

Chapter 21.

Measures of Central Tendency

Exercise 21-1

1. → Calculate the arithmetic mean of 5, 7, 6.6, 7.2, 9.3, 6.2

→ Given observations are 5, 7, 6.6, 7.2, 9.3, 6.2

$$\text{Sum of observations} = (5 + 7 + 6.6 + 7.2 + 9.3 + 6.2) = 35$$

$$\text{Arithmetic Mean} = \frac{35}{5} = 7$$

$\boxed{\text{A. Mean} = 7}$ is the required arithmetic mean.

2. → The marks obtained by 15 students in a class test are 12, 14, 07, 09, 23, 11, 08, 13, 11, 19, 16, 24, 17, 03, 20. Find

i) the mean of their marks

ii) the mean of their marks when the marks of each student are increased by 4.

iii) the mean of their marks when 2 marks are deducted from the marks of each student.

→ Given that, 15 students in a class test are 12, 14, 07, 09, 23, 11, 08, 13, 11, 19, 16, 24, 17, 03, 20.

$$\begin{aligned} \text{Sum of observations} &= (12 + 14 + 07 + 09 + 23 + 11 + 08 + 13 + \\ & 11 + 19 + 16 + 24 + 17 + 03 + 20) \\ &= 207 \end{aligned}$$

i) Mean = $207/15$

$\boxed{\text{Mean} = 13.8}$ is the required mean.

ii) If mark of each student is increased by 4.

Then total marks increased = $4 \times 15 = 60$

Total sum of marks of all students = $207 + 60 = 267$

Then, Mean = $\frac{267}{15} = 17.8$

$\boxed{\text{Mean} = 17.8}$ is the required mean when marks of each student are increased by 4.

iii) If mark of each student is deducted by 2.

Then total marks deducted = $15 \times 2 = 30$

Total sum of marks of all students = $-30 + 207 = 177$

Then, Mean = $\frac{177}{15} = 11.8$

$\boxed{\text{Mean} = 11.8}$ is the required mean when marks of each student are deducted by 2.

3.) a) The mean of the numbers 6, y, 7, x, 14 is 8. Express y in terms of x.

b) The mean of 9 variates is 11. If eight of them are 7, 12, 9, 14, 21, 3, 8 and 15. Find the 9th variate.

→ a) Given observations: 6, y, 7, x, 14

Sum of observations = $6 + y + 7 + x + 14 = 27 + x + y$

Mean = 8

But, $\frac{(27 + x + y)}{5} = 8$

$27 + x + y = 40$

$\boxed{y = 13 - x}$ is the required answer.

b) Mean of 9 variates = 11

Sum of given 8 variates = $(7 + 12 + 9 + 14 + 21 + 3 + 8 + 15) = 89$

Mean = $\frac{(\text{8 variates sum} + \text{9th variate})}{9} \Rightarrow \frac{89 + 9^{\text{th}}}{9} = 11$

$$99 = 89 + 9^{\text{th}} \text{ variate}$$

$$\Rightarrow 9^{\text{th}} \text{ variate} = 99 - 89$$

$\boxed{9^{\text{th}} \text{ variate} = 10}$ is the required answer.

5.) find the mean of 25 given numbers when the mean of 10 of them is 13 and the mean of remaining numbers is 18.

→ Given that, Mean of 10 numbers = 13

Mean of remaining 15 numbers = 18

$$(\text{sum of } 10 \text{ no.}) + (\text{sum of } 15 \text{ no.}) = \text{total sum}$$

$$(10 \times 13) + (18 \times 15) = \text{total sum}$$

$$130 + 270 = \text{total sum}$$

$$\Rightarrow \boxed{\text{total sum} = 400}$$

$$\text{Then, Mean of 25 numbers} = \frac{400}{25} = 16$$

$\boxed{\text{Mean} = 16}$ is the required answer.

6.) find the mean of the following distribution:

Number	5	10	15	20	25	30	35
Frequency	1	2	5	6	3	2	1

→ from given distribution we can make following table:

x	f	fx
5	1	5
10	2	20
15	5	75
20	6	120
25	3	75
30	2	60
35	1	35
Total	20	390

$$\text{Thus, Mean} = \frac{\sum fx}{\sum f} = \frac{390}{20} = 19.5$$

$$\boxed{\text{Mean} = 19.5}$$

is the required mean for given distribution.

7) The contents of 100 match boxes were checked to determine the number of matches they contained

No. of matches	35	36	37	38	39	40	41
No. of boxes	6	10	18	25	21	12	8

i) Calculate, correct to one decimal place, the mean of no. of matches per box.

ii) Determine how many extra matches would have to be added to the total contents of the 100 boxes to bring the mean up to exactly 39 matches.

→ From given distribution we can make following table

No. of matches (x)	No. of boxes (f)	fx
35	6	210
36	10	360
37	18	666
38	25	950
39	21	819
40	12	480
41	8	328
Total	100	3813

Thus,

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$= \frac{3813}{100}$$

$$\text{Mean} = 38.13$$

is the required mean.

ii) Given that, New mean = 39

$$\sum fx = 39 \times 100 = 3900$$

Then, no. of extra matches we have to add = $3900 - 3813 = 87$

Thus, the no. of extra matches we have to add are found to be 87.

8) Find the mean for the following distribution by short cut method:

Numbers	60	61	62	63	64	65	66
Cumulative freq.	8	18	33	40	49	55	60

From given table we can make following frequency distribution table.

Numbers (x)	Cumulative freq. c.f.	frequency f	fx
60	8	8	480
61	18	10	610
62	33	15	990
63	40	7	441
64	49	9	576
65	55	6	390
66	60	5	330
Total		60	3757

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{3757}{60} = 62.616 = 62.62$$

$\boxed{\text{Mean} = 62.62}$ is the required mean for given distribution

10) If the mean of the following distribution is 7.5, find the missing frequency f.

Variate	5	6	7	8	9	10	11	12
frequency	20	17	f	10	8	6	7	6

From given table we can make following distribution table.

Variate (x)	frequency (f)	fx
5	20	100
6	17	102
7	f	7f
8	10	80
9	8	72
10	6	60
11	7	77
12	6	72
Total	$\sum f = 74 + f$	$563 + 7f$

$$\text{Thus, Mean} = \frac{\sum fx}{\sum f}$$

$$7.5 = \frac{563 + 7f}{74 + f}$$

$$555 + 7.5f = 563 + 7f$$

$$0.5f = 8$$

$$\boxed{f = 16}$$

is the required missing frequency.

11) Marks obtained by 40 students in a short assessment are given below, where a and b are two missing data. If the mean of the distribution is 7.2 , find a and b .

Marks	5	6	7	8	9
No. of students	6	a	16	13	b

→ From given distribution table, we can make following frequency distribution table.

