Chapter 11. Section Formula

1.) find the coordinates of the mid-point of the line segments joining the following pairs of points:

i) (2,-3), (-6,7)

ii) (5,-11), (4,3)

iii) (a+3,5b), (2a-1,3b+4)

→ i) Given two points are (2,-3) and (-6,7).

Mid-point formula is given by,

$$(x,y) \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$(x,y) \equiv \left(\frac{(2-6)}{2}, \frac{(-3+7)}{2}\right) \equiv (-2,2)$$

Thus, the coordinates of the mid-point of line segment joining the points (2,-3) & (-6,7) is found to be (-2,2).

ii) Given two points are (5,-11) and (4,3).

Mid-point formula is given by

$$(x,y) \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$(x,y) \equiv \left(\frac{5+4}{2}, \frac{-11+3}{2}\right) \equiv \left(\frac{9}{2}, \frac{-8}{2}\right) \equiv (4.5,-4)$$

Thus, the co-ordinates of the mid-point of line segment joining the points (5,-11) and (4,3) is found to be (4.5,-4).

iii) Given two points are $(a+3, 5b)$ and $(2a-1, 3b+4)$

Mid-point formula is given by,

$$(x, y) \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\equiv \left(\frac{a+3+2a-1}{2}, \frac{5b+3b+4}{2}\right) \equiv \left(\frac{3a+2}{2}, \frac{8b+4}{2}\right)$$

$$(x, y) \equiv \left(\frac{3a+2}{2}, 4b+2\right)$$

Thus, the co-ordinates of mid-point of line-segment joining the points $(a+3, 5b)$ and $(2a-1, 3b+4)$ is found to be $\left(\frac{3a+2}{2}, 4b+2\right)$.

2.) P divides the distance between $A(-2,1)$ and $B(1,4)$ in the ratio of 2:1. Calculate the co-ordinates of the point P.

→ Given that, point P divides the distance between $A(-2,1)$ and $B(1,4)$ in the ratio of 2:1.

Let us consider the point $P(x,y)$ divides AB in the ratio of $m_1 : m_2$ i.e. 2:1.

$$x = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}\right)$$

$$x = \left(\frac{2\times1 + 1\times(-2)}{2+1}\right) = \left(\frac{2-2}{3}\right) = \frac{0}{3} = 0 \Rightarrow \boxed{x=0}$$

And $$y = \left(\frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right) = \left(\frac{2\times4 + 1\times1}{2+1}\right) = \left(\frac{8+1}{3}\right) = \frac{9}{3} = 3 \Rightarrow \boxed{y=3}$$

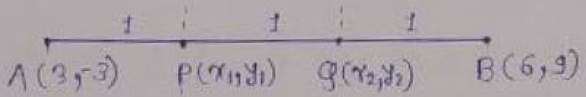
Hence, the co-ordinates of point P are found to be $P \equiv (0,3)$.

3) i) Find the co-ordinates of the points of trisection of the line segment joining the point $(3,-3)$ and $(6,9)$.

ii) The line segment joining the points $(3,-4)$ and $(1,2)$ is trisected at the points P and Q. If the co-ordinates of P and Q are $(p,-2)$ and $(5/3, q)$ respectively, find the values p and q.

→ i) Given that,

- The points of line segment joining the points $(3,-3)$ and $(6,9)$.
- Let us consider the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are the points which trisect the line segment joining points $A(3,-3)$ and $B(6,9)$.



- Here, point $P(x_1, y_1)$ divides AB in the ratio 1:2.

Thus, $x_1 = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}\right) = \left(\frac{1 \times 6 + 2 \times 3}{1+2}\right) = \frac{(6+6)}{3} = \frac{12}{3} = 4$

$\Rightarrow \boxed{x_1 = 4}$

$y_1 = \left(\frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right) = \left(\frac{1 \times 9 + 2 \times (-3)}{1+2}\right) = \left(\frac{9-6}{3}\right) = \frac{3}{3} = 1 \Rightarrow \boxed{y_1 = 1}$

- Thus, the coordinates of point $P(x_1, y_1)$ are found to be $P(4,1)$.
- Let us consider, $Q(x_2, y_2)$ divides the line segment AB in the ratio 2:1.

