4)+4(p-4)=0$$
$$(p-4)(7p+4)=0$$
$$\boxed{p=4} \quad \text{or} \quad \boxed{p=-4/7}$$

These are the required values of p for given equation (1).

$\boxed{p=4} \Rightarrow$

$$9x^2-30x+25=0$$
$$9x^2-15x-15x+25=0$$
$$3x(3x-5)-5(3x-5)=0$$
$$(3x-5)(3x-5)=0$$
$$3x-5=0 \quad \text{or} \quad 3x-5=0$$
$$\boxed{x=5/3} \quad \text{or} \quad \boxed{x=5/3}$$

These are the roots of given equation (1).

9.) Find the values of p for which the equation $2x^2+3x+p=0$ has real roots.

$\longrightarrow$ Given quadratic equation is $2x^2+3x+p=0$ —— ①

On compairing ① with $ax^2+bx+c=0$

$\therefore a=2, \quad b=3, \quad c=p$

Then, $D=b^2-4ac$

$= 9-4(2)p$

$\boxed{D=9-8p}$ But given that roots of equ^n ① are equal.

$\Rightarrow D=0$

$9-8p=0$

$8p=+9$

$\boxed{p=9/8}$ is the required value of p when roots are real & equal.

But, when roots are real only $\Rightarrow D \geq 0$

$9-8p \geq 0$

$9 \geq 8p$

$\boxed{p \leq 9/8}$ is the required value of p.

10.) Find the least positive value of k for which the equ^n $x^2+kx+4=0$ has real roots.

$\longrightarrow$ Given quadratic equation is $x^2+kx+4=0$ —— ①

On compairing equ^{n}① with $ax^2+bx+c=0$

$\therefore a=1, \quad b=k, \quad c=4$

$D=b^2-4ac = k^2-16$

But, given that roots are real $\Rightarrow D \geq 0$

$\Rightarrow \quad k^2-16 \geq 0$

$k^2 \geq 16$

$k \geq 4$

$\boxed{k=4}$ is the required least positive value of for equation ①.

11.) Find the values of p for which the equation $3x^2-px+5=0$ has real roots.

→ Given quadratic equation is $3x^2-px+5=0$ — ①

On comparing equ^n ① with $ax^2+bx+c=0$

$\Rightarrow a=3,\ b=-p,\ c=5$

$D=b^2-4ac=p^2-4(3)(5)$

$D=p^2-60$

But, given that roots are real $\Rightarrow D\geq 0$

$\Rightarrow p^2-60\geq 0$

$p^2\geq 60$

$p\geq \pm\sqrt{60}$

$p\geq \pm 2\sqrt{15}$

$\boxed{p\geq +2\sqrt{15}}$ or $\boxed{p\leq -2\sqrt{15}}$

These are the required values of p for given equ^n ①.

Exercise 5.5

1.) Find the two consecutive natural numbers such that the sum of their squares is 61.

→ Let us consider 'x' is the first natural no.

Then $(x+1)$ is next consecutive natural no.

From given condition, we can write

$x^2+(x+1)^2=61$

$x^2+x^2+2x+1=61$

$2x^2+2x+1=61$

$2x^2+2x-60=0$

$x^2+x-30=0$

$x^2+6x-5x-30=0$

$x(x+6)-5(x+6)=0$

$(x+6)(x-5)=0$

$\boxed{x=-6}$ or $\boxed{x=5}$ Then $x+1=5+1=6$

Thus, the required consecutive natural no. are 5 & 6.

2.) i) If the product of two consecutive even integers is 224, find the integers.

→ Let us consider first even integer be '$2x$'.