Marks (x)	No. of students (f)	fx
5	6	30
6	a	$6a$
7	16	112
8	13	104
9	b	$9b$
	$35+a+b=40$	$246+6a+9b$

Total no. of students = 40

$$\Sigma f = 35 + a + b$$

$$40 = 35 + a + b$$

$$\boxed{a = 5 - b} \text{ --- ①}$$

$$\text{Again, Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$7.2 = \frac{(246 + 6a + 9b)}{40}$$

$$\Rightarrow 246 + 6a + 9b = 40 \times 7.2$$

$$246 + 6a + 9b = 288$$

$$6a + 9b = 288 - 246$$

$$6a + 9b = 288 - 246$$

$$\Rightarrow \boxed{2a + 3b = 14} \text{ --- ②}$$

from ① & ② \Rightarrow

$$2(5-b) + 3b = 14$$

$$-10 - 2b + 3b = 14$$

$$-10 + b = 14$$

$$\boxed{b = 4} \text{ put in ①}$$

$$\text{①} \Rightarrow a = 5 - b$$

$$a = 5 - 4$$

$$\boxed{a = 1}$$

Thus, required values of a and b are found to be $a=1, b=4$

12) Calculate the mean of the following distribution:

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
frequency	8	5	12	35	24	16

from given distribution table we can make following frequency distribution table.

Class interval	frequency f_i	Class mark x_i	$f_i x_i$
0-10	8	5	40
10-20	5	15	75
20-30	12	25	300
30-40	35	35	1225
40-50	24	45	1080
50-60	16	55	880
Total	$f_i = 100$		$f_i x_i = 3600$

Here, Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{3600}{100}$

Mean = 36 is the required mean for given distribution.

13. > Calculate the mean of the following distribution using step deviation method.

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	10	9	25	30	16	10

→ We can make following distribution table:

Marks	Mid Value (x_i)	No. of students (f_i)	$d_i = x_i - A$	$h_i = d_i/c$	$f_i h_i$
0-10	5	10	-20	-2	-20
10-20	15	9	-10	-1	-9
20-30	25	25 = A	0	0	0
30-40	35	30	10	1	30
40-50	45	16	20	2	32
50-60	55	10	30	3	30
		$\sum f_i = 100$			$\sum f_i h_i = 63$

Then, by step deviation method mean is given by

$$\begin{aligned} \text{Mean} = \bar{x} &= A + h \frac{\sum f_i d_i}{\sum f_i} \\ &= 25 + 10 \times \left(\frac{63}{100}\right) \\ &= 25 + 6.3 \end{aligned}$$

$\boxed{\text{Mean} = 31.3}$ is the required mean for given distribution by step deviation method.

14) The data on the number of patients attending a hospital in a month are given below. Find the average (mean) no. of patients attending the hospital in a month by using the shortcut method.
Take assumed mean as 45. Give your answer correct to 2 decimal places.

No. of patients	10-20	20-30	30-40	40-50	50-60	60-70
No. of days	5	2	7	9	2	5

→ We can make following distribution table to find mean:

No. of patients	No. of days (f_i)	Class marks	$d_i = x_i - a$	$f_i d_i$
10-20	5	15	-30	-150
20-30	2	25	-20	-40
30-40	7	35	-10	-70
40-50	9	$a = 45$	0	0
50-60	2	55	10	20
60-70	5	65	20	100
	$\sum f_i = 30$			$\sum f_i d_i = -140$

$$\begin{aligned} \text{Then, Mean} = \bar{x} &= a + \frac{\sum f_i d_i}{\sum f_i} \\ &= 45 + \left(\frac{-140}{30}\right) = 45 - 4.67 = 40.33 \end{aligned}$$

$\boxed{\text{Mean} = 40.33}$ is the required mean for given distribution.

16.) Calculate the mean of the distribution given below using the short cut method.

Marks	11-20	21-30	31-40	41-50	51-60	61-70	71-80
No. of students	2	6	10	12	9	7	4

→ From given distribution table we can make following table:

Marks	frequency (f)	Mid Value (x)	$d_i = x_i - A$ $A = 45.5$	$u_i = \frac{x_i - A}{10}$	$f \times u_i$
11-20	2	15.5	A - 30	-3	-6
21-30	6	25.5	-20	-2	-12
31-40	10	35.5	-10	-1	-10
41-50	12	45.5 = A	0	0	0
51-60	9	55.5	10	1	9
61-70	7	65.5	20	2	14
71-80	4	75.5	30	3	12
	$\Sigma f = 50$				$\Sigma fu_i = 7$

Then, by short cut method mean is given by

$$\text{Mean} = \bar{x} = A + \frac{\Sigma f d_i}{\Sigma f_i}$$

$$= 45.5 + \frac{70}{50}$$

$$= 45.5 + 1.4$$

Mean = 46.9 is the required mean for given distribution.

17.) A class teacher has the following absentee record of 40 students of a class for the whole term. find the mean no. of days a student was absent.

No. of days	0-6	6-10	10-14	14-20	20-28	28-38	38-40
No. of students	11	10	7	4	4	3	1

→ From above table we can make following distribution table:

$$\text{Mean} = \frac{\Sigma f_i x_i}{f_i}$$

No. of days	Mid value (x)	No. of students (f)	$f \times x$
0-6	3	11	33
6-10	8	10	80
10-14	12	7	84
14-20	17	4	68
20-28	24	4	96
28-38	33	3	99
38-40	39	1	39
Total		40	499

Then, Mean is given by

$$\text{Mean} = \frac{\sum f_i x_i}{f_i} = \frac{499}{40}$$

$\boxed{\text{Mean} = 12.475}$ is the required mean for given distribution

18.) If the mean of the following distribution is 24. Find the value of 'a'.

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	7	a	8	10	5

→ From above table we can make following distribution table:

Marks	Class Marks (x_i)	No. of Students (f_i)	$f_i x_i$
0-10	5	7	35
10-20	15	a	15a
20-30	25	8	200
30-40	35	10	350
40-50	45	5	225
Total		30+a	15a+810

$$\text{Mean} = 24$$

$$\therefore \frac{15a + 810}{30 + a} = 24 \Rightarrow$$

$$15a + 810 = 24(30 + a)$$

$$24a - 15a = 810 - 720$$

$$9a = 90$$

$\boxed{a = 10}$ is the required value of 'a' for given distribution.

20.) The following table gives the lifetime in days of 100 electricity tubes of a certain make:

Life time in days	No. of tubes
less than 50	8
less than 100	23
less than 150	55
less than 200	81
less than 250	93
less than 300	100

Find the mean lifetime of electricity tubes.

