Chapter 7. Ratio and proportion

1.) An alloy consists of $27\frac{1}{2}$ kg of copper and $2\frac{3}{4}$ kg of tin. Find the ratio by weight of tin to the alloy.

→ An alloy consists of copper $= 27\frac{1}{2}$ kg $= \frac{55}{2}$ kg

and Tin $= 2\frac{3}{4}$ kg $= 11/4$ kg

Total weight of alloy $= \frac{55}{2} + \frac{11}{4} = \frac{110+11}{4} = \frac{121}{4}$ kg

Then, $\frac{\text{weight of tin}}{\text{weight of alloy}} = \frac{11/4}{121/4} = \frac{11}{121} = \frac{1}{11}$

Thus, the required ratio of weight of tin to alloy is 1:11.

2.) Find the compounded ratio of:

i) 2 : 3 and 4 : 9

→ Here,

Compound ratio $= \left(\frac{2}{3}\right) \times \left(\frac{4}{9}\right)$

$= \frac{8}{27}$

$\boxed{\text{Compound ratio} = 8:27}$

ii) 4 : 5, 5 : 7 and 9 : 11

→ Here,

Compound ratio $= \left(\frac{4}{5}\right) \times \left(\frac{5}{7}\right)\left(\frac{9}{11}\right)$

$= \frac{36}{77}$

$\boxed{\text{Compound ratio} = 36:77}$

iii) $(a-b):(a+b)$, $(a+b)^2:(a^2+b^2)$ and $(a^4-b^4):(a^2-b^2)^2$

→ Here, Compound ratio $= \frac{(a-b)}{(a+b)} \times \frac{(a+b)^2}{(a^2+b^2)} \times \frac{(a^4-b^4)}{(a^2-b^2)^2}$

$= \frac{(a-b)(a+b)(a^4-b^4)}{(a^2+b^2)(a-b)^2(a+b)^2} = \frac{(a^4-b^4)}{(a^2+b^2)(a-b)(a+b)}$

$\because a^2-b^2 = (a-b)(a+b)$

$$\text{Compound ratio} = \frac{a^4-b^4}{(a^2+b^2)(a-b)(a+b)}$$

$$\because a^2-b^2 = (a-b)(a+b) \Rightarrow a^4-b^4 = (a^2-b^2)(a^2+b^2)$$

$$\Rightarrow \text{Compound ratio} = \frac{(a^2-b^2)(a^2+b^2)}{(a^2+b^2)(a-b)(a+b)}$$

$$= \frac{(a-b)(a+b)}{(a-b)(a+b)}$$

$$\boxed{\text{Compound ratio} = \frac{1}{1} = 1:1}$$

3.) find the duplicate ratio of

i) 2:3

→ Duplicate ratio of 2:3 $\Rightarrow (2)^2:(3)^2 = 4:9$

ii) $\sqrt{5}:7$

→ Duplicate ratio of $\sqrt{5}:7 \Rightarrow (\sqrt{5})^2:7^2 = 5:49$

iii) 5a:6b

→ Duplicate ratio of 5a:6b $\Rightarrow (5a)^2:(6b)^2 = 25a^2:36b^2$

4.) find the triplicate ratio of

i) 3:4

→ Triplicate ratio of 3:4 $\Rightarrow (3)^3:(4)^3 = \cancel{9}\ 27:64$

ii) $1/2 : 1/3$

→ Triplicate ratio of $1/2 : 1/3 \Rightarrow (1/2)^3:(1/3)^3 = \frac{1}{8}:\frac{1}{27} = 27:8$

iii) $1^3:2^3$

→ Triplicate ratio of $1^3:2^3 \Rightarrow (1^3):(2^3) = 1^3:8^3 = 1:512$

5) Find the sub-duplicate ratio of

i) $9:16$

→ Subduplicate ratio of $9:16 \Rightarrow \sqrt{9}:\sqrt{16} = \sqrt{9}:\sqrt{16} = 3:4$

ii) $\frac{1}{4}:\frac{1}{9}$

→ Sub-duplicate ratio of $\frac{1}{4}:\frac{1}{9} \Rightarrow \frac{1}{\sqrt{4}}:\frac{1}{\sqrt{9}} = \frac{1}{2}:\frac{1}{3} = 3:2$

iii) $27a^3 : 9a^2 : 49b^2$

→ Sub-duplicate ratio of $9a^2 : 49b^2 \Rightarrow \sqrt{9a^2} : \sqrt{49b^2} = 3a:7b$

6) Find the subtriplicate ratio of

i) $1:216$

→ Sub-triplicate ratio of $1:216 \Rightarrow \sqrt[3]{1} : \sqrt[3]{216} = 1:6$

ii) $\frac{1}{8}:\frac{1}{125}$

→ Sub-triplicate ratio of $\frac{1}{8}:\frac{1}{125} \Rightarrow \left(\frac{1}{8}\right)^{1/3} : \left(\frac{1}{125}\right)^{1/3} = \frac{1}{2} : \frac{1}{5} = 5:2$

iii) $27a^3 : 64b^3$

→ Sub-triplicate ratio of $27a^3 : 64b^3 \Rightarrow (27a^3)^{1/3} : (64b)^{3} = 3a:4b$

7) Find the reciprocal ratio of

i) $4:7$

→ Reciprocal ratio of $4:7 \Rightarrow 7:4$

ii) $3^2:4^2$

→ Reciprocal ratio of $3^2:4^2 \Rightarrow 4^2:3^2 = 16:9$

iii) $\frac{1}{9}:2$

→ Reciprocal ratio of $\frac{1}{9}:2 \Rightarrow 2:\frac{1}{9} = 18:1$

8.) Arrange the following ratios in ascending order of magnitude:
2:3, 17:21, 11:14 and 5:7

→ Given ratios are ⇒ 2:3, 17:21, 11:14 and 5:7

Ratios in fractions ⇒ $\frac{2}{3}, \frac{17}{21}, \frac{11}{14}$ & $\frac{5}{7}$

Here, the LCM of 3, 21, 14 & 7 is 42

Thus, we convert ratio as equivalent.

$$\Rightarrow \frac{2}{3} = \frac{2\times14}{3\times14} = \frac{28}{42}$$

$$\frac{17}{21} = \frac{17\times2}{21\times2} = \frac{34}{42}$$

$$\frac{11}{14} = \frac{11\times3}{14\times3} = \frac{33}{42}$$

$$\frac{5}{7} = \frac{5\times6}{7\times6} = \frac{30}{42}$$

Now, the ratios in ascending order will be as follows:

$$\frac{28}{42}, \frac{30}{42}, \frac{33}{42}, \frac{34}{42} \Rightarrow \frac{2}{3}, \frac{5}{7}, \frac{11}{14}, \frac{17}{21}$$

9.) i) If A:B = 2:3, B:C = 4:5 and C:D = 6:7, find A:D

→ Given that, A:B = 2:3, B:C = 4:5 and C:D = 6:7

Then, $\frac{A}{B} = \frac{2}{3}$, $\frac{B}{C} = \frac{4}{5}$ and $\frac{C}{D} = \frac{6}{7}$

We find A:D ⇒ $\frac{A}{B} \times \frac{B}{C} \times \frac{C}{D} = \frac{A}{D}$

$$\left(\frac{2}{3}\right) \times \left(\frac{4}{5}\right) \times \left(\frac{6}{7}\right) = \frac{A}{D}$$

$$\frac{16}{35} = \frac{A}{D}$$

$$\boxed{A:D = 16:35}$$

9) If $x : y = 2 : 3$ and $y : z = 4 : 7$, find $x : y : z$.