Then next consecutive even integer be $(2x+2)$.

from given condition,

$$2x(2x+2) = 224$$
$$4x^2+4x = 224$$
$$4x^2+4x-224=0$$
$$x^2+x-56=0$$
$$x^2+8x-7x-56=0$$
$$x(x+8)-7(x+8)=0$$
$$(x+8)(x-7)=0$$
$$x+8=0 \text{ or } x-7=0$$
$$\boxed{x=-8} \text{ or } \boxed{x=7}$$ Then $2x = 14$

& $2x+2 = 14+2 = 16$

Thus, the required two consecutive even integers are 14 & 16 respectively.

ii) Find the two consecutive odd integers such that the sum of their squares is 394.

→ Let us consider the first odd integer be $(2x+1)$

Then next consecutive odd integer be $(2x+3)$.

from given condition,

$$(2x+1)^2+(2x+3)^2 = 394$$
$$4x^2+4x+1+4x^2+12x+9 = 394$$
$$8x^2+16x+10-394=0$$
$$8x^2+16x-384=0$$
$$x^2+2x-48=0$$
$$x^2+8x-6x-48=0$$
$$x(x+8)-6(x+8)=0$$
$$(x+8)(x-6)=0$$
$$x+8=0 \text{ or } x-6=0$$
$$\boxed{x=-8} \text{ or } \boxed{x=6}$$

Then, first odd integer is $(2x+1) = (12+1) = 13$
And next consecutive odd integer is $(2x+3) = (12+3) = 15$
Thus, the required two consecutive odd integers are 13 & 15 respectively.

3) The sum of the two numbers is 9 and the sum of their squares is 41. Taking one number as x, from all equation in x and solve it to find the numbers.

→ Given that, The sum of two numbers is 9.
And sum of their squares is 41.
Let us consider the first number be 'x'.
Then next or second number be $(9-x)$.

Then,
$$x^2 + (9-x)^2 = 41$$
$$x^2 + 81 - 18x + x^2 - 41 = 0$$
$$2x^2 - 18x + 40 = 0$$
$$x^2 - 9x + 20 = 0$$
$$x^2 - 4x - 5x + 20 = 0$$
$$x(x-4) - 5(x-4) = 0$$
$$(x-4)(x-5) = 0$$
$$\boxed{x=4} \text{ or } \boxed{x=5}$$

Then, the first number be $x = 4$
And second number be $= 9 - x = 9 - 4 = 5$.
Thus, the two required numbers are found to be 4 & 5.

4) Five times a certain whole number is equal to three less than twice the square of the number. Find the no.

→ Let us consider 'x' be the required no.
From given condition, $5x = 2x^2 - 3$
$$2x^2 - 3 - 5x = 0$$
$$2x^2 - 5x - 3 = 0$$
$$2x^2 - 6x + x - 3 = 0$$
$$2x(x-3) + 1(x-3) = 0$$

$(2x+1)(x-3)=0$

$2x+1=0$ or $x-3=0$

$\boxed{x=-1/2}$ or $\boxed{x=3}$

Then, the required no. is found to be $x=3$.

5) Sum of two natural numbers is 8 and the difference of their reciprocal is 2/15. Find the numbers.

→ Let us consider the two natural no. x & y.

From given condition,

$x+y=8 \Rightarrow y=8-x$

$\frac{1}{x}-\frac{1}{y}=\frac{2}{15}$ —— ①

put $y=8-x$ in eqn ① $\frac{1}{x}-\frac{1}{(8-x)}=\frac{2}{15}$

$$\frac{(8-x)-x}{x(8-x)}=\frac{2}{15}$$

$$\frac{(8-2x)}{(8x-x^2)}=\frac{2}{15}$$

$$120-30x=16x-2x^2$$

$$2x^2-16x-30x+120=0$$

$$2x^2-46x+120=0$$

$$x^2-23x+60=0$$

$$x^2-20x-3x+60=0$$

$$x(x-20)-3(x-20)=0$$

$$(x-20)(x-3)=0$$

$x-20=0$ or $x-3=0$

$\boxed{x=20}$ or $\boxed{x=3}$

~~put~~ $x\neq 20$ · put $x=3$ in $x+y=8$

$3+y=8$

$\boxed{y=5}$

Thus, the required two numbers are 3 & 5 respectively.