→ From the given table we can make following distribution table:

Lifetime (In days) (Class intervals)	c.f.	Frequency (f)	Class Mark (x)	$u = \frac{x-A}{h}$	fu
0 - 50	8	8	25	-3	-24
50 - 100	23	15	75	-2	-30
100 - 150	55	32	125	-1	-32
150 - 200	81	26	175	0	0
200 - 250	93	12	225	1	12
250 - 300	100	7	275	2	14
Total		100			-60

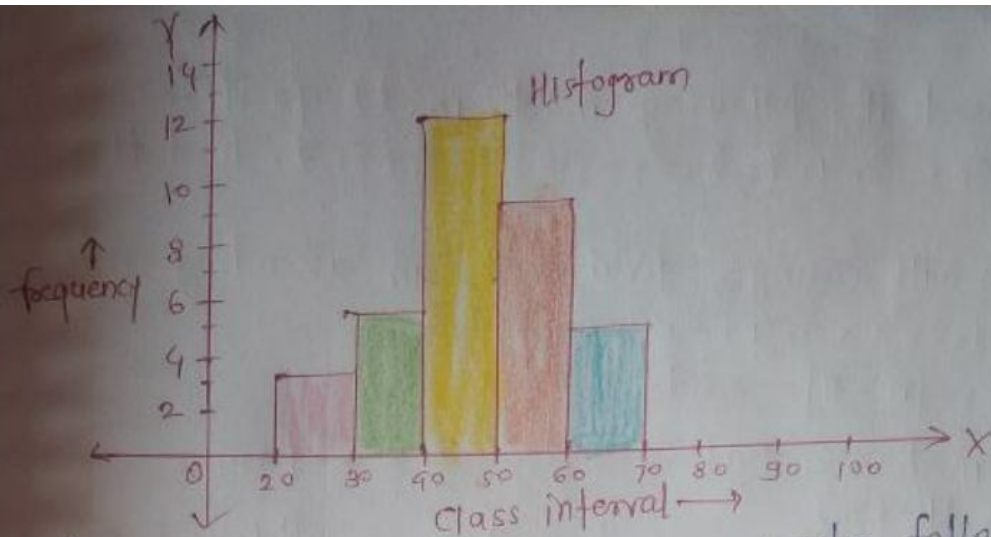
Here, Class Mark = (upper limit + lower limit) / 2

Let us consider (A) = 175, Class size (h) = 50

$$\begin{aligned} \text{Then, Mean} &= 175 + 50 \times (-0.6) \\ &= 175 + 50 \times (-60/100) \\ &= 175 - 30 \end{aligned}$$

Mean = 145 is the required mean of lifetime of electricity tubes.

21.) Using the information given in the histogram, calculate the mean correct to one decimal place.



→ From the given histogram we can make following distribution table.

Class Interval	frequency (f)	Class Mark (x)	f x
20-30	3	25	75
30-40	5	35	175
40-50	12	45	540
50-60	9	55	495
60-70	4	65	260
Total	33		1545

Mean by shortcut method is given by,

$$\begin{aligned} \text{Mean} = \bar{x} &= A + \frac{\sum fidi}{\sum fi} \\ &= 45 + \frac{60}{33} \\ &= 45 + 1.81 \end{aligned}$$

Mean = 46.81 is the required mean for given distribution

Exercise 21.2

1.) A student scored the following marks in 11 questions of a question paper: 3, 4, 7, 2, 5, 6, 1, 8, 2, 5, 7. Find the median marks.

→ Initially, we will arrange given marks in ascending order as follows: 1, 2, 2, 3, 4, 5, 5, 6, 7, 7, 8.

Thus, $n = 11$ → odd number

$$\text{Then, median} = \frac{n+1}{2} = \frac{11+1}{2} = \frac{12}{2} = 6^{\text{th}} \text{ term}$$

(middle term)

Thus, the required median is 6th term = 5.

2.) For the following set of the number, find the median:
10, 75, 3, 81, 17, 27, 4, 48, 12, 47, 9, 15.

→ Initially, we will arrange given numbers in ascending order as follows:

3, 4, 9, 10, 12, 15, 17, 27, 47, 48, 75, 81.

Here, $n = 12$ → even number

$$\begin{aligned} \text{Thus, Median} &= \left\{ \frac{(n/2)^{\text{th}} \text{ term} + (n/2+1)^{\text{th}} \text{ term}}{2} \right\} \\ &= \left\{ \frac{(12/2)^{\text{th}} \text{ term} + (12/2+1)^{\text{th}} \text{ term}}{2} \right\} \\ &= (6^{\text{th}} \text{ term} + 7^{\text{th}} \text{ term}) / 2 \\ &= (15 + 17) / 2 = 32 / 2 = 16 \end{aligned}$$

$$\boxed{\text{Median} = 16}$$

3.) Calculate the mean and median of the numbers:
2, 1, 0, 3, 1, 2, 3, 4, 3, 5

→ Initially, we will write given numbers in ascending order as follows:

0, 1, 1, 2, 2, 3, 3, 3, 4, 5

Here, $n = 10$ → even number

$$\begin{aligned} \text{Median} &= \left\{ \left(\frac{n}{2} \right)^{\text{th}} \text{ term} + \left(\frac{n}{2} + 1 \right)^{\text{th}} \text{ term} \right\} / 2 \\ &= \left\{ 10/2^{\text{th}} \text{ term} + \left(\frac{10}{2} + 1 \right)^{\text{th}} \text{ term} \right\} / 2 \\ &= \left\{ 5^{\text{th}} \text{ term} + 6^{\text{th}} \text{ term} \right\} / 2 \\ &= (2+3)/2 = 5/2 \end{aligned}$$

$\boxed{\text{Median} = 2.5}$ is the required median for given numbers.

Then, $\text{Mean} = \frac{\text{sum of all numbers / observations}}{\text{No. of observations}}$

$$\begin{aligned} &= \frac{\sum x_i}{n} \\ &= (0+1+1+2+2+3+3+3+4+5)/10 \\ &= 24/10 = 2.4 \end{aligned}$$

$\boxed{\text{Mean} = 2.4}$ is the required mean for given numbers.

4.) The median of the observations 11, 12, 14, $(x-2)$, $(x+4)$, $(x+9)$, 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean.

→ Given observations in ascending order as follows:

11, 12, 14, $(x-2)$, $(x+4)$, $(x+9)$, 32, 38, 47 $\Rightarrow \boxed{n=9}$ \hookrightarrow odd

$$\text{Median} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ term} = \left(\frac{9+1}{2} \right) = \frac{10}{2} = 5^{\text{th}} \text{ term}$$

$$\text{But, } 5^{\text{th}} \text{ term} = x+4$$

$$\Rightarrow \text{Median} = x+4$$

$$24 = x+4$$

$$\boxed{x = 20}$$

$$\begin{aligned} \text{Now, sum of all observations} &= [11 + 12 + 14 + (x-2) + (x+4) + (x+9) + 32 + 38 + 47] \\ &= 165 + 3x \end{aligned}$$

$$\text{Mean} = \frac{165 + 3x}{9} = \frac{(165 + 3 \times 20)}{9} = \frac{225}{9} = 25$$

$\boxed{\text{Mean} = 25}$ is the required mean for given observations.