$\longrightarrow$ Given that, $x : y = 2 : 3$, $y : z = 4 : 7$,

Then, $\frac{x}{y} = \frac{2}{3}$ and $\frac{y}{z} = \frac{4}{7}$

LCM of 3 & 4 is 12. $\Rightarrow$ By making equals of y as 12.

$\Rightarrow \frac{x}{y} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \Rightarrow x : y = 8 : 12$

$\frac{y}{z} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21} \Rightarrow y : z = 12 : 21$

Thus, $\boxed{x : y : z = 8 : 12 : 21}$

10) If $A : B = 1/4 : 1/5$ and $B : C = 1/7 : 1/6$, find $A : B : C$

$\longrightarrow$ Given that, $A : B = \frac{1}{4} : \frac{1}{5} = 5 : 4$

and $B : C = \frac{1}{7} : \frac{1}{6} = 6 : 7$

Hence, $\frac{A}{B} = \frac{5}{4}$ and $\frac{B}{C} = \frac{6}{7}$

Making equals of B as 12. (LCM of 6 & 4 is 12)

$\Rightarrow \frac{A}{B} = \frac{5 \times 3}{4 \times 3} = \frac{15}{12} \Rightarrow A : B = 15 : 12$

$\frac{B}{C} = \frac{6 \times 2}{7 \times 2} = \frac{12}{14} \Rightarrow B : C = 12 : 14$

Thus, $\boxed{A : B : C = 15 : 12 : 14}$

11) If $3A = 4B = 6C$, find $A : B : C$

$\longrightarrow$ Given that $3A = 4B = 6C$

Now, $3A = 4B$ and $4B = 6C$

$\frac{A}{B} = \frac{4}{3}$ and $\frac{B}{C} = \frac{6}{4} = \frac{3}{2}$

Thus, $\boxed{A : B : C = 4 : 3 : 2}$

11.) i) If $\frac{3x+5y}{3x-5y} = \frac{7}{3}$, find $x:y$

$\rightarrow$ Given that, $\frac{(3x+5y)}{(3x-5y)} = \frac{7}{3}$

$$9x+15y = 21x-35y$$
$$12x = 50y$$
$$\frac{x}{y} = \frac{50}{12} = \frac{25}{6} \Rightarrow \boxed{x:y = 25:6}$$

ii) If $a:b = 3:11$, find $(15a-3b):(9a+5b)$

$\rightarrow$ Given that, $a:b = 3:11$

$$\Rightarrow a/b = 3/11$$

Now, $\frac{(15a-3b)}{(9a+5b)} = \frac{(15a/b-3)}{(9a/b+5)}$ dividing N^{or} & D^{er} by b

$$= \frac{(15\times 3/11-3)}{(9\times\frac{3}{11}+5)} \quad \because \frac{a}{b} = \frac{3}{11}$$

$$\frac{(15a-3b)}{(9a+5b)} = \frac{(45-33)}{(27+55)} = \frac{(12)}{(82)} = \frac{6}{41}$$

Thus, $\boxed{(15a-3b):(9a+5b) = 6:41}$

12.) i) If $(4x^2+xy):(3xy-y^2) = 12:5$, find $(x+2y):(2x+y)$.

$\rightarrow$ Given that, $(4x^2+xy):(3xy-y^2) = 12:5$

$$\frac{(4x^2+xy)}{(3xy-y^2)} = \frac{12}{5}$$
$$20x^2+5xy = 36xy-12y^2$$
$$20x^2-31xy+12y^2 = 0$$
$$20x^2/y^2 - 31xy/y^2 + 12y^2/y^2 = 0$$
$$20(x/y)^2 - 31(x/y) + 12 = 0$$
$$20(x/y)^2 - 15(x/y) - 16(x/y) + 12 = 0$$
$$5(x/y)[4(x/y)-3] - 4[4(x/y)-3] = 0$$

$[4(x/y)-3][5(x/y)-4]=0$

$4(x/y)-3=0$ or $5(x/y)-4=0$

$4(x/y)=3$ or $5(x/y)=4$

$\boxed{\frac{x}{y}=\frac{3}{4}}$ or $\boxed{x/y=\frac{4}{5}}$

Now, $\frac{(x+2y)}{(2x+y)}=\frac{(x/y+2)}{(2x/y+1)}$

a) If $\frac{x}{y}=\frac{3}{4}$ then $\frac{(x/y+2)}{(2x/y+1)}=\frac{(3/4+2)}{(6/4+1)}=\frac{(3+8)}{(6+4)}=\frac{11}{20}$

Thus, $\boxed{(x+2y):(2x+y)=11:20}$

b) If $\frac{x}{y}=\frac{4}{5}$ then $\frac{(x/y+2)}{(2x/y+1)}=\frac{(4/5+2)}{(8/5+1)}=\frac{(4+10)}{(8+5)}=\frac{14}{13}$

Thus, $\boxed{(x+2y):(2x+y)=14:13}$

13.) i) If $(x-9):(3x+6)$ is the duplicate ratio of $4:9$, find the value of x.

→ Given that, $(x-9):(3x+6)$ is the duplicate ratio of $4:9$.

But, duplicate ratio of $4:9 \Rightarrow 4^2:9^2=16:81$

Thus, $\frac{(x-9)}{(3x+6)}=\frac{16}{81}$

$81x-9(81)=48x+96$

$81x-48x=96+9(81)$

$33x=96+729$

$33x=825$

$x=825/33$

$\boxed{x=25}$

ii) If $(3x+1):(5x+3)$ is the triplicate ratio of $3:4$, find the value of x.

→ Given that, $(3x+1):(5x+3)$ is the triplicate ratio of $3:4$.

But, the triplicate ratio of $3:4 = 3^3:4^3 = 27:64$

Thus, $$\frac{(3x+1)}{(5x+3)} = \frac{27}{64}$$

$$192x+64 = 135x+81$$

$$192x-135x = 81-64$$

$$57x = 17$$

$\boxed{x = 17/57}$ is the required value of x.

iii) If $(x+2y):(2x-y)$ is equal to the duplicate ratio of $3:2$, find $x:y$

→ Given that, $(x+2y):(2x-y)$ is equal to duplicate ratio of $3:2$.