Thus, $x_2 = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}\right) = \left(\frac{2 \times 6 + 1 \times 3}{2+1}\right) = \left(\frac{12+3}{3}\right) = \frac{15}{3} = 5$

And $y_2 = \left(\frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right) = \left(\frac{2 \times 9 + 1(-3)}{2+1}\right) = \frac{(18-3)}{3} = \frac{15}{3} = 5$

Thus, the coordinates of point $Q(x_2, y_2)$ are found to be $Q(5,5)$.

ii) Given that, the line segment joining the points $(3,-4)$ and $(1,2)$ is trisected at the points P and Q.
And the co-ordinates of point P and Q are $(p,-2)$ and $(5/3, q)$ respectively.

• Here, point $P(p, -2)$ divides lines AB in the ratio 1:2 and point q divides line AB in the ratio 2:1.

Here, $p = \left(\frac{mx_2 + nx_1}{m+n}\right)$ and $q = \left(\frac{my_2 + my_1}{m+n}\right)$

$= \left(\frac{1\times1 + 2\times3}{1+2}\right)$ $= \left(\frac{2\times2 + 1(-4)}{2+1}\right)$

$= \left(\frac{1+6}{3}\right)$ $= \left(\frac{4-4}{2}\right)$

$\boxed{p = 7/3}$ $\boxed{q = 0}$

Hence, the co-ordinates of point q are found to be $q(5/3, 0)$ and $P(7/3, -2)$.

4) The line segment joining the points $A(3,2)$ and $B(5,1)$ is divided at the point P in the ratio 1:2 and it lies on the line $3x - 18y + k = 0$ find the value of k.

→ Let us consider the point $P(x,y)$ divides the line segment joining the points $A(3,2)$ and $B(5,1)$ in the ratio 1:2.

Thus, $x = \left(\frac{mx_2 + nx_1}{m+n}\right)$ and $y = \left(\frac{my_2 + ny_1}{m+n}\right)$

$= \left(\frac{1\times5 + 2\times3}{1+2}\right)$ $= \left(\frac{1\times1 + 2\times2}{1+2}\right)$

$= \left(\frac{5+6}{2}\right)$ $= \left(\frac{1+4}{3}\right)$

$\boxed{x = 11/3}$ $\boxed{y = 5/3}$

Given that point P is on the line $3x - 18y + k = 0$ —— ①

put $x = 11/3$ and $y = 5/3$ in equn ①.

$\Rightarrow 3(11/3) - 18(5/3) + k = 0$

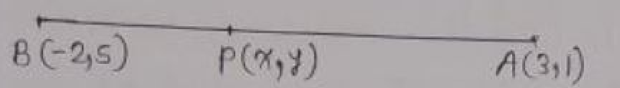
$11 - 30 + k = 0$

$-19 + k = 0$

$\boxed{k = 19}$

5.) find the coordinates of the point which is three-fourth of the way from A(3,1) to B(-2,5).

$\rightarrow$



• Given that, the points having coordinates A(3,1) and B(-2,5) are the end points of the line segment.

Let us consider, P(x,y) be the point which is three-fourth of the way from A to B.

$$\Rightarrow \frac{AP}{AB} = \frac{3}{4} \Rightarrow \frac{AP}{(AP+PB)} = \frac{3}{4}$$

$$4AP = 3AP + 3PB$$

$$4AP - 3AP = 3PB$$

$$\Rightarrow \boxed{AP = 3PB}$$

Hence, $\frac{AP}{PB} = \frac{3}{1} \Rightarrow m_1 = 3, m_2 = 1$

Now, to find the co-ordinates of point P(x,y):

$$x = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}\right) \quad \text{and} \quad y = \left(\frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{3\times(-2) + 1\times 3}{3+1}\right) \quad\quad = \left(\frac{3\times 5 + 1\times 1}{3+1}\right)$$

$$= \left(\frac{-6+3}{4}\right) \quad\quad = \left(\frac{15+1}{4}\right) = 16/4$$

$$\boxed{x = -3/4} \quad\quad \boxed{y = 4}$$

Thus, the coordinates of point P will be $(-3/4, 4)$.