5) The mean of the numbers 1, 7, 5, 3, 4, 4 is m . The numbers 3, 2, 4, 2, 3, 3, p have mean $(m-1)$ and median q .
find i) p ii) q iii) the mean of p and q .

→ i) Mean of given numbers 1, 7, 5, 3, 4, 4 is m .

$$\text{Here, } \boxed{n=6}$$

$$\text{Then, mean} = m = (1+7+5+3+4+4)/6$$

$$m = 24/6$$

$\boxed{m=4}$ is the required mean

Again, the numbers 3, 2, 4, 2, 3, 3, p have mean $(m-1)$ and median q .

$$\text{Thus, } (m-1) = (3+2+4+2+3+3+p)/7$$

$$(m-1) = (17+p)/7$$

$$(4-1) = (17+p)/7 \quad \therefore m=4$$

$$3 \times 7 = 17+p$$

$$p = 21 - 17 \Rightarrow \boxed{p=4} \text{ is the required value of } p.$$

ii) Also, given that the no. 3, 2, 4, 2, 3, 3, 4 has median q .

We arrange given no. in ascending order as follows:

$$2, 2, 3, 3, 3, 4, 4 \quad \text{and } \boxed{n=7} \rightarrow \text{odd no.}$$

$$\text{Then, Median} = (n+1)/2^{\text{th}} \text{ term}$$

$$q = (7+1)/2^{\text{th}} \text{ term} = 8/2^{\text{th}} \text{ term}$$

$$q = 4^{\text{th}} \text{ term}$$

$$\boxed{q=3} \text{ is the required value of } q.$$

iii) Mean of p and q is given by

$$\text{Mean} = \frac{p+q}{2} = \frac{4+3}{2} = \frac{7}{2}$$

$$\boxed{\text{Mean} = 3.5} \text{ is the required mean.}$$

6.) Find the median for the following distribution

Wages per day (Rs)	38	45	48	55	62	65
No. of workers	14	8	7	10	6	2

→ From above distribution table we will prepare following cumulative frequency table.

Wages per day (in Rs)	No. of workers (f)	c.f.
38	14	14
45	8	22
48	7	29
55	10	39
62	6	45
65	2	47

Here, $n = 47 \rightarrow$ odd

$$\begin{aligned} \text{Median} &= \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} \\ &= \left(\frac{48}{2}\right)^{\text{th}} \text{ term} \\ &= 24^{\text{th}} \text{ term} \end{aligned}$$

$$\boxed{\text{Median} = 48}$$

is the required median for given distribution.

7.) Marks obtained by 70 students are given below:

Marks	20	70	50	60	75	90	40
No. of students	8	12	18	6	9	5	12

Calculate the median marks.

→ From above table we will make c.f. table as given below:

Marks	No. of students (f)	c.f.
20	8	8
40	12	20
50	18	38
60	6	44
70	12	56
75	9	65
90	5	70

Here $n = 70$

$$\begin{aligned} \text{Median} &= \frac{\left[\frac{n}{2} \text{ term} + \left(\frac{n}{2} + 1\right) \text{ term}\right]}{2} \\ &= \frac{\left(\frac{70}{2} \text{ term} + \left(\frac{70}{2} + 1\right) \text{ term}\right)}{2} \\ &= \frac{(35^{\text{th}} \text{ term} + 36^{\text{th}} \text{ term})}{2} \\ &= \frac{(50 + 50)}{2} \\ &= 50 \end{aligned}$$

Thus, $\boxed{\text{Median} = 50}$

is the required median for given distribution.

8.) Calculate the mean and median of the following distribution:

Number	5	10	15	20	25	30	35
Frequency	1	2	5	6	3	2	1

→ from given distribution table we will make c.f. table as below:

Numbers (x)	frequency (f)	c.f.	fx
5	1	1	5
10	2	3	20
15	5	8	75
20	6	14	120
25	3	17	75
30	2	19	60
35	1	20	35
Total	20		390

Then, Mean is found to be

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$= \frac{390}{20}$$

$$\boxed{\text{Mean} = 19.5}$$

is the required mean for given distribution.

Here, $\boxed{n = 20}$ → even

Then, median is given by,

$$\text{Median} = \frac{\left[\frac{n}{2} \text{th term} + \left(\frac{n}{2} + 1 \right) \text{th term} \right]}{2}$$

$$= \frac{\left[\frac{20}{2} \text{th term} + \left(\frac{20}{2} + 1 \right) \text{th term} \right]}{2}$$

$$= \frac{(10 \text{th term} + 11 \text{th term})}{2} = \frac{(20 + 20)}{2}$$

$\boxed{\text{Median} = 20}$ is the required median for the given distribution.

9.) The daily wages (in Rs) of 19 workers are,

41, 21, 38, 29, 31, 45, 23, 26, 29, 30, 28, 25, 35, 42, 47, 53, 29, 31, 35. Find

i) the median iii) upper quartile range
ii) lower quartile iv) interquartile range

→ Initially, we will arrange given observation in ascending order
21, 23, 25, 26, 27, 28, 29, 29, 30, 31, 31, 35, 35, 38, 41, 42, 45, 47, 53. Here $\boxed{n = 19}$ → odd

i) Then, median is given by

$$\text{Median} = \left(\frac{n+1}{2} \right) \text{th term} = \left(\frac{19+1}{2} \right) \text{th term} = 5 \text{th term}$$

$$\boxed{\text{Median} = 27}$$

ii) Upper quartile is given by,

$$\begin{aligned} Q_3 &= \left[3 \left(\frac{n+1}{4} \right)^{\text{th}} \text{ term} \right] \\ &= \left[3 \left(\frac{20}{4} \right) \right]^{\text{th}} \text{ term} \\ &= (3 \times 5)^{\text{th}} \text{ term} \\ &= 15^{\text{th}} \text{ term} \end{aligned}$$

Upper quartile = 41 is the required answer.

iii) Lower quartile is given by,

$$\begin{aligned} Q_1 &= \left(\frac{n+1}{4} \right)^{\text{th}} \text{ term} = \left(\frac{19+1}{4} \right)^{\text{th}} \text{ term} \\ Q_1 &= 20/4 = 5^{\text{th}} \text{ term} \end{aligned}$$

$Q_1 = 27$ is the required lower quartile

iv) The interquartile range is given by

$$Q_3 - Q_1 = 41 - 27 = 14$$

10.) From the following frequency distribution, find i) the median
ii) Lower quartile iii) upper quartile iv) inter quartile range

Variate	15	18	20	22	25	27	30
frequency	4	6	8	9	7	8	6

→ From above distribution table we will make c.f. table as below:

Variates	frequency	c.f.
15	4	4
18	6	10
20	8	18
22	9	27
25	7	34
27	8	42
30	6	48

i) Here, $n = 48$ → even

Then, median is given by

$$\begin{aligned} \text{Median} &= \frac{\left[\left(\frac{n}{2} \right)^{\text{th}} \text{ term} + \left(\frac{n}{2} + 1 \right)^{\text{th}} \text{ term} \right]}{2} \\ &= \frac{(24^{\text{th}} \text{ term} + 25^{\text{th}} \text{ term})}{2} \\ &= \frac{(22 + 22)}{2} \end{aligned}$$

Median = 22 is the required median for given distribution.

i) Lower quartile is given by

$$Q_1 = \frac{n}{4} \text{th term} \\ = \left(\frac{48}{4}\right) \text{th term}$$

$$Q_1 = 12^{\text{th}} \text{ term}$$

$$Q_1 = 20$$

is the required lower quartile.

ii) Upper quartile is given by

$$Q_3 = \left(\frac{3n}{4}\right) \text{th term} \\ = \left(3 \times \frac{48}{4}\right) \text{th term} \\ = (3 \times 12) \text{th term}$$

$$Q_3 = 36^{\text{th}} \text{ term}$$

$$Q_3 = 27$$

is the required upper quartile.

iv) Now, the interquartile range is given by,

$$Q_3 - Q_1 = 27 - 20$$

$$Q_3 - Q_1 = 7 \text{ is the required interquartile range.}$$

11.) for the following frequency distribution, find:

i) the median ii) lower quartile iii) upper quartile

Variate	25	31	34	40	45	48	50	60
frequency	3	8	10	15	10	9	6	2

→ From above distribution table we will make c.f. table as below:

Variate	frequency (f)	c.f.
25	3	3
31	8	11
34	10	21
40	15	36
45	10	46
48	9	55
50	6	61
60	2	63

i) Here, $n = 63 \rightarrow$ odd
then median is given by
Median = $\left(\frac{n+1}{2}\right) \text{th term}$
 $= \left(\frac{63+1}{2}\right) \text{th term}$
 $= 64/2 = 32^{\text{th}} \text{ term}$

$$\text{Median} = 40$$

is the required median for given distribution.

ii) Lower quartile is given by,

$$Q_1 = 3 \left(\frac{n+1}{4}\right) \text{th term} = 3 \times \left(\frac{63+1}{4}\right) \text{th term} \\ = (3 \times 16) \text{th term} = 48^{\text{th}} \text{ term}$$

$$Q_1 = 48$$

iii) Lower quartile is given by $Q_1 = \left(\frac{n+1}{4}\right)^{\text{th}} \text{ term}$
 $= \left(\frac{63+1}{4}\right)^{\text{th}} \text{ term} = 16^{\text{th}} \text{ term}$

$Q_1 = 34$ is the required lower quartile.

Exercise 21.3

1.) find the mode of the following sets of numbers:

i) 5, 7, 6, 8, 9, 0, 6, 8, 1, 8

ii) 9, 0, 2, 8, 5, 3, 5, 4, 1, 5, 2, 7

→ i) Given numbers are 5, 7, 6, 8, 9, 0, 6, 8, 1, 8
 Here, the no. 8 occurs maximum times.

Thus, $\text{Mode} = 8$

ii) Given numbers are 9, 0, 2, 8, 5, 3, 5, 4, 1, 5, 2, 7
 Here, the no. 5 occurs maximum times.

Thus, $\text{Mode} = 5$

2.) find the mean, median and mode of the following distribution:
 8, 10, 7, 6, 10, 11, 6, 13, 10.

→ Given observations we can write in ascending order as follows:
 6, 6, 7, 8, 10, 10, 10, 11, 13

Then, Mean is given by, $\text{Mean} = \frac{(6+6+7+8+10+10+10+11+13)}{9}$

$$\text{Mean} = \frac{81}{9}$$

$\text{Mean} = 9$ is the required mean

Now, $n = 9 \rightarrow \text{odd}$

Median is given by, $\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = \left(\frac{9+1}{2}\right)^{\text{th}} \text{ term}$

$$\text{Median} = 5^{\text{th}} \text{ term}$$

$\text{Median} = 10$ is the required median

Then, the no. 10 occurs maximum times.

Hence, $\text{mode} = 10$ is the required mode

3) Calculate the mean, the median and the mode of the following no. 3, 1, 5, 6, 3, 4, 5, 3, 7, 2.

→ Initially, we will arrange the given numbers in ascending order as follow: 1, 2, 3, 3, 3, 4, 5, 5, 6, 7.

Mean is given by,
$$\text{Mean} = \frac{(1+2+3+3+3+4+5+5+6+7)}{10}$$

$$\text{Mean} = 39/10$$

$\boxed{\text{Mean} = 3.9}$ is the required mean

Here, $\boxed{n=10}$ → even

Then,
$$\text{Median} = \left[\frac{n}{2}^{\text{th}} \text{ term} + \left(\frac{n}{2} + 1 \right)^{\text{th}} \text{ term} \right] / 2$$

$$= [5^{\text{th}} \text{ term} + 6^{\text{th}} \text{ term}] / 2$$

$$= (3+4)/2$$

$\boxed{\text{Median} = 3.5}$ is the required median.

In given observations, the no. 3 occurs maximum times.

Thus, $\boxed{\text{mode} = 3}$ is the required mode.

4) The marks of 10 students of a class in an examination arranged in ascending order are as follows:

13, 35, 43, 46, x , $x+4$, 55, 61, 71, 80.

If the median marks is 48, find the value of x . Hence, find the mode of the given data.

→ Given marks arranged in ascending order as follows:

13, 35, 43, 46, x , $x+4$, 55, 61, 71, 80

Here, $\boxed{n=10}$ → even and $\boxed{\text{median} = 48}$

Then, median is given by,

$$\text{Median} = \left[\frac{n}{2}^{\text{th}} \text{ term} + \left(\frac{n}{2} + 1 \right)^{\text{th}} \text{ term} \right] / 2$$

$$48 = [5^{\text{th}} \text{ term} + 6^{\text{th}} \text{ term}] / 2$$

$$48 = (x + x + 4) / 2$$

$$48 = x + 2$$

$$\boxed{x = 46}$$

$$\boxed{x + 4 = 50}$$

Thus, the required no. in ascending order are found to be
13, 35, 43, 46, 46, 50, 55, 61, 71, 80.

Here, we can see the no. 46 occurs maximum times.

Thus, $\boxed{\text{mode} = 46}$ is the required mode.

5) Find the mode and median of the following frequency distribution:

x	10	11	12	13	14	15
f	1	4	7	5	9	3

→ From given distribution table we will make c.f. table as belows:

x	f	Cumulative freq.
10	1	1
11	4	5
12	7	12
13	5	17
14	9	26
15	3	29

Here, $\boxed{n = 29} \rightarrow \text{odd}$

Then, median is given by

$$\begin{aligned} \text{Median} &= \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} \\ &= \left(\frac{29+1}{2}\right)^{\text{th}} \text{ term} \\ &= 15^{\text{th}} \text{ term} \end{aligned}$$

$$\boxed{\text{Median} = 13}$$

is the required median for given distribution

Also, the no. 14 occurs maximum times.