But, duplicate ratio of $3:2 = 3^2:2^2 = 9:4$

Thus, $$\frac{(x+2y)}{(2x-y)} = \frac{9}{4}$$

$$4x+8y = 18x-9y$$

$$18x-4x = 8y+9y$$

$$14x = 17y$$

$\frac{x}{y} = 17/14$ Thus $\boxed{x:y = 17:14}$

Q.14) i) Find two numbers in the ratio of $8:7$ such that when each is decreased by $12\frac{1}{2}$, they are in the ratio $11:9$.

→ Given that, two numbers in the ratio of $8:7$ such that when each is decreased by $12\frac{1}{2}$, they are in the ratio $11:9$.

→ Let us consider the numbers $8x$ & $7x$.

From given condition,

$$\frac{(8x-25/2)}{(7x-25/2)} = \frac{11}{9}$$

After taking LCM, $\frac{(16x-25)/2}{(14x-25)/2} = \frac{11}{9}$

$(16x-25)/(14x-25) = 11/9$

$154x - 144x = 275 - 225$

$10x = 50$

$x = 50/10 = 5$ $\boxed{x=5}$

Thus, $8x = 8 \times 5 = 40$

$7x = 7 \times 5 = 35$

Thus, the required numbers are 40 & 35 respectively.

ii) The income of a man is increased in the ratio of 10:11, if the increase in his income is Rs. 600 per month, find his new income.

→ Let us consider the present income be '$10x$'.

and increased income be '$11x$'.

Increase in his income per month = 600

Thus, $11x - 10x = x = 600$ Rs.

Hence, new income $= 11x = 11 \times 600 =$ Rs. 6600.

15) i) A woman reduces her weight in the ratio 7:5, What does her weight become if originally it was 91 kg.

ii) A school collected Rs. 2100 for charity. It was decided to divide the money between an orphanage and a blind school in the ratio of 3:4. How much money did each receive?

→ i) Given that, ratio between the original weight & reduced weight = 7:5

Let us consider the original weight = $7x$

and reduced weight = $5x$

If original weight is 91 kg then $91 kg = 7x$

$\boxed{x = 13}$

Hence, the reduced weight $= 5x = 13 \times 5 = 65$ kg

ii) Amount collected for charity = Rs. 2100

Given that, the ratio between orphanage & a blind school is 3:4.

Then sum of ratios = $3+4=7$

Thus, orphanage schools share $= 2100 \times \frac{3}{7} =$ Rs. 900

and Blind schools share $= 2100 \times 4/7 =$ Rs. 1200

16) The sides of a triangle are in the ratio 7:5:3 & its perimeter is 30cm. Find the lengths of sides.

→ Given that, Perimeter of triangle = 30 cm

Ratio among the sides = 7:5:3

Sum of ratios = $7+5+3 = 15$

Then, Length of first side $= 30 \times \frac{7}{15} = 14$cm

Length of second side $= 30 \times 5/15 = 10$cm

Length of third side $= 30 \times 3/15 = 6$ cm

Thus, the sides of triangle are 14cm, 10cm, 6cm.

17) Three numbers are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. If the sum of their squares is 244, find the numbers.

→ Given that, Ratio of three numbers $= \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

$= (6:4:3)/12$

$= 6:4:3$

Then, first number $= 6x$

Second number $= 4x$

Third number $= 3x$

from given condition,

$(6x)^2 + (4x)^2 + (3x)^2 = 244$

$36x^2 + 16x^2 + 9x^2 = 244$

$$61x^2 = 244$$

$$x^2 = 244/61 = 4 = 2^2$$

$$\boxed{x=2}$$

Thus, first number $= 6x = 6 \times 2 = 12$

Second number $= 4x = 4 \times 2 = 8$

Third number $= 3x = 3 \times 2 = 6$

18.) i) A certain sum was divided among A, B and C in the ratio 7:5:4. If B got 500 more than C, find the total sum divided.

→ Given that, Ratio betwen A, B and C = 7:5:4

Let us consider, A's share $= 7x$

B's share $= 5x$

and C's share $= 4x$

Then, total sum $= 7x + 5x + 4x = 16x$

$$5x - 4x = 500$$

$$\boxed{x = 500}$$

Hence, the total sum $= 16x = 16 \times 500 =$ Rs. 8000.

19.) i) In a mixture of 45 litres, the ratio of milk to water is 13:2. How much water must be added to this mixture to make the ratio of milk to water as 3:1.

→ Given that, mixture of milk & water = 45 litres.

And ratio of milk & water = 13:2

Sum of ratio = 13 + 2 = 15

Then, quantity of milk $= (45 \times 13)/15 = 39$ litres.

quantity of water $= 45 \times 2/15 = 6$ litres.

Let us consider, x be the water to be added.

Thus, water $= (6+x)$ litres

Thus, the new ratio $= 3:1$

$$39 : (6+x) = 3:1$$

$$39/(6+x) = 3/1$$

$$39 = 18+3x$$

$$3x = 39-18 = 21$$

$$x = 21/3 = 7 \text{ litres}$$

Thus, 7 litre water is added to make the ratio of milk to water as 3:1.

20) The monthly pocket money of Ravi & Sanjeev are in the ratio 5:7. Their expenditures are in the ratio 3:5. If each saves Rs. 80 every month, find their monthly pocket money?

→ Let us consider the monthly pocket money of Ravi & Sanjeev be $5x$ and $7x$ respectively.

Then, their expenditure be $3y$ and $5y$ respectively.

Thus, $5x - 3y = 80$ —— ①

$7x - 5y = 80$ —— ②

① × 7 → $35x - 21y = 560$

② × 5 → $35x - 25y = 400$

② − ① ⇒ $4y = 160$

$$\boxed{y = 40}$$

Thus, $5x = 80+3\times 40 = 200$

$$\boxed{x = 40}$$

Thus, monthly pocket money of Ravi is $= 5\times 40 = 200$.

21.) In an examination, the ratio of passes to failures was 4:1. If 30 less had appeared and 20 less passed, the ratio of passes to failures would have been 5:1. How many students appeared for the examination.

→ Let us consider the no. of passes $= 4x$

and no. of failures $= x$

Thus, the total no. of students appeared $= 4x + x = 5x$

from second condition,

The no. of students appeared $= 5x - 30$

and no. of passes $= 4x - 20$

Thus, the no. of failures found to be $= (5x-30) - (4x-20)$

$= 5x - 30 - 4x + 20$

$= x - 10$

Thus, $\frac{(4x-20)}{(x-10)} = \frac{5}{1}$

$5x - 50 = 4x - 20$

$5x - 4x = -20 + 50$

$\boxed{x = 30}$

Hence, the no. of students appeared are $= 5x = 5 \times 30 = 150$.