6.) The line segment joining points A(-3,1) and B(5,-4) is a diameter of a circle whose centre is C. find the co-ordinates of the point C.

$\rightarrow$ Given that, the line segment joining points A(-3,1) and B(5,-4) is a diameter of a circle whose centre is C.

i.e. C is the midpoint of AB.

Let us consider the point C(x,y).

$$\therefore x = \frac{(-3+5)}{2} = \frac{(2)}{2} = \frac{2}{2} = 1$$

$$y = \frac{(1-4)}{2} = -3/2$$

Thus, the coordinates of point C are found to be $(1, -3/2)$.

7) The mid-point of the line segment joining the points $(3m, 6)$ and $(-4, 3n)$ is $(1, 2m-1)$. find the values of m and n.

→ Let us consider, the point $P(1, 2m-1)$ is the mid-point of the line segment joining the points $(3m, 6)$ and $(-4, 3n)$.

$\Rightarrow 1 = \frac{(x_1+x_2)}{2}$

$1 = \frac{(3m-4)}{2}$

$\Rightarrow 2 = 3m-4$

$m = 6/3$

$\boxed{m=2}$

and

$2m-1 = (6+3n)/2$

$4m-2 = 6+3n$

$\Rightarrow 4\times2-2 = 6+3n$

$8-2 = 6+3n$

$3n = 8-2-6$

$\boxed{n=0}$

Thus, Here the values of m and n are found to be $\boxed{m=2}$ and $\boxed{n=0}$.

8) The coordinates of the mid-point of the line segment PQ are $(1, -2)$. The coordinates of P are $(-3, 2)$. find the coordinates of Q.

→ Let us consider the co-ordinates of point Q be (x, y)

Given that, the coordinates of the mid-point of the line segment PQ are $(1, -2)$. and $P(-3, 2)$

$\Rightarrow 1 = \frac{(-3+x)}{2}$

$-3+x = 2$

$x = 2+3 = 5$

$\boxed{x=5}$

and

$-2 = (2+y)/2$

$-4 = 2+y$

$y = -4-2$

$\boxed{y=-6}$

Thus, the co-ordinates of point Q are found to be $Q(5, -6)$.

9.) AB is a diameter of a circle with centre C(-2,5). If point A is (3,-7). Find i) the length of radius AC.
ii) the co-ordinates of B.

→ Here, given that
AB is a diameter of a circle with centre C(-2,5).
and point A having coordinates A(3,-7).

i) Thus, By distance formula

$$AC = \sqrt{(3+2)^2 + (-7-5)^2} = \sqrt{5^2 + 12^2} = \sqrt{25+144} = \sqrt{169} = 13$$

Thus, the length of radius AC is found to be 13 units.

ii) Let us consider, co-ordinates of point B are (x, y).

$\therefore \frac{(3+x)}{2} = -2$ and $\frac{(y-7)}{2} = 5$

$3 + x = -4$ $\quad$ $y = 10 + 7$

$\boxed{x = -7}$ $\quad$ $\boxed{y = 17}$

Thus, the co-ordinates of point B are found to be B(-7, 17).

10.) Find the reflection (image) of the point (5,-3) in the point (-1,3).

→ Let us consider, the co-ordinates of the image of the point A(5,-3) be A'(x,y) in the point (-1,3).

⇒ Point (-1,3) is the midpoint of AA'.