Hence, $\boxed{\text{mode} = 14}$ is the required mode.

6) In a class of 40 students, marks obtained by the students in a class test (out of 10) are given below:

Calculate the following for given distribution:

i) Median ii) Mode

Marks	1	2	3	4	5	6	7	8	9	10
no. of students	1	2	3	3	6	10	5	4	3	3

→ Here, given that $\boxed{N = 40} \Rightarrow \frac{N}{2} = \frac{40}{2} = 20$

Marks	1	2	3	4	5	6	7	8	9	10	Total
No. of students	1	2	3	3	6	10	5	4	3	3	40
Cumulative freq.	1	3	6	9	15	25	30	34	37	40	

Thus, Marks corresponding to c.f. is found to be 20 \rightarrow 6
Hence, the required median is 6.

$$\boxed{\text{Median} = 6}$$

And the no. 6 occurs maximum times.

Thus, $\boxed{\text{mode} = 6}$ is the required mode.

8.) The distribution given below shows the marks obtained by 25 students in an aptitude test. Find the mean, median and mode of the distribution.

Marks obtained	5	6	7	8	9	10
No. of students	3	9	6	4	2	1

\rightarrow We will make c.f. table for given distribution table as below:

Marks obtai (x_i)	No. of students (f_i)	$f_i \cdot x_i$	c.f
5	3	15	3
6	9	54	12
7	6	42	18
8	4	32	22
9	2	18	24
10	1	10	25
Total	$\sum f_i = 25$	$\sum f_i x_i = 171$	

Here, $\boxed{n = 25} \rightarrow$ odd

Then, Median is given by $\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term} = \left(\frac{25+1}{2}\right)^{\text{th}} \text{ term}$

$$\text{Median} = 26/2 = 13^{\text{th}} \text{ term}$$

$$\boxed{\text{Median} = 7}$$

Here, the no. 6 occurs maximum times.

Hence, $\boxed{\text{Mode} = 6}$

9) The following table gives the weekly wages (in Rs.) of workers in a factory.

Weekly wages (in Rs.)	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
No. of workers	5	20	10	10	9	6	12	8

Calculate: i) The mean

ii) The modal class

iii) the no. of workers getting weekly wages below Rs. 80

iv) the no. of workers getting Rs. 65 or more but less than Rs. 85 as weekly wages.

→ From given distribution table we will make c.f. table as below:

Weekly wages	No. of workers (f)	Class Marks (x)	c.f.	f x x
50-55	5	52.5	5	262.5
55-60	20	57.5	25	1150.0
60-65	10	62.5	35	625.0
65-70	10	67.5	45	675.0
70-75	9	72.5	54	652.5
75-80	6	77.5	60	465.0
80-85	12	82.5	72	990.0
85-90	8	87.5	80	700.0
Total	80			5520.0

i) Mean = $\frac{\sum fx}{\sum f} = \frac{5520}{80} = 69$ Mean = 69 is the required mean

ii) Modal class:

The frequency of class (55-60) is found to be maximum.

Thus, class (55-60) is the modal class.

iii) The no. of workers getting weekly wages below Rs. 80 are found to be 60.

iv) No. of workers getting Rs. 65 or more but less than 85 as weekly wages are found to be = $72 - 35 = 37$.

Exercise 2.4

1.) Draw a histogram for the following frequency distribution and find the mode from the graph:

Class	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	2	5	18	14	8	5

→ From given distribution table we will draw a histogram as given below:



Thus, from the histogram we will found that,
mode = 14

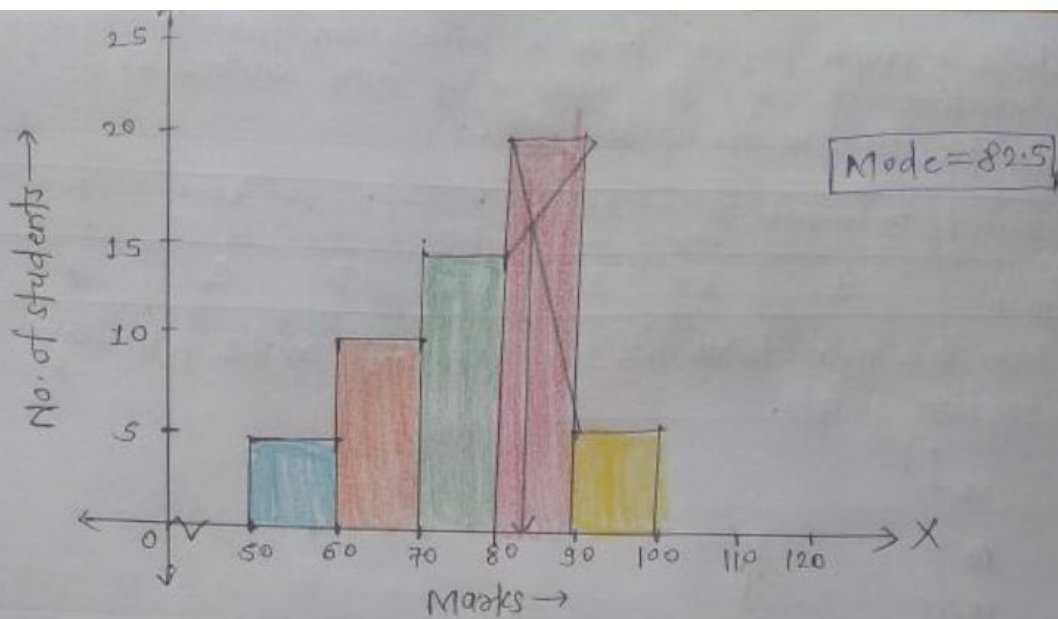
2.) A mathematics aptitude test of 50 students was recorded as follows:

Marks	50-60	60-70	70-80	80-90	90-100
No. of students	4	8	14	19	5

Draw a histogram for the above data using a graph paper and locate the mode.