Exercise 7.2

1.) find the value of x in the following proportions:

i) $10:35 = x:42$

→ Given that, $10:35 = x:42$

$$\frac{10}{35} = \frac{x}{42}$$

$$35x = 420$$

$$x = \frac{420}{35}$$

$$\boxed{x = 12}$$

This is the required value.

ii) $3:x = 24:2$

→ Given that, $3:x = 24:2$

$$\frac{3}{x} = \frac{24}{2}$$

$$6 = 24x$$

$$x = 6/24$$

$$\boxed{x = 1/4}$$

This is the required value.

iii) $2.5:1.5 = x:3$

→ Given that, $2.5:1.5 = x:3$

$$\frac{2.5}{1.5} = \frac{x}{3}$$

$$7.5 = 1.5x$$

$$x = \frac{7.5}{15} = \frac{75}{15}$$

$$\boxed{x = 5}$$

This is the required value.

iv) $x:50 = 3:2$

→ Given that $x:50 = 3:2$

$$\frac{x}{50} = \frac{3}{2}$$

$$2x = 150$$

$$x = \frac{150}{2}$$

$$\boxed{x = 75}$$

This is the required value.

2.) find the fourth proportional to

i) $3, 12, 15$

→ Let us consider fourth proportional to $3, 12, 15$ be x.

Then, $3:12::15:x$

$$3x = 12 \times 15$$

$$\boxed{x = 60}$$

ii) $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

→ Let us consider fourth proportional to $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ be x

Then $\frac{1}{3} : \frac{1}{4} :: \frac{1}{5} : x$

~~4 : 3 :: x : 5~~

~~20 = 3x~~

~~x =~~

$\frac{1}{3} : \frac{1}{4} :: \frac{1}{5} : x$

$\frac{1}{3}(x) = (\frac{1}{4})(\frac{1}{5})$

$\frac{x}{3} = \frac{1}{20}$

$\boxed{x = 3/20}$

This is the required value.

iii) 1.5, 2.5, 4.5

→ Let us consider fourth proportional to 1.5, 2.5, 4.5 be x.

Then, $1.5 : 2.5 :: 4.5 : x$

$1.5(x) = 2.5(4.5)$

$x = (2.5 \times 4.5)/1.5$

$\boxed{x = 7.5}$

This is the required value.

3) Find the third proportional to

i) 5, 10

→ Let us consider third proportional to 5, 10 be x.

Then $5 : 10 :: 10 : x$

$\Rightarrow (5x) = 10 \times 10$

$x = 100/5$

$\boxed{x = 20}$

Thus, the third proportional to 5, 10 is 20.

ii) Rs. 3, Rs. 12

→ Let us consider third proportional to 3, 12 be x.

Then, $3 : 12 :: 12 : x$

$\Rightarrow 3x = 12 \times 12$

$x = 144/3$

$\boxed{x = 48}$

Thus, the third proportional to 3, 12 is 48.

4) Find the mean proportion of:

i) 5 and 80

→ Let us consider mean proportion of 5 & 80 be x.

Then, $5 : x :: x : 80$

$x^2 = 5 \times 80 = 400$

$\boxed{x = 20}$

Thus, the mean proportion of 5 & 80 is 20.

ii) 1/12 & 1/75

→ Let us consider mean proportion of 1/12 & 1/75 be x.

Then, $\frac{1}{12} : x :: x : \frac{1}{75}$

$x^2 = \frac{1}{12} \times \frac{1}{75} = \frac{1}{900}$

$\boxed{x = \frac{1}{30}}$

Thus, the mean proportion of $\frac{1}{12}$ & $\frac{1}{75}$ is 1/30.

5) If $a, 12, 16$ and b are in continued proportion find a and b.

→ Given that, $a, 12, 16$ & b are in the continued propostion.

Then, $\frac{a}{12} = \frac{12}{16} = \frac{16}{b}$

$\frac{a}{12} = \frac{12}{16}$ and $\frac{12}{16} = \frac{16}{b}$

$16a = 144$ and $12b = 256$

$a = 144/16$ $\qquad$ $b = 256/12$

$\boxed{a = 9}$ $\qquad$ $\boxed{b = 64/3}$

Thus, the required values of a and b found to be 9 & $64/3$ respe.

6) What number must be added to each of the no. 5, 11, 19 & 37 so that they are in proportion.

→ Let us consider 'x' be the number added to 5, 11, 19 & 37 to make in proportion.

Then, $(5+x) : (11+x) :: (19+x) : (37+x)$

$$(5+x)(37+x) = (11+x)(19+x)$$

$$\Rightarrow 185 + 5x + 37x + x^2 = 209 + 11x + 19x + x^2$$

$$185 + 42x + x^2 = 209 + 30x + x^2$$

$$42x - 30x + x^2 - x^2 = 209 - 185$$

$$12x = 24$$

$\boxed{x = 2}$ This is the required number.

7) What number should be subtracted from each of the numbers 23, 30, 57 and 78 so that the remainders are in proportion.

→ Let us consider 'x' be the number subtracted from each term.

Then, $(23-x)$, $(30-x)$, $(57-x)$ and $(78-x)$ are proportional.

Thus, $(23-x) : (30-x) :: (57-x) : (78-x)$

$$\frac{(23-x)}{(30-x)} = \frac{(57-x)}{(78-x)}$$

$$(23-x)(78-x) = (57-x)(30-x)$$

$$1794 - 23x - 78x + x^2 = 1710 - 30x - 57x + x^2$$

$$x^2 - 101x + 1794 - x^2 + 87x - 1710 = 0$$

$$-14x + 84 = 0$$

$$14x = 84$$

$$x = 84/14 = 6 \quad \boxed{x=6}$$

This is the required number.

8) The following numbers $(k+3)$, $(k+2)$, $(3k-7)$ and $(2k-3)$ are in proportion. find k.

→ Given that, $(k+3)$, $(k+2)$, $(3k-7)$ and $(2k-3)$ are in proportion.

Then, $\frac{(k+3)}{(k+2)} = \frac{(3k-7)}{(2k-3)}$

$$\Rightarrow (k+3)(2k-3) = (k+2)(3k-7)$$

$$2k^2 - 3k + 6k - 9 = 3k^2 - 7k + 6k - 14$$

$$k^2 - 4k - 5 = 0$$

$$(k-5)(k+1) = 0$$

$$\boxed{k=5} \text{ or } \boxed{k=-1}$$

is the required value of k.

9.) If $(x+5)$ is the mean proportion between $(x+2)$ and $(x+9)$, find the value of x.

$\rightarrow$ Given that, $(x+5)$ is the mean proportion between $(x+2)$ and $(x+9)$.

Then, $(x+5)^2 = (x+2)(x+9)$

$x^2 + 10x + 25 = x^2 + 11x + 18$

and $x^2 + 10x - x^2 - 11x = 18 - 25$

Thus, $-x = -7$

$\boxed{x=7}$ is the required value.

10.) What number must be added to each of the numbers 16, 26 and 40 so that the resulting numbers may be in continued proportion?

$\rightarrow$ Let us consider 'x' be the number added to each term.

Then, $(16+x)$, $(26+x)$ and $(40+x)$ are in continued proportion.

$\Rightarrow (16+x)/(26+x) = (26+x)/(40+x)$

$(16+x)(40+x) = (26+x)(26+x)$

$640 + 16x + 40x + x^2 = 676 + 26x + 26x + x^2$

$640 + 56x + x^2 = 676 + 52x + x^2$

$56x + x^2 - 52x - x^2 = 676 - 640$

$4x = 36$

$x = 36/4 = 9$

$\boxed{x=9}$ is the required value.