⇒ $-1 = (5+x)/2$ and $3 = (-3+y)/2$

$5 + x = -2$ $\quad$ $-3 + y = 6$

$\boxed{x = -7}$ $\quad$ $\boxed{y = 9}$

Hence, the co-ordinates of the image A will be A(-7,9).

11.) The line segment joining $A(-1, 5/3)$ the points $B(a, 5)$ is divided in the ratio 1:3 at P, the point where the line segment AB intersects y-axis. Calculate i) the value of a
ii) the coordinates of P.

→ Given that, the line segment joining $A(-1, 5/3)$ the points $B(a, 6)$ is divided in the ratio 1:3 at P.

i) Thus, $x = \left(\frac{1 \times a + 3(-1)}{1+3}\right)$ and $y = \frac{(1 \times a + 3(-1))}{(1+3)}$

$x = \frac{(a-3)}{4}$ $\quad = \frac{(a-3)}{4} = \frac{(5+5)}{4} = \frac{10}{4}$

$\boxed{y = 5/2}$

ii) Given that, the line segment AB intersects Y-axis at P.

$\therefore x = 0$

$\Rightarrow \frac{(a-3)}{4} = 0 \quad \Rightarrow a - 3 = 0 \quad \Rightarrow \boxed{a = 3}$

And hence the coordinates of point P are found to be $P(0, 5/2)$

12.) The point $P(-4, 1)$ divides the line segment joining the points $A(2, -2)$ and B in the ratio 3:5. Find the point B.

→ Given that,
The point $P(-4, 1)$ divides the line segment joining the points $A(2, -2)$ and B in the ratio 3:5.
Let us consider, B be any point whose coordinates are found to be $B(x, y)$.

$\Rightarrow -4 = \left[\frac{3x + 5(2)}{3+5}\right] = \frac{(3x+10)}{8}$ and $1 = \left[\frac{3y + 5(-2)}{3+5}\right]$.

Also, $3x + 10 = -32$ $\qquad 1 = (3y - 10)/8$

$3x = -32 - 10$ $\qquad 8 = 3y - 10$

$3x = -42$ $\qquad 3y = 8 + 10$

$\therefore \boxed{x = -14}$ $\qquad y = 18/3$

$\boxed{y = 6}$

Thus, the coordinates of point B are found to be $B(-14, 6)$

13.) i) In what ratio does the point (5,4) divide the line segment joining the points (2,1) and (7,6)?

ii) In what ratio does the point (-4,b) divide the line segment joining the points P(2,-2), Q(-14,6)? Hence find the value of b.

→ i) Given that, the point (5,4) divide the line segment joining the points (2,1) and (7,6).

Let us consider the points be A(2,1) & B(7,6) and P(5,4).

Let us consider point P divides AB in the ratio $m_1 : m_2$.

Then, $5 = \left(\frac{m_1 \times 7 + m_2 \times 2}{m_1 + m_2}\right)$

$5m_1 + 5m_2 = 7m_1 + 2m_2$

$5m_2 - 2m_2 = 7m_1 - 5m_1$

$3m_2 = 2m_1$

$m_1/m_2 = 3/2$

$\Rightarrow \boxed{m_1 : m_2 = 3:2}$

Thus, point P(5,4) divides the line segment A(2,1) & B(7,6) joining points in the ratio 3:2.

ii) • Given that, point (-4,b) divide the line segment joining the points P(2,-2), Q(-14,6).

• Let us consider point A(-4,b) divide the line segment joining points P & Q in the ratio $m_1 : m_2$,

Then, $-4 = \left(\frac{m_1 \times (-14) + m_2 \times 2}{m_1 + m_2}\right)$

$-4m_1 - 4m_2 = -14m_1 + 2m_2$

$-4m_1 + 14m_1 = 2m_2 + 4m_2$

$10m_1 = 6m_2$

$m_1/m_2 = 6/10 = 3/5$

$\boxed{m_1 : m_2 = 3:5}$

Also, $b = \left(\frac{m_1 \times 6 + m_2(-2)}{m_1 + m_2}\right)$

$= \left(\frac{6m_1 - 2m_2}{m_1 + m_2}\right)$

$b = \frac{6 \times 3 - 2 \times 5}{(3+5)}$

$b = 8/8$

$\boxed{b = 1}$

Thus, point A divide the line segment joining points P and Q in the ratio 3:5.