→ From the given distribution table we will draw a histogram as below:

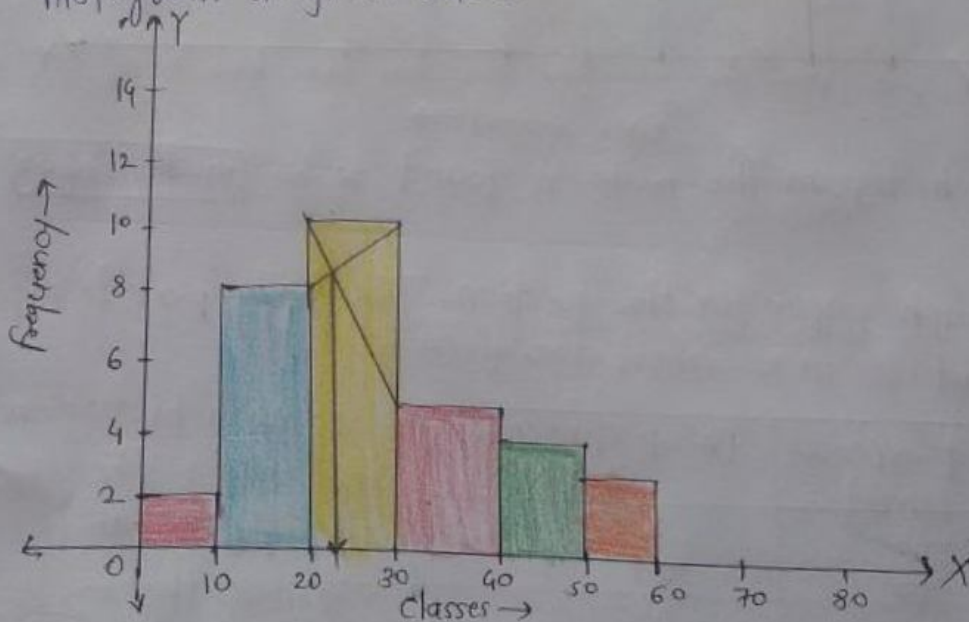
Here, the mode from the histogram is found to be mode = 82.5



3) Draw a histogram and estimate the mode for the following frequency distribution:

Class	0-10	10-20	20-30	30-40	40-50	50-60
frequency	2	8	10	5	4	3

→ From the given frequency distribution table we will draw a histogram as given below:



Here, from histogram graph we got mode as 23.

Mode = 23

4) Using a graph paper draw a histogram for the given distribution showing the number of runs scored by 50 batsmen. Estimate the mode of the data.

Runs Scored	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000
No. of Batsmen	4	18	9	6	7	2	4

→ From the given distribution table we will make following histogram graph.

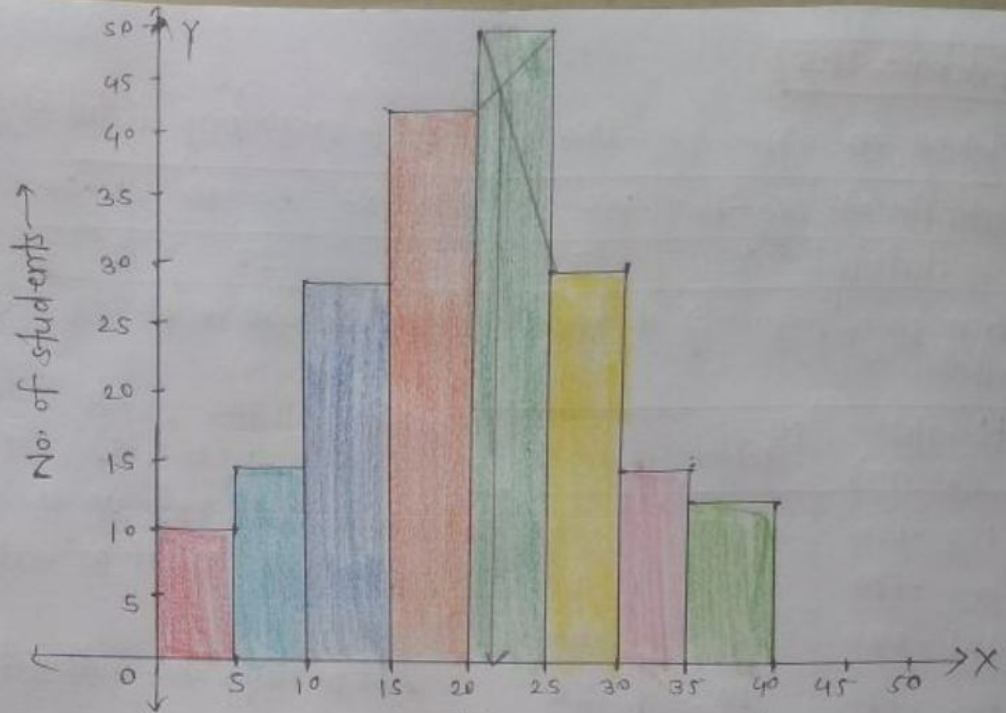


Here, from histogram the mode is found to be $\boxed{\text{mode} = 4600}$

5) Use a graph paper for this question. The daily pocket expenses of 200 students in a school are given below.

Pocket expenses (in Rs)	No. of students (frequency)
0-5	10
5-10	14
10-15	28
15-20	42
20-25	50
25-30	30
30-35	14
35-40	12

Draw a histogram representing the above distribution and estimate the mode from the graph.

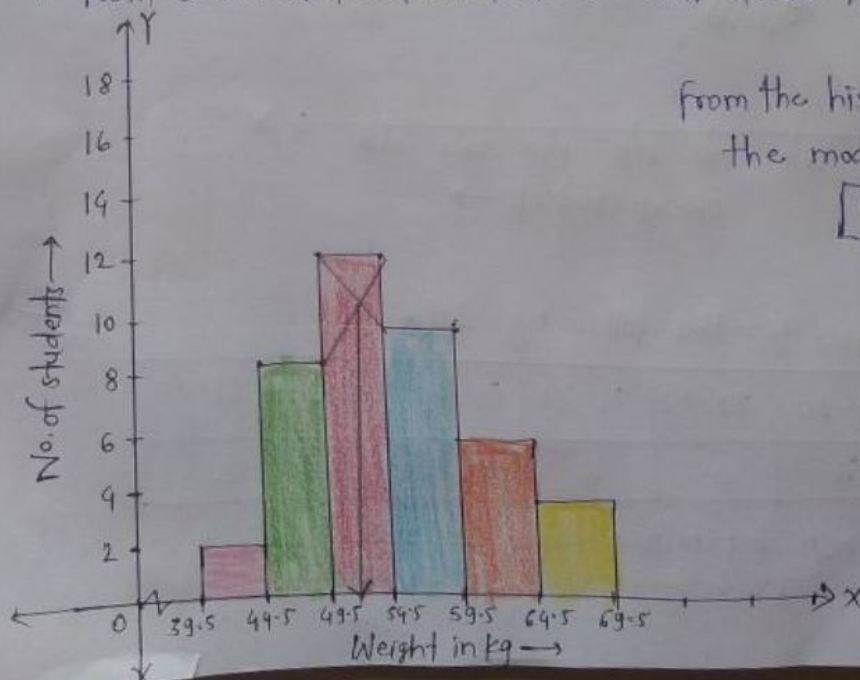


Pocket expenses in Rs. \rightarrow
 from the above histogram graph, the mode is found to be 21
Mode = 21

6) Draw a histogram for the following distribution. Hence estimate the modal weight.