7) There are three consecutive positive integers such that the sum of the square of the first & the product of the other two is 154. What are the integers?

→ Let us consider the first integer be 'x'.

Then second consecutive integer be '$(x+1)$'.

Then Third consecutive integer be '$(x+2)$'.

from given condition,

$$x^2 + (x+1)(x+2) = 154$$
$$x^2 + (x^2+2x+x+2) = 154$$
$$2x^2+3x+2 = 154$$
$$2x^2+3x+2-154 = 0$$
$$2x^2+3x-152 = 0$$
$$2x^2+19x-16x-152 = 0$$
$$x(2x+19) - 8(2x+19) = 0$$
$$(2x+19)(x-8) = 0$$
$$2x+19 = 0 \quad \text{or} \quad x-8 = 0$$
$$\boxed{x = -19/2} \quad \text{or} \quad \boxed{x=8}$$

$\boxed{x=8}$ is the first positive integer.

Then nex two consecutive positive integers are 9 & 10 respect.

9.) In a certain positive fraction, the denominator is greater than the numerator by 3. If 1 is subtracted from both the numerator & denominator, the fraction is decreased by $\frac{1}{14}$. Find the fraction.

→ Let us consider the numerator of a fraction be 'x'.

Then from given condition, denominator be $(x+3)$.

from given condition,

fraction → $\frac{x}{x+3}$ $\qquad \frac{x-1}{x+2} = F - \frac{1}{14}$

$$\frac{x-1}{x+2} = \frac{x}{x+3} - \frac{1}{14}$$

$$\frac{x-1}{x+2} - \frac{x}{x+3} = -\frac{1}{14}$$

$$\frac{(x-1)(x+3) - x(x+2)}{(x+2)(x+3)} = -\frac{1}{14}$$

$$\frac{(x^2+3x-x-3) - (x^2+2x)}{(x^2+3x+2x+6)} = -\frac{1}{14}$$

$$\frac{(3x-x-3-2x)}{(x^2+5x+6)} = -\frac{1}{14}$$

$$\frac{-3}{x^2+5x+6} = -\frac{1}{14}$$

$$42 = x^2+5x+6$$

$$x^2+5x+6-42=0$$

$$x^2+5x-36=0$$

$$x^2+9x-4x-36=0$$

$$x(x+9)-4(x+9)=0$$

$$(x+9)(x-4)=0$$

$$x+9=0 \text{ or } x-4=0$$

$\boxed{x=-9}$ or $\boxed{x=4}$ Then denominator $= x+3$
$= 7$

Thus, the required fraction is $4/7$.

{ since, $\frac{4-1}{7-1} = \frac{3}{6}$ & $\frac{4}{7} - \frac{3}{6} = \frac{24-21}{42} = \frac{3}{42} = \frac{1}{14}$ }

11.) A two digit no. contains the bigger at ten's place. The product of the digits is 27 and the difference between two digits is 6. find the number.

→

Let us consider unit digit of that number is 'x'.

Then ten's digit of that number is '$x+6$'.

Number $= x+10(x+6)$

$= x+10x+60$

$\boxed{N = 11x+60}$

From given condition,

$x(x+6) = 27$

$x^2+6x-27=0$

$x^2+9x-3x-27=0$

$x(x+9)-3(x+9)=0$

$(x+9)(x-3)=0$

$x+9=0$ or $x-3=0$

$\boxed{x=-9}$ or $\boxed{x=3}$

$x=-9$ is not possible.

Thus put $x=3$ in $N=11x+60$

$= 11(3)+60 = 33+60 = 93$

Hence, the required number is found to be 93.

12.) A two digit positive number is such that the product of its digit is 6. If 9 is added to the number, the digits interchange their places. Find the number.