And the value of b is found to be $\boxed{b=1}$.

14) The line segment joining A(2,3) and B(6,-5) is intercepted by the X-axis at the point K. Write the ordinate of point K. Hence, find the ratio in which K divides AB. Also, find the coordinates of the point K.

→ Given that,

- The line segment joining A(2,3) and B(6,-5) is intercepted by the X-axis at the point K.
- Let us consider the coordinates of point K be $K(x,0)$ which intersects X-axis.
- Let us consider point K divides AB in the ratio $m_1 : m_2$.

$$\Rightarrow \quad 0 = \left(\frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

$$0 = \left(\frac{m_1 \times (-5) + m_2 \times 3}{m_1 + m_2}\right)$$

$$-5m_1 + 3m_2 = 0$$

$$-5m_1 = -3m_2$$

$$m_1/m_2 = 3/5$$

$$\boxed{m_1 : m_2 = 3:5}$$

And

$$x = \left(\frac{m_2 x_2 + m_2 x_1}{m_1 + m_2}\right)$$

$$= \left(\frac{3\times 6 + 5\times 2}{3+5}\right)$$

$$= \left(\frac{18+10}{8}\right)$$

$$\boxed{x = 7/2}$$

Thus, coordinates of point K are found to be $K(7/2, 0)$.

16) In what ratio does the line $x-y-2=0$ divide the line segment joining the points (3,-1) and (8,9)? Also, find the co-ordinates of the point of division?

→ Given that,

- The line $x-y-2=0$ divide the line segment the points (3,-1) and (8,9).
- Let us consider it divide the segment in the ratio $m_1 : m_2$ at point $P(x,y)$.

Thus,

$$x = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}\right)$$

$$x = \left(\frac{m_1 \times 8 + m_2 \times 3}{m_1 + m_2}\right)$$

and

$$y = \left(\frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{m_1 \times 9 + m_2 (-1)}{m_1 + m_2}\right)$$

$$y = \left(\frac{9m_1 - m_2}{m_1 + m_2}\right)$$

Given that, point $P(x,y)$ is on the line $x-y-2=0$

$$\Rightarrow \frac{(8m_1+3m_2)}{(m_1+m_2)} - \frac{(9m_1-m_2)}{(m_1+m_2)} - 2 = 0$$

$$8m_1+3m_2-9m_1+m_2-2m_1-2m_2=0$$

$$-3m_1+2m_2=0$$

$$3m_1=2m_2$$

$$\therefore \boxed{m_1/m_2=2/3}$$

Thus, the co-ordinates of point P are found to be $P(5,3)$.

17) Given a line segment AB joining the points $A(-4,6)$ and $B(8,-3)$. Find i) the ratio in which AB is divided by the Y-axis.

ii) find the co-ordinates of the point of intersection.

iii) the length of AB

→ Given that, a line segment AB joining the points $A(-4,6)$ & $B(8,-3)$.

i) Let us consider AB is divided by the Y-axis in the ratio $m:1$.

$$\Rightarrow \quad 0 = \left(\frac{m\times 8 - 4\times 1}{m+1}\right)$$

$$\Rightarrow 8m-4=0$$

$$8m=4$$

$$\boxed{m=1/2}$$

Thus, AB is divided by the Y-axis in the ratio $1/2:1$ or $1:2$

ii) Here, $y = \frac{(1\times(-3)+2\times 6)}{(1+2)} = \frac{9}{3} = 3$

Thus, co-ordinates of the point of intersection are found to be $(0,3)$.

iii) Here, By distance formula,

$$AB = \sqrt{(8-4)^2+(-3-6)^2}$$

$$= \sqrt{144+81}$$

$$AB = \sqrt{225} = 15 \text{ units}$$

Thus, the length of line segment AB is found to be 15 units.