12.) If b is the mean proportional between a and c, prove that $a, c, (a^2+b^2)$ and (b^2+c^2) are proportional.

→ Given that, b is the mean proportional between a and c. Then, $b^2 = ac$

And, $a, c, (a^2+b^2)$ and (b^2+c^2) are proportional.

Thus, $a/c = (a^2+b^2)/(b^2+c^2)$

$a(b^2+c^2) = c(a^2+b^2)$

$\Rightarrow a(ac+c^2) = c(a^2+ac)$

$ac(a+c) = a^2c + ac^2$

Thus, $\boxed{ac(a+c) = ac(a+c)}$

14.) If y is mean proportional between x and z, prove that $xyz(x+y+z)^3 = (xy+yz+zx)^3$.

→ Given that, y is mean proportional between x and z.

Then, $y^2 = xz$

Now, L.H.S. $= xyz(x+y+z)^3$

$= xz \cdot y(x+y+z)^3$

$= y^2 \cdot y(x+y+z)^3$

$= y^3(x+y+z)^3$

$= [y(x+y+z)]^3$

$= (xy+y^2+yz)^3$

$= (xy+xz+yz)^3$

$\boxed{(x+y+z)^3 = (xy+yz+zx)^3}$

Hence proved.

15) If $(a+c) = mb$ and $\left(\frac{1}{b}+\frac{1}{d}\right) = \frac{m}{c}$, prove that a, b, c and d are in proportion.

→ Given that, $(a+c) = mb$ and $\left(\frac{1}{b}+\frac{1}{d}\right) = \frac{m}{c}$

divide by b ⇒ $\frac{(a+c)}{b} = m$ multiply by c,

$$\frac{a}{b}+\frac{c}{b} = m \quad \text{—— (1)} \qquad \frac{c}{b}+\frac{c}{d} = m \quad \text{—— (2)}$$

from (1) & (2) ⇒ $\frac{a}{b}+\frac{c}{b} = \frac{c}{b}+\frac{c}{d}$

$\boxed{\frac{a}{b} = \frac{c}{d}}$ Thus, a, b, c and d are in proportion.

16) If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ Prove that $\frac{x^3}{a^2}+\frac{y^3}{b^2}+\frac{z^3}{c^2} = \frac{(x+y+z)^3}{(a+b+c)^2}$

→ Let us consider, $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$

Then, $x = ak$, $y = bk$ and $z = ck$

$$L.H.S. = \frac{x^3}{a^2}+\frac{y^3}{b^2}+\frac{z^3}{c^2}$$

$$L.H.S. = \frac{a^3k^3}{a^2}+\frac{b^3k^3}{b^2}+\frac{c^3k^3}{c^2} = (a+b+c)k^3 \quad \text{—— (1)}$$

Now, $R.H.S. = \frac{(x+y+z)^3}{(a+b+c)^2}$.

$$= \frac{(ak+bk+ck)^3}{(a+b+c)^2} = \frac{k^3(a+b+c)^3}{(a+b+c)^2}$$

$$R.H.S. = k^3(a+b+c) \quad \text{—— (2)}$$

from (1) and (2) ⇒ $\boxed{\frac{x^3}{a^2}+\frac{y^3}{b^2}+\frac{z^3}{c^2} = \frac{(x+y+z)^3}{(a+b+c)^2}}$

Hence proved.

17> If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then prove that $(b^2+d^2+f^2)(a^2+c^2+e^2) = (ab+cd+ef)^2$

$\longrightarrow$ Let us consider, $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

Then, $a = bk$, $c = dk$ and $e = fk$

Now, $L.H.S. = (b^2+d^2+f^2)(a^2+c^2+e^2)$

$= (b^2+d^2+f^2)(b^2k^2+d^2k^2+f^2k^2)$

$= k^2(b^2+d^2+f^2)(b^2+d^2+f^2)$

$L.H.S. = k^2(b^2+d^2+f^2)^2$ —— ①

Now, $R.H.S. = (ab+cd+ef)^2$

$= [(bk)b + (dk)d + (fk)f]^2$

$= (b^2k + d^2k + f^2k)^2$

$R.H.S. = k^2(b^2+d^2+f^2)^2$ —— ②

from ① & ② $\Rightarrow$ $(b^2+d^2+f^2)(a^2+c^2+e^2) = (ab+cd+ef)^2$

Hence proved.

18> If $ax = by = cz$ then prove that $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = \frac{bc}{a^2} + \frac{ca}{b^2} + \frac{ab}{c^2}$

$\rightarrow$ Let us consider, $ax = by = cz = k$

Then, $x = k/a$, $y = k/b$ and $z = k/c$

$$L.H.S = \frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = \frac{k^2/a^2}{(k/b)(k/c)} + \frac{(k^2/b^2)}{(k/c)(k/a)} + \frac{(k^2/c^2)}{(k/a)(k/b)}$$

$$= \frac{1/a^2}{1/bc} + \frac{1/b^2}{1/ac} + \frac{1/c^2}{1/ab}$$

$$= \frac{bc}{a^2} + \frac{ac}{b^2} + \frac{ab}{c^2}$$

$L.H.S. = R.H.S.$

Hence proved.

19.) If a, b, c and d are in proportion. Prove that

i) $(5a+7b)(2c-3d) = (5c+7d)(2a-3b)$

→ Given that a, b, c and d are in proportion.

Let us consider, $\frac{a}{b} = \frac{c}{d} = k$

$\Rightarrow a = bk,\ c = dk$

Now, $L.H.S. = (5a+7b)(2c-3d)$

$= (5bk+7b)(2dk-3d)$

$= k^2(5b+7b)(2d-3d)$

$= k^2(12b)(-d)$

$L.H.S. = -12bdk^2$ —— ①

And $R.H.S. = (5c+7d)(2a-3b)$

$= (5dk+7d)(2bk-3b)$

$= k^2(5d+7d)(2b-3b)$

$= k^2(12d)(-b)$

$R.H.S. = -12bdk^2$ —— ②

from ① & ② ⇒ $(5a+7b)(2c-3d) = (5c+7d)(2a-3b)$

Hence proved

20.) If x, y, z are in continued proportion, prove that

$\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$.

→ Given that, x, y, z are in continued proportion

Let us consider $\frac{x}{y} = \frac{y}{z} = k$

Then, $y = kz$

$x = yk = (kz)k = k^2z$

Now, $LHS = (x+y)^2/(y+z)^2$

$= (k^2z+kz)^2/(kz+z)^2$

$\therefore$ L.H.S. $= [kz(k+1)^2] / [z^2(k+1)^2]$

$\boxed{L.H.S = k^2}$ —— ①

Now, R.H.S $= \frac{x}{z} = \frac{k^2 z}{z} = k^2$ —— ②

from ① & ② $\Rightarrow$ $\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$ Hence proved.