→ Let us consider the two digit no. is $xy = 10x+y$

After reversing the digits $= yx = 10y+x$

From given condition,

$10x+y+9 = 10y+x$ and $xy=6$

put $y=6/x \Rightarrow 10x+6/x+9 = 10(6/x)+x$

$10x^2+6+9x = 60+x^2$

$9x^2+9x-54=0$

$x^2+x-6=0$

$x^2+3x-2x-6=0$

$$x^2+3x-2x-6=0$$
$$x(x+3)-2(x+3)=0$$
$$(x+3)\ (x-2)=0$$
$$x+3=0 \text{ or } x-2=0$$
$$\boxed{x=-3} \text{ or } \boxed{x=2}$$

The required value of x is 2.

Then $y = 6/2 = 3$

Hence, the requred number is $= 10x+y$
$= 20+3$
$\boxed{N=23}$

14) A rectangular garden 10m by 16m is to be surrounded by a concrete walk of uniform width. Given that the area of the walk is 120 square meters, assuming the width of the walk to be x, form an equation in x and solve it to find the value of x.

→ Given that, Length of garden $=16$m
width of garden $=10$m

Let us consider the width of the walk be 'x'm

Then, outer length $=16+2x$
and outer width $=10+2x$

From given condition,

$$(16+2x)\ (10+2x) - 16(10) = 120$$
$$160+32x+20x+4x^2-160-120=0$$
$$4x^2+52x-120=0$$
$$x^2+13x-30=0$$
$$x^2+15x-2x-30=0$$
$$x(x+15)-2(x+15)=0$$
$$(x+15)(x-2)=0$$

$x+15=0$ or $x-2=0$

$\boxed{x=-15}$ or $\boxed{x=2}$

$x=2$ is the requred value

16.) The perimeter of a rectangular plot is 180m and its area is 1800 m^2. Take the length of the plot as x m. Use the perimeter 180m to write the value of breadth in terms of x. Use the values of length, breadth and the area to write an equation in x. Solve the equation to calculate the length & breadth of the plot.

→ Given that, the perimeter of a rectangular field = 180m.

Area = 1800 m^2

Let us consider length of rectangular plot be 'x'.

Then, Perimeter = 2 (length + breadth)

$180 = 2(x + \text{breadth})$

$x + \text{breadth} = 90$

$\boxed{\text{breadth} = 90 - x}$

Then, Area = length × breadth

$1800 = x(90-x)$

$90x - x^2 = 1800$

$x^2 - 90x + 1800 = 0$

$x^2 - 60x - 30x + 1800 = 0$

$x(x-60) - 30(x-60) = 0$

$(x-60)(x-30) = 0$

$\boxed{x=30}$ or $\boxed{x=60}$

Thus, length > breadth

Hence, length = 60 m

Breadth = 90 − 60 = 30m

18.) If the perimeter of a rectangular plot is 68m and the length of its diagonal is 26m, find its area.

→ Given that, The perimeter of rectangular plot = 68m.

Length of diagonal of plot = 26 m.

Perimeter = $2(l+b)$

length + breadth = 68/2

$\boxed{\text{length + breadth} = 34m}$

Let 'x' be the length of rectangular plot

Then breadth = $34-x$

Now, Area = length x breadth

Area = $x(34-x)$ —— ①

But, $\text{length}^2 + \text{breadth}^2 = \text{diagonal}^2$ ∵ By Pythagoras thm

$$x^2 + (34-x)^2 = 26^2$$

$$x^2 + 1156 + x^2 - 68x = 676$$

$$2x^2 - 68x + 1156 - 676 = 0$$

$$2x^2 - 68x + 480 = 0$$

$$x^2 - 34x + 240 = 0$$

$$x^2 - 24x - 10x + 240 = 0$$

$$x(x-24) - 10(x-24) = 0$$

$$(x-24)(x-10) = 0$$

$$x - 24 = 0 \text{ or } x - 10 = 0$$

$$\boxed{x = 24} \text{ or } \boxed{x = 10}$$

But length > breadth

Thus, length = 24m

breadth = 34 − 24 = 10m

put in ① ⇒ Area = $24 \times 10 = 240 m^2$

Thus, the area of required rectangular plot is found to be $240 m^2$.