18) Calculate the length of the median through the vertex A of the triangle ABC with vertices A(7,-3), B(5,3) and C(3,-1).

→ Given that, In triangle ABC,
the vertices are having co-ordinates
A(7,-3), B(5,3) and C(3,-1).

Let us consider the point D(x, y) be the midpoint of BC.

Hence, co-ordinates of point D are calculated by,

$x = \frac{(5+3)}{2} = \frac{8}{2} = 4$ and $y = \frac{(3-1)}{2} = \frac{2}{2} = 1$

Hence, Co-ordinates of point D are found to be D(4,1).

Thus, length of $DA = \sqrt{(7-4)^2 + (-3-1)^2}$

$= \sqrt{(3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25}$

$\boxed{l(DA) = 5 \text{ units}}$

Thus, the length of the median is found to be 5 units.

19) Three consecutive vertices of a parallelogram ABCD are A(1,2), B(1,0) and C(4,0). Find the fourth vertex D.

→ Given that, the three consecutive vertices of a parallelogram ABCD are A(1,2), B(1,0) and C(4,0).

Let us consider Point 'P' is the mid-point of diagonal AC.

Thus, coordinates of point $P \equiv \left(\frac{1+4}{2}, \frac{2+0}{2}\right) \equiv \left(\frac{5}{2}, 1\right)$

Let us consider the coordinates of point D(x, y).

Thus, $\frac{5}{2} = \frac{(1+x)}{2}$ And $1 = (0+y)/2$

$\Rightarrow 10 = 2 + 2x$ $\boxed{y=2}$

$2x = 10 - 2 = 8$

$\boxed{x=4}$

Thus, coordinates of point D are found to be $D \equiv (4,2)$.

20.) If the points $A(-2,-1)$, $B(1,0)$, $C(p,3)$ and $D(1,q)$ form a parallelogram ABCD, find the values of p and q.

→ Given that, the points $A(-2,-1)$, $B(1,0)$, $C(p,3)$ and $D(1,q)$ forms a parallelogram ABCD.

- Let us consider, the diagonals of parallelogram ABCD which are AC and BD bisects each other at point P.
- Thus, By the properties of a parallelogram P is the mid-point of AC as well as BD.
- Let us consider point P has coordinates $P(x,y)$.

Then, $x = \frac{(p-2)}{2}$ and $y = \frac{(3-1)}{2} = \frac{2}{2} = 1 \Rightarrow x = \frac{(p-2)}{2}$, $y = 1$

[Since, point P is the mid-point of diagonal AC.]

- Now, $x = \frac{(1+1)}{2} = \frac{2}{2} = 1$ and $y = \frac{(0+q)}{2} = \frac{q}{2}$

$\Rightarrow \frac{(p-2)}{2} = 1$ and $\frac{q}{2} = 1$

$p-2 = 2$ $\boxed{q=2}$

$p = 2+2 = 4$

$\boxed{p=4}$

Thus, the values of p and q are found to be $\boxed{p=4}$ and $\boxed{q=2}$.

21.) If two vertices of a parallelogram are $(3,2)$, $(-1,0)$ and its diagonals meet at $(2,-5)$. Find the other two vertices of the parallelogram.

→ Given that, the two vertices of a parallelogram are $(3,2)$, $(-1,0)$ and its diagonals meet at $(2,-5)$.

Let us consider, $A(3,2)$, $B(-1,0)$ & $P(2,-5)$.

Then, P is the mid-point of diagonals AC & BD.

Let us take, the coordinates of point C be (x,y).

$\Rightarrow 2 = (x+3)/2$

$x+3 = 4 \Rightarrow \boxed{x=1}$

And, $-5 = \frac{(y+2)}{2}$

$y = -10 - 2$

$\boxed{y = -12}$

Hence co-ordinates of point 'C' are found to be $C(1, -12)$.