22.) If a, b, c are in continued proportion then prove that

$\frac{(a+b)}{(b+c)} = \frac{a^2(b-c)}{b^2(a-b)}$

$\rightarrow$ Given that, a, b, c are in continued proportion.

Let us consider, $\frac{a}{b} = \frac{b}{c} = k$ $\Rightarrow$ $a = bk$, $b = ck$

Then, $\frac{(a+b)}{(b+c)} = \frac{(bk+b)}{(ck+c)} = \frac{(k+1)b}{(k+1)c} = \frac{b}{c}$ —— ①

Now, R.H.S. $= \frac{a^2(b-c)}{b^2(a-b)} = \frac{b^2k^2(b-c)}{b^2(b^2k^2-b)} = \frac{k^2(b-c)}{b(bk-1)} = k$ —— ②

from ① & ② $\Rightarrow$ $\boxed{\frac{(a+b)}{(b+c)} = \frac{a^2(b-c)}{b^2(a-b)}}$ Hence proved.

23.) If a, b, c, d are in continued proportion, prove that

$\frac{(a^3+b^3+c^3)}{(b^3+c^3+d^3)} = \frac{a}{d}$

$\rightarrow$ Given that, a, b, c, d are in continued proportion.

Let us consider $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$

$\therefore a = bk$, $c = dk$, $b = ck = dk^2$

$a = bk = ck^2 = dk^3$

$\boxed{a = dk^3}$ $\boxed{b = dk^2}$

Now, L.H.S. $= \frac{(a^3+b^3+c^3)}{(b^3+c^3+d^3)}$

$$= \frac{(dk^3)^3 + (dk^2)^3 + (dk)^3}{(dk^2)^3 + (dk)^3 + d^3}$$

$$= \frac{d^3k^9 + d^3k^6 + d^3k^3}{d^3k^6 + d^3k^3 + d^3} = \frac{k^9+k^6+k^3}{k^6+k^3+1}$$

$$= \frac{k^3(k^6+k^3+1)}{(k^6+k^3+1)}$$

L.H.S. $= k^3$ —— ①

And R.H.S. $= \frac{a}{d} = \frac{dk^3}{d} = k^3$ —— ②

from ① & ② $\Rightarrow$ $\frac{(a^3+b^3+c^3)}{(b^3+c^3+d^3)} = \frac{a}{d}$ Hence proved.

$\boxed{\text{Exercise 7.3}}$

1.) If $a:b::c:d$, prove that

i) $\frac{(2a+5b)}{(2a-5b)} = \frac{(2c+5d)}{(2c-5d)}$

→ Given that, $a:b::c:d$

Then, $\frac{a}{b} = \frac{c}{d}$

$\frac{2a}{5b} = \frac{2c}{5d}$

By componendo & dividendo,

$$\boxed{\frac{(2a+5b)}{(2a-5b)} = \frac{(2c+5d)}{(2c-5d)}}$$

Hence proved.

ii) $\frac{(5a+11b)}{(5c+11d)} = \frac{(5a-11b)}{(5c-11d)}$

→ Given that, $a:b::c:d$

Then, $\frac{a}{b} = \frac{c}{d}$

$\frac{5a}{11b} = \frac{5c}{11d}$

By Componendo & dividendo,

$$\frac{(5a+11b)}{(5a-11b)} = \frac{(5c+11d)}{(5c-11d)}$$

$$\Rightarrow \boxed{\frac{(5a+11b)}{(5c+11d)} = \frac{(5a-11b)}{(5c-11d)}}$$

Hence proved

2) If $\frac{(5x+7y)}{(5u+7v)} = \frac{(5x-7y)}{(5u-7v)}$, show that $\frac{x}{y} = \frac{u}{v}$.

$\longrightarrow$ Given that, $\frac{(5x+7y)}{(5u+7v)} = \frac{(5x-7y)}{(5u-7v)}$

Then, by alternendo,

$$\frac{(5x+7y)}{(5u+7y)} = \frac{(5x-7y)}{(5u-7v)}$$

By componendo and dividendo,

$$\frac{(5x+7y+5x-7y)}{(5x+7y-5x+7y)} = \frac{(5u+7v+5u-7v)}{(5u+7u-5u+7v)}$$

$$\frac{10x}{14y} = \frac{10u}{14v}$$

$$\boxed{\frac{x}{y} = \frac{u}{v}}$$ Hence proved.

3) If $(4a+5b)(4c-5d) = (4a-5b)(4c+5d)$, prove that a, b, c, d are in proportion.

$\longrightarrow$ Given that, $(4a+5b)(4c-5d) = (4a-5b)(4c+5d)$

$$\Rightarrow \frac{(4a+5b)}{(4a-5b)} = \frac{(4c+5d)}{(4c-5d)}$$

By componendo & dividendo,

$$\frac{(4a+5b+4a-5b)}{(4a+5b-4a+5b)} = \frac{(4c+5d+4c-5d)}{(4c+5d-4c+5d)}$$

$$\frac{8a}{10b} = \frac{8c}{10d}$$

$$\boxed{\frac{a}{b} = \frac{c}{d}}$$

Hence, a, b, c are in proportion proved.

4) If $(pa+qb):(pc+qd)::(pa-qb):(pc-ad)$ prove that $a:b::c:d$

→ Given that, $(pa+qb):(pc+qd)::(pa-qb):(pc-ad)$

$$\Rightarrow \frac{(pa+qb)}{(pc+qd)} = \frac{(pa-qb)}{(pc-ad)}$$

$$\Rightarrow \frac{(pa+qb)}{(pa-qd)} = \frac{(pc+qd)}{(pc-qd)}$$

By componendo and dividendo,

$$\frac{(pa+qb+pa-qb)}{(pa+qb-pa+qb)} = \frac{(pc+qd+pc-qd)}{(pc-qd-pc+qd)}$$

$$\frac{(2pa)}{(2qb)} = \frac{(2pc)}{(2qd)}$$

$$\Rightarrow \boxed{\frac{a}{b} = \frac{c}{d}}$$

Thus, $a:b::c:d$ Hence proved.

5) If $(ma+nb):b::(mc+nd):d$, prove that a, b, c, d are in proportion.

→ Given that, $(ma+nb):b::(mc+nd):d$

$$\frac{(ma+nb)}{b} = \frac{(mc+nd)}{d}$$

$$mad+nbd = mbc+nbd$$

$$mad = mbc$$

$$ad = bc$$

$$\boxed{\frac{a}{b} = \frac{c}{d}}$$

Thus, $a:b::c:d$

Hence, a, b, c, d are in proportion.

6) If $(11a^2+13b^2)(11c^2-13d^2) = (11a^2-13b^2)(11c^2+13d^2)$, then prove that $a:b::c:d$.