19.) If the sum of two simillar sides of a right-angle triangle is 17cm and the perimeter is 30cm, then find the area of the triangle.

→ Given that, Perimeter of the triangle = 30 cm

Let 'x' be the length of one of two small sides of a triangle.

Then, other side = $17-x$

Area of a triangle $= \frac{1}{2}(5 \times 12) = 60/2 = 30\,cm^2$

Thus, the required area of a given triangle is $30\,cm^2$.

21.) Mohini wishes to fit three rods together in the shape of a right triangle. If the hypotenuse is 2 cm longer than the base and 4cm longer than the shortest side, find the lengths of the rod.

→ Let us consider the length of hypotenuse be 'x' cm.

Then base = $(x-2)$ cm

and shortest side = $(x-4)$ cm

from given condition,

$$x^2 = (x-2)^2 + (x-4)^2$$
$$x^2 = x^2 - 4x + 4 + x^2 - 8x + 16$$
$$x^2 = 2x^2 - 12x + 20$$
$$2x^2 - 12x + 20 - x^2 = 0$$
$$x^2 - 10x - 2x + 20 = 0$$
$$x(x-10) - 2(x-10) = 0$$
$$(x-10)(x-2) = 0$$
$$x-10=0 \quad \text{or} \quad x-2=0$$
$$\boxed{x=10} \quad \text{or} \quad \boxed{x=2}$$

Here, Hypotenuse = 10 cm

Base = $10-2 = 8$ cm

And Shortest side = $10-4 = 6$ cm

22.) In a P.T. display, 480 students are arranged in rows and columns. If there are 4 more students in each row than the number of rows, find the number of students in each row.

→ Given that, the total no. of students = 480

Let 'x' be the number of students in each row.

Then, the no. of rows = $480/x$.

from given condition,

$$x = (480/x) + 4$$

$$x^2 = 480 + 4x$$

$$x^2 - 4x - 480 = 0$$

$$x^2 - 24x + 20x - 480 = 0$$

$$x(x-24) + 20(x-24) = 0$$

$$(x-24)(x+20) = 0$$

$$\boxed{x = 24} \quad \text{or} \quad \boxed{x = -20}$$

Thus, the total no. of students in each row = 24.

24.) At an annual function of a school, each student gives the gift to every other student. If the number of gifts is 1980, find the number of students.

→ Let us consider the total no. of students are 'x'.

Then no. of gifts given = $x - 1$

Total no. of gifts = $x(x-1)$

From given condition,

$$x(x-1) = 1980$$

$$x^2 - x - 1980 = 0$$

$$x^2 - 45x + 44x - 1980 = 0$$

$$x(x-45) + 44(x-45) = 0$$

$$(x-45)(x+44) = 0$$

$$x - 45 = 0 \quad x + 44 = 0$$

$\boxed{x = 45}$ or $\boxed{x = -44}$

Thus, the total no. of students $= 45$.

26.) The speed of an express train is x km/hr and the speed of an ordinary train is 12 km/hr less than that of the express train. If the ordinary train takes one hour longer than the express train to cover ~~to~~ a distance of 240 km, find the speed of the express train.

→ Let us consider the speed of the express train $= x$ km

Then, the speed of ordinary train $= (x-12)$ km

Time taken to cover 240 km by the express.

Then, time require to cover for each train is $240/x$ & $\frac{240}{(x-12)}$ respectively.

from given condition,

$$240x - 12 - 240x = 1$$

$$240x - 240(x-12)\ (x-12)\ x = 1$$

$$240x - 240(x-12) = x(x-12)$$

$$x^2 - 12x - 2880 = 0$$

$$(x-60)\ (x+48) = 0$$

$$\boxed{x = 60 \text{ km/hr}}$$

Thus, the speed of the express train is 60 km/hr.

30) An aeroplane flying with a wind of 30 km/hr takes 40 minutes less to fly 3600 km, than what it would have taken to fly against the same wind. find the planes speed of flying in still air.