Let us consider, coordinates of point 'D' be $D(x, y)$.

$2 = \frac{(x-1)}{2}$

$x - 1 = 4$

$\boxed{x = 5}$

and $-5 = (y+0)/2$

$\boxed{y = -10}$

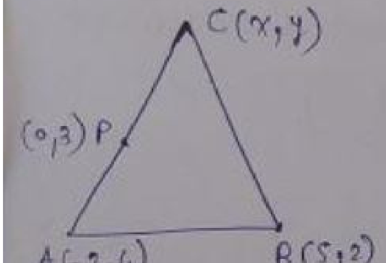
Thus, the co-ordinates of point D are found to be $D(5, -10)$.

22.) Find the third vertex of a triangle if its two vertices are $(-1, 4)$ and $(5, 2)$ and mid-point of one side is $(0, 3)$.

→ Given that,

The third vertex of a triangle if its two vertices are $A(-1, 4)$ and $B(5, 2)$.

And also, the mid-point of one side is $P(0, 3)$.



Thus, By mid-point formula,

$0 = (x-1)/2$ & $3 = (y+4)/2$

$\boxed{x = 1}$ $\quad y + 4 = 6$

$\boxed{y = 2}$

Hence, coordinates of third vertex be $C(1, 2)$.

But, if we considered point $P(0, 3)$ is the mid-point of side BC then, By mid-point formula

$0 = (5+x)/2$ and $3 = (2+y)/2$

$x + 5 = 0$ $\quad 2 + y = 6$

$\boxed{x = -5}$ $\quad \boxed{y = 4}$

Hence, co-ordinates of third vertex C be $C(-5, 4)$.

Finally, the coordinates of point C found to be $(1, 2)$ or $(-5, 4)$.

24) Show that by section formula the points $(3,-2)$, $(5,2)$ and $(8,8)$ are collinear.

→ Given points are $(3,-2)$, $(5,2)$ and $(8,8)$.

To show the given points are colinear:

Let us consider $(5,2)$ divides the line joining points $(3,-2)$ & $(8,8)$ in the ratio $m:n$.

Then, $5 = \left(\frac{m\times 8 + n\times 3}{m+n}\right)$ and $2 = \frac{(8m-2n)}{(m+n)}$

$8m+3n = 5m+5n$ | $2m+2n = 8m-2n$

$8m-5m = 5n-3n$ | $8m-2m = 2n+2n$

$3m = 2n$ | $6m = 4n$

$\boxed{\frac{m}{n} = \frac{2}{3}}$ —— ① | $\frac{m}{n} = \frac{4}{6} = \frac{2}{3} \Rightarrow \boxed{\frac{m}{n} = \frac{2}{3}}$ ②

from ① & ②, it is proved that point $(5,2)$ lies on the line joining the points $(3,-2)$ & $(8,8)$.

Hence, the points $(3,-2)$, $(5,2)$ & $(8,8)$ are colinear.

Hence proved.

25) Find the value of p for which the points $(-5,1)$, $(1,p)$ & $(4,-2)$ are collinear.

→ Given that, the points $(-5,1)$, $(1,p)$ & $(4,-2)$ are collinear.

Let us take, $A(-5,1)$, $B(1,p)$ & $C(4,-2)$.

And point $A(-5,1)$ divides BC in the ratio $m:n$.