→ Given that,

$$(11a^2+13b^2)(11c^2-13d^2) = (11a^2-13b^2)(11c^2+13d^2)$$

$$\Rightarrow \frac{(11a^2+13b^2)}{(11a^2-13b^2)} = \frac{(11c^2+13d^2)}{(11c^2-13d^2)}$$

By componendo & dividendo,

$$\frac{(11a^2+13b^2+11a^2-13b^2)}{(11a^2+13b^2-11a^2+13b^2)} = \frac{(11c^2+13d^2+11c^2-13d^2)}{(11c^2+13d^2-11c^2+13d^2)}$$

$$\frac{(22a^2)}{(26b^2)} = \frac{22c^2}{26d^2}$$

$$a^2/b^2 = c^2/d^2 \Rightarrow \boxed{\frac{a}{b}=\frac{c}{d}}$$

Thus, $a:b::c:d$ Hence proved.

7.) If $x = \frac{2ab}{(a+b)}$ find the value of $\frac{(x+a)}{(x-a)} + \frac{(x+b)}{(x-b)}$.

→ Given that, $x = \frac{2ab}{(a+b)}$

$$\frac{x}{a} = \frac{2b}{(a+b)}$$

By componendo & dividendo,

$$\frac{(x+a)}{(x-a)} = \frac{(2b+a+b)}{(2b-a-b)} = \frac{(3b+a)}{(b-a)} \quad \text{—— ①}$$

Now, $\frac{x}{b} = \frac{2a}{(a+b)}$

By componendo & dividendo,

$$\frac{(x+b)}{(x-b)} = \frac{(2a+a+b)}{(2a-a-b)} = \frac{(3a+b)}{(-b+a)} \quad \text{—— ②}$$

from ① & ② ⇒

$$\frac{(x+a)}{(x-a)} + \frac{(x+b)}{(x-b)} = \frac{(3b+a)}{(b-a)} + \frac{(3a+b)}{(-b+a)}$$

$$= \frac{-(3b+a)}{(a-b)} + \frac{(3a+b)}{(a-b)}$$

$$= \frac{-3b-a+3a+b}{(a-b)}$$

$$\Rightarrow \frac{(-2b+2a)}{(a-b)} = \frac{2(a-b)}{(a-b)} = 2$$

$$\boxed{\frac{(x+a)}{(x-a)} + \frac{(x+b)}{(x-b)} = 2}$$

8.) If $x = \frac{8ab}{(a+b)}$ find the value of $\frac{(x+4a)}{(x-4a)} + \frac{(x+4b)}{(x-4b)}$.

$\rightarrow$ Given that, $x = \frac{8ab}{(a+b)}$

$$\frac{x}{4a} = \frac{2b}{(a+b)} \quad \text{By componendo \& dividendo,}$$

$$\frac{(x+4a)}{(x-4a)} = \frac{(2b+a+b)}{(2b-a-b)} = \frac{(3b+a)}{(b-a)} \quad \text{--- (1)}$$

Now, $\frac{x}{4b} = \frac{2a}{(a+b)}$ By componendo & dividendo,

$$\frac{(x+4b)}{(x-4b)} = \frac{(2a+a+b)}{(2a-a-b)} = \frac{(3a+b)}{(a-b)} \quad \text{--- (2)}$$

Now, $\frac{(x+4a)}{(x-4a)} + \frac{(x+4b)}{(x-4b)} = \frac{(3b+a)}{(b-a)} + \frac{(3a+b)}{(a-b)}$

$$= \frac{-3b-a+3a+b}{(a-b)}$$

$$= \frac{(-2b+2a)}{(a-b)} = \frac{2(a-b)}{(a-b)} = 2$$

Thus, $\boxed{\frac{(x+4a)}{(x-4a)} + \frac{(x+4b)}{(x-4b)} = 2}$

9.) If $x = \frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$, then find the value of $\frac{(x+2\sqrt{2})}{(x-2\sqrt{2})} + \frac{(x+2\sqrt{3})}{(x-2\sqrt{3})}$

$\rightarrow$ Given that, $x = \frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

$$x = \frac{4\sqrt{2}\sqrt{3}}{(\sqrt{2}+\sqrt{3})}$$

Then, $\frac{x}{2\sqrt{2}} = \frac{2\sqrt{3}}{(\sqrt{2}+\sqrt{3})}$,

By componendo & dividendo,

$$\frac{(x+2\sqrt{2})}{(x-2\sqrt{2})} = \frac{(2\sqrt{3}+\sqrt{2}+\sqrt{3})}{(2\sqrt{3}-\sqrt{2}-\sqrt{3})} = \frac{(3\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})} \quad \text{--- (1)}$$

Now,

$$\frac{x}{2\sqrt{3}} = \frac{2\sqrt{2}}{(\sqrt{2}+\sqrt{3})}$$

By componendo & dividendo,

$$\frac{(x+2\sqrt{3})}{(x-2\sqrt{3})} = \frac{(2\sqrt{2}+\sqrt{2}+\sqrt{3})}{(2\sqrt{2}-\sqrt{2}-\sqrt{3})} = \frac{(3\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})} \quad \text{—— (2)}$$

from (1) & (2) $\Rightarrow$

$$\frac{(x+2\sqrt{2})}{(x-2\sqrt{2})} + \frac{(x+2\sqrt{3})}{(x-2\sqrt{3})} = \frac{(3\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})} + \frac{(3\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})}$$

$$= \frac{(-3\sqrt{3}-\sqrt{2}+3\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})}$$

$$= \frac{(-2\sqrt{3}+2\sqrt{2})}{(\sqrt{2}-\sqrt{3})}$$

$$= \frac{2(\sqrt{2}-\sqrt{3})}{(\sqrt{2}-\sqrt{3})}$$

Thus, $\boxed{\frac{(x+2\sqrt{2})}{(x-2\sqrt{2})} + \frac{(x+2\sqrt{3})}{(x-2\sqrt{3})} = 2}$.

10.) Using properties of proportion, find x from the following:

i) $\frac{\sqrt{2-x}+\sqrt{2+x}}{\sqrt{2-x}-\sqrt{2+x}} = 3$

→ By applying componendo & dividendo here,

$$\frac{(\sqrt{2-x}+\sqrt{2+x}+\sqrt{2-x}-\sqrt{2+x})}{(\sqrt{2-x}+\sqrt{2+x}-\sqrt{2-x}+\sqrt{2+x})} = \frac{3+1}{3-1}$$

$$\Rightarrow \frac{(2\sqrt{2-x})}{(2\sqrt{2+x})} = \frac{4}{2} = 2$$

$$\frac{\sqrt{2-x}}{\sqrt{2+x}} = 2$$ squaring on both sides,

$$\Rightarrow \frac{(2-x)}{(2+x)} = 4$$

$$\Rightarrow 2-x = 8+4x$$

$$-6 = 5x$$

$\boxed{x = -6/5}$ is the required value.

ii) $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}}=\frac{5}{2}$

→ Given that, $\frac{(\sqrt{x+4}+\sqrt{x-10})}{(\sqrt{x+4}-\sqrt{x-10})}=\frac{5}{2}$

By dividendo & componendo,

$$\frac{(\sqrt{x+4}+\sqrt{x-10}+\sqrt{x+4}-\sqrt{x-10})}{(\sqrt{x+4}+\sqrt{x-10}-\sqrt{x+4}+\sqrt{x-10})}=\frac{(5+2)}{(5-2)}$$

$$\Rightarrow \frac{(2\sqrt{x+4})}{(2\sqrt{x-10})}=\frac{7}{3}$$

$$\Rightarrow \frac{(\sqrt{x+4})}{(\sqrt{x-10})}=\frac{7}{3}$$ squaring on both sides,

$$\frac{(x+4)}{(x-10)}=\frac{49}{9} \Rightarrow 9x+36=49x-490$$

$$36+490=40x$$

$$526=40x$$

$$x=\frac{526}{40}=\frac{263}{20}$$

$\boxed{x=\frac{263}{20}}$ is the required value.