Wt. in kg	40-44	45-49	50-54	55-59	60-64	65-69
No. of Students	2	8	12	10	6	4

\rightarrow from above distribution table we will draw histogram as below:



from the histogram graph,
 the mode is found to be
mode = 51.5

Exercise 21.5

1.) Draw an ogive for the following frequency distribution:

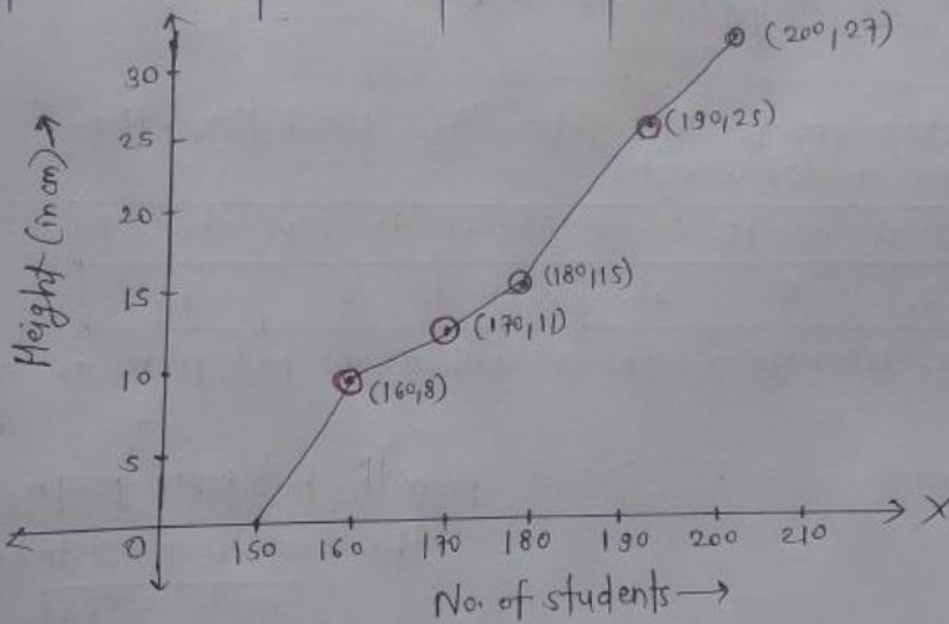
Height (in cm)	150-160	160-170	170-180	180-190	190-200
No. of students	8	3	4	10	2

→ From given frequency distribution table we will make c.f. table as below:

Height (In cm)	No. of students (f)	c.f.
150-160	8	8
160-170	3	11
170-180	4	15
180-190	10	25
190-200	2	27

We plot the points $(160, 8)$, $(170, 11)$, $(180, 15)$, $(190, 25)$ and $(200, 27)$ on the graph & we will join them with free hand.

And finally, we will get an ogive as shown.



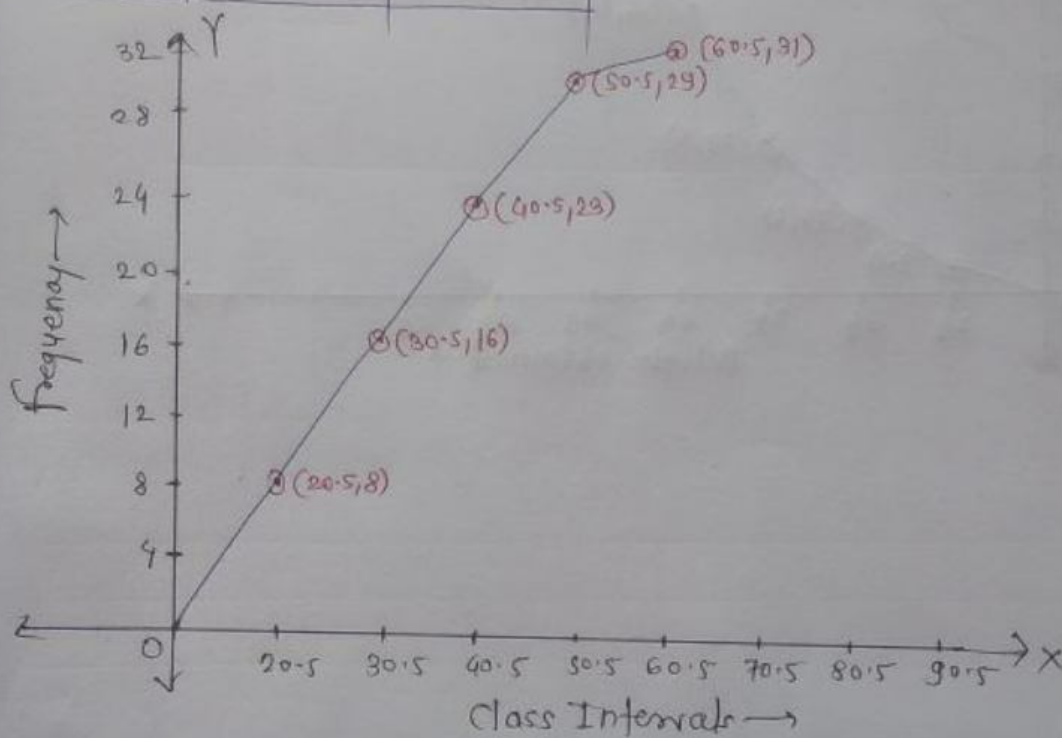
2.) Draw an ogive for the following data:

Class Interval	1-10	11-20	21-30	31-40	41-50	51-60
frequency	3	5	8	7	6	2

→ From the given distribution table we will make c.f. table as given below & from that we will draw ogive.

Class Interval	frequency	c.f.
0.5-10.5	3	3
10.5-20.5	5	8
20.5-30.5	8	16
30.5-40.5	7	23
40.5-50.5	6	29
50.5-60.5	2	31

We plot the points $(10.5, 3)$, $(20.5, 8)$, $(30.5, 16)$, $(40.5, 23)$, $(50.5, 29)$, $(60.5, 31)$ on the graph as shown below and we will join these points with a free hand. And finally, we will get an ogive as shown below.



3. > Draw a cumulative frequency curve for the following data:

Marks Obtained	24-29	29-34	34-39	39-44	44-49	49-54	54-59
No. of Students	1	2	5	6	4	3	2

→ From given distribution table we will make c.f table as below:

Marks Obtained	24-29	29-34	34-39	39-44	44-49	49-54	54-59
No. of Students	1	2	5	6	4	3	2
c.f	1	3	8	14	18	21	23

We plot the points $(29, 1)$, $(34, 3)$, $(39, 8)$, $(44, 14)$, $(49, 18)$, $(54, 21)$ and $(59, 23)$ on the graph paper as shown and after that we will join these points with free hand to get an ogive as shown.