Then, $x = \frac{(mx_2+nx_1)}{(m+n)}$ and $y = \frac{(m\times(-2)+n\times p)}{(m+n)}$

$-5 = \frac{(4m+n)}{(m+n)}$ | $1 = \frac{-2m+np}{m+n}$

$-5m-5n = 4m+n$ | $m+n = -2m+np$

$-9m = 6n$ | $3m = -n+np$

$\frac{m}{n} = \frac{-6}{9} = -\frac{2}{3}$ | $3m = (p-1)n$

$\boxed{\frac{m}{n} = \frac{2}{3}}$ —— ① | $\boxed{\frac{m}{n} = \frac{(p-1)}{3}}$ —— ②

from ① & ② ⇒ $\frac{(p-1)}{3} = \frac{-2}{3}$

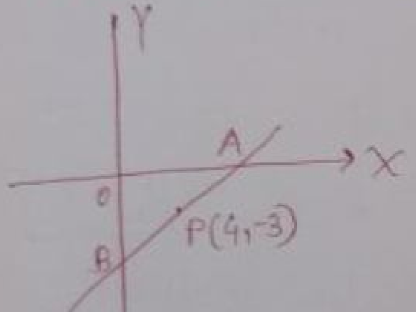
$\Rightarrow -3p+3 = 6$

$-3p = 3$

$p = -3/3$

$\boxed{p=-1}$ is the required value of p.

26.) The mid-point of the line segment AB shown in the adjoining diagram is (4,-3). Write down the co-ordinates of A and B.



→ Given that,

The mid-point of the line segment AB shown in the adjoining diagram is (4,-3).

A lies on the X-axis & B on the Y-axis as shown in fig.

Then, Co-ordinates of A and B are found to be:

A (x,0) and B (0,y).

As P(+4,-3) is the midpoint of AB, then by mid-point formula,

$4 = \frac{(x+0)}{2}$ and $-3 = (0+y)/2$

$\boxed{x=8}$ $\boxed{y=-6}$

Thus, the co-ordinates of point A and B are found to be:

A (8,0) and B (0,-6).

27.) Find the co-ordinates of the centroid of a triangle whose vertices are A(-1,3), B(1,-1) and C(5,1).

→ Given that, the three vertices of a triangle are

A(-1,3), B(1,-1) and C(5,1).

Then, By Centroid formula,

$C(x,y) \equiv \left(\frac{-1+1+5}{3}, \frac{3-1+1}{3}\right) \equiv \left(\frac{5}{3}, \frac{3}{3}\right) \equiv (5/3, 1)$

Thus, the co-ordinates of a centroid of a given triangle is found to be C(5/3,1).

28.) Two vertices of a triangle are $(3,-5)$ and $(-7,4)$. find the third vertex given that the centroid is $(2,-1)$.

→ Given that, two vertices of a triangle are $A(3,-5)$ & $B(-7,4)$

And co-ordinates of a centroid of a triangle are $C(2,-1)$.

Let us consider, the coordinates of third vertex of a triangle are $D(x,y)$.

Then, By centroid formula,

$$2 = \frac{(3-7+x)}{3} \quad \text{and} \quad -1 = \frac{(-5+4+y)}{3}$$

$$\frac{(x-4)}{3} = 2 \qquad -3 = -1+y$$

$$x-4 = 6 \qquad y = -3+1$$

$$\boxed{x=10} \qquad \boxed{y=-2}$$

Thus, the coordinates of third vertex of a given triangle are found to be $D(10,-2)$.

29.) The vertices of a triangle are $A(-5,3)$, $B(p,-1)$ and $C(6,q)$ find the values of p and q, if the centroid of the triangle ABC is the point $(1,-1)$.

→ Given that, the vertices of a triangle are $A(-5,3)$, $B(p,-1)$ and $C(6,q)$.

And centroid of a triangle is found to be $D(1,-1)$.

Then, By centroid formula,

$$(x,y) \equiv \left(\frac{-5+p+6}{3}, \frac{3-1+q}{3}\right) \equiv \left(\frac{1+p}{3}, \frac{2+q}{3}\right)$$

$$\Rightarrow \frac{1+p}{3} = 1 \quad \text{and} \quad \frac{(2+q)}{3} = -1$$

$$1+p = 3 \qquad 2+q = -3$$

$$\boxed{p=2} \qquad \boxed{q=-5}$$

Thus, the values of p and q are found to be

$p=2$ and $q=-5$