11.) Using properties of proportion, solve for x.

i) $\frac{3x+\sqrt{9x^2-5}}{3x-\sqrt{9x^2-5}}=5$

→ Given that, $\frac{(3x+\sqrt{9x^2-5})}{(3x-\sqrt{9x^2-5})}=5$

By componendo & dividendo,

$$\frac{(3x+\sqrt{9x^2-5}+3x-\sqrt{9x^2-5})}{(3x+\sqrt{9x^2-5}-3x+\sqrt{9x^2-5})}=\frac{(5+1)}{(5-1)}=\frac{6}{4}=\frac{3}{2}$$

$$\frac{(6x)}{2\sqrt{9x^2-5}}=\frac{3}{2}$$

$12x = 6\sqrt{9x^2-5}$

$2x = \sqrt{9x^2-5}$ squaring on both sides

$4x^2 = 9x^2-5$

$9x^2-4x^2-5=0$

$5x^2 = 5$

$x^2 = 1$

$\therefore \boxed{x = \pm 1}$

But, when $x=1 \Rightarrow \quad \frac{3(1)+\sqrt{9-5}}{3(1)-\sqrt{9-5}} = \frac{3+\sqrt{4}}{3-\sqrt{4}} = \frac{3+2}{3-2} = \frac{5}{1}$

Hence, the required value of x is found to $\boxed{x=+1}$

ii) $\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}} = 4$

$\rightarrow$ Given that, $\frac{(2x+\sqrt{4x^2-1})}{(2x-\sqrt{4x^2-1})} = 4$

By componendo & dividendo

$\Rightarrow \quad \frac{(2x+\sqrt{4x^2-1}+2x-\sqrt{4x^2-1})}{(2x+\sqrt{4x^2-1}-2x+\sqrt{4x^2-1})} = \frac{(4+1)}{(4-1)} = \frac{5}{3}$

$\Rightarrow \quad \frac{(4x)}{2\sqrt{4x^2-1}} = \frac{5}{3}$

$\Rightarrow \quad \frac{12x}{\sqrt{4x^2-1}} = 10 \quad \Rightarrow \quad 6x = 5\sqrt{4x^2-1}$

squaring on both sides,

$36x^2 = 25(4x^2-1)$

$\Rightarrow 36x^2 - 100x^2 = -25$

$-64x^2 = -25$

$x^2 = +25/64 \qquad \because x$ is positive

$\Rightarrow \boxed{x = 5/8}$

This is the required value of x

12.) Solve: $\dfrac{(1+x+x^2)}{(1-x+x^2)} = \dfrac{62(1+x)}{63(1-x)}$

$\rightarrow$ Given that, $\dfrac{(1+x+x^2)}{(1-x+x^2)} = \dfrac{62(1+x)}{63(1-x)}$

$$\frac{(1-x)(1+x+x^2)}{(1+x)(1-x+x^2)} = \frac{62}{63}$$

$$\frac{(1+x^3)}{(1-x^3)} = \frac{63}{62} \quad \because \text{By simplifying}$$

By componendo & dividendo,

$$\frac{(1+x^3+1-x^3)}{(1+x^3-1+x^3)} = \frac{(63+62)}{(63-62)}$$

$$\frac{2}{2x^3} = \frac{125}{1}$$

$$\frac{1}{x^3} = 125$$

$$\therefore x^3 = \frac{1}{125} \Rightarrow \boxed{x = \frac{1}{5}}$$

This is the required value of x.

13.) Solve for x: $16\left(\dfrac{a-x}{a+x}\right)^3 = \dfrac{(a+x)}{(a-x)}$

$\longrightarrow$ Given that, $16\left(\dfrac{a-x}{a+x}\right)^3 = \dfrac{(a+x)}{(a-x)}$

$$\Rightarrow \frac{(a+x)^3(a+x)}{(a-x)(a-x)^3} = 16$$

$$\Rightarrow \left[\frac{(a+x)}{(a-x)}\right]^4 = (\pm 2)^4$$

$$\boxed{\frac{(a+x)}{(a-x)} = \pm 2}$$

When $\dfrac{(a+x)}{(a-x)} = \dfrac{2}{1}$ By componendo & dividendo

$$\frac{(a+x+a-x)}{(a+x-a+x)} = \frac{3}{1} \Rightarrow \frac{2a}{2x} = 3$$

$$\Rightarrow \frac{a}{x} = 3 \quad \boxed{x = a/3}$$

When $\frac{(a+x)}{(a-x)} = \frac{-2}{1}$ By componendo & dividendo

$$\frac{(a+x+a-x)}{(a+x-a+x)} = \frac{(-2+1)}{(-2-1)} = \frac{-1}{-3} = \frac{1}{3}$$

$$\frac{2a}{2x} = \frac{1}{3} \Rightarrow \frac{a}{x} = \frac{1}{3} \Rightarrow \boxed{x = 3a}$$

Hence, the required values of x are found to be $a/3$, $3a$.

14) If $x = \frac{(\sqrt{a+x} + \sqrt{a-1})}{(\sqrt{a+1} - \sqrt{a-1})}$, using properties of proportion, show that $(x^2 - 2ax + 1) = 0$.

→ Given that, $x = \frac{(\sqrt{a+x} + \sqrt{a-1})}{(\sqrt{a+1} - \sqrt{a-1})}$ By componendo & dividendo

$$\Rightarrow \left(\frac{x+1}{x-1}\right) = \frac{2\sqrt{a+1}}{2\sqrt{a-1}}$$

Now, $\frac{(x+1)^2}{(x-1)^2} = \frac{(a+1)}{(a-1)}$ ∵ squaring on both sides

$\frac{(x+1)^2 + (x-1)^2}{(x+1)^2 - (x-1)^2} = \frac{(a+1+a-1)}{(a+1-a+1)}$ ∵ By componendo & dividendo

$$\frac{(x^2+1+2x+x^2+1-2x)}{(x^2+1+2x-x^2-1+2x)} = \frac{2a}{2} = a$$

$$\frac{(2x^2+2)}{4x} = a$$

$$\frac{2(x^2+1)}{4x} = a \Rightarrow \frac{(x^2+1)}{2x} = a$$

$$\Rightarrow \frac{(x^2+1)}{2x} = a$$

$$2ax = x^2+1$$

$$\boxed{x^2 - 2ax + 1 = 0}$$

Hence proved.