→ Let us consider the speed of the plane in still air = x km/hr

And speed of wind = 30 km/hr

Distance = 3600 km

Hence, Time taken with the wind = $\frac{3600}{x+30}$

Time taken against the wind = $\frac{3600}{x-30}$

from given condition,

$$\frac{3600}{(x-30)} - \frac{3600}{(x+30)} = 40 \text{ minutes} = \frac{2}{3} \text{ hour}$$

$$3600\left(\frac{1}{x-30} - \frac{1}{x+30}\right) = \frac{2}{3}$$

$$3600\left(\frac{x+30-x+30}{(x-30)(x+30)}\right) = \frac{2}{3}$$

$$\frac{3600 \times 60}{x^2-900} = \frac{2}{3}$$

$$2x^2 - 1800 = 3 \times 3600 \times 60$$

$$2x^2 - 1800 = 64,8000$$

$$2x^2 - 1800 - 648000 = 0$$

$$2x^2 - 649800 = 0$$

$$x^2 - 324900 = 0$$

$$x^2 - (570)^2 = 0$$

$$(x+570)(x-570) = 0$$

$$x+570 = 0 \quad \text{or} \quad x-570 = 0$$

$$x = -570 \quad \text{or} \quad \boxed{x = 570}$$

Thus, the speed of the plane in still air = 570 km/hr

32.) A boat can cover 10 km up the stream and 5 km down the stream in 6 hours. If the speed of the stream is 1.5 km/hr. Find the speed of the boat in still water.

→ Given that, Distance up stream = 10 km

Distance down stream = 5 km

Total time taken = 6 hours

Speed of the stream = 1.5 km/hr

Speed of the boat in still water is

let us consider the speed of the boat in still water is x km/hr

from given condition,

$$\frac{10}{x-1.5} + \frac{5}{x+1.5} = 6$$

$$10x + 15 + 5x + 5x - 7.5 = 6(x-15)(x+15)$$

$$15x + 7.5 = 6(x^2 - 2.25)$$

$$15x + 7.5 = 6x^2 - 13.5$$

$$6x^2 - 15x - 13.5 - 7.5 = 0$$

$$6x^2 - 15x - 21 = 0$$

$$2x^2 - 5x - 7 = 0$$

$$2x^2 - 7x + 2x - 7 = 0$$

$$x(2x-7) + 1(2x-7) = 0$$

$$(2x-7)(x+1) = 0$$

$$2x = 7 \text{ or } \boxed{x = -1}$$

$$\boxed{x = 7/2}$$

Thus, $x = 7/2 = 3.5$

Thus, the speed of the boat in still water is found to be 3.5 km/hr

36) A trader buys x articles for a total cost of Rs. 600.
i) Write down the cost of one article in terms of x.
If the cost per article were Rs. 5 more, the number of articles that I can be bought for Rs. 600 would be four less.
ii) Write down the equation in x for the above situation and solve it to find x.

→ Given that, The total cost of x articles = 600 Rs.

No. of articles bought = x

Then, Cost of one article = $600/x$

from given condition,

$$\frac{600}{(x-4)} - \frac{600}{x} = 5$$

$$\Rightarrow x^2 - 4x - 480 = 0$$

$$x^2 - 24x + 20x - 480 = 0$$

$$x(x-24) + 20(x-24) = 0$$

$$(x-24)(x+20) = 0$$

$$\boxed{x = 24} \text{ or } \boxed{x = -20}$$

Thus, the total no. of articles are found to be 24.

39.) The hotel bill for a number of people for an overnight stay is Rs. 4800. If there were 4 more, the bill each person had to pay would have reduced by Rs. 200. Find the no. of people staying overnight.

→ Let us consider the no. of people staying overnight be 'x'.

Amount of bill = 4800 Rs.

$\Rightarrow$ Bill for each person = $4800/x$

From second condition,

The total no. of people = $x+4$

Then bill for each person = $4800/(x+4)$

From the given condition, we can write

$$\frac{4800}{x} - \frac{4800}{(x+4)} = 200$$

$$4800\left[\frac{1}{x} - \frac{1}{(x+4)}\right] = 200$$

$$\Rightarrow 19200 = 200x^2 + 800x$$

$$200x^2 + 800x - 19200 = 0$$

$$x^2 + 4x - 96 = 0$$

$$x^2 + 12x - 8x - 96 = 0$$

$$x(x+12) - 8(x+12) = 0$$

$$(x+12)(x-8) = 0$$

$$\boxed{x = -12} \text{ or } \boxed{x = 8}$$

Thus, the total no. of people staying in hotel are found to be 8.

42.) The sum of the ages of Vivek and his younger brother Amit is 47 years. The product of their ages in years is 550. Find their ages.

→ Let us consider the Vivek's present age be 'x'. years.

Then, His brother's age will be $(47-x)$ years.

From given condition,

$$x(47-x) = 550$$

$$47x - x^2 = 550$$

$$x^2 - 47x + 550 = 0$$

$$x^2 - 25x - 22x + 550 = 0$$

$$x(x-25) - 22(x-25) = 0$$

$$(x-22)(x-25) = 0$$

$$\boxed{x = 22} \text{ or } \boxed{x = 25}$$

When $x=25$ then $47-x=47-25=22$
When $x=22$ then $47-x=47-22=25$
Hence, the Vivek's age is found to be $x=25$ years.
His younger brother's age be 22 years.

44.) Two years ago, a man's age was three times the square of his daughter's age. Three years hence, his age will be four times his daughter's age. Find their present ages.

→ Given that, 2 years ago
Let us consider the age of daughter be 'x'.
Then age of the man $=3x^2$

At present, the age of daughter be $(x+2)$
and age of man $=(3x^2+2)$

After 3 years again, the age of daughter $=x+2+3$
$=(x+5)$
and the age of man $=3x^2+2+3=(3x^2+5)$
From given condition,

$$3x^2+5=4(x+5)$$
$$3x^2+5=4x+20$$
$$3x^2-4x+5-20=0$$
$$3x^2-4x-15=0$$
$$3x^2-9x+5x-15=0$$
$$3x(x-3)+5(x-3)=0$$
$$(x-3)(3x+5)=0$$
$$\boxed{x=3} \text{ or } \boxed{x=-5/3}$$

If $x=3$ then present age of man $=3x^2+2$

present age of man $= 3x^2+2 = 27+2 = 29$ years

And age of daughter $= x+2 = 3+2 = 5$ years.

45.) The length of the hypotenuse of a right-angled triangle exceeds the length of one side by 2 cm and exceeds twice the length of another side by 1cm. Find the length of each side. Also, find the perimeter and the area of the triangle.

$\rightarrow$ Let us consider the length of one side of triangle be 'x' cm.

And of another side be y cm.

Then, hypotenuse $= x+2$ and $(2y+1)$.

$$\therefore \quad x+2 = 2y+1$$
$$x-2y-1+2=0$$
$$x-2y=-1$$
$$\boxed{x=2y-1} \quad \text{—— (1)}$$

By Pythagoras Theorem,

$$x^2+y^2 = (2y+1)^2$$
$$x^2+y^2 = 4y^2+4y+1$$
$$(2y-1)^2+y^2 = 4y^2+4y+1$$
$$4y^2-4y+1+y^2-4y^2-4y-1=0$$
$$y^2-8y=0$$
$$y(y-8)=0$$
$$\boxed{y=8} \text{ put in (1)} \Rightarrow x = 2y-1$$
$$= 16-1$$
$$\boxed{x=15}$$

Hence, Length of one side $= 15$ cm

length of another side $= 8$ cm

Hypotenuse $= x+2 = 15+2 = 17$ cm

Area of triangle $= \frac{1}{2} \times (8)(15) = 60 \text{ cm}^2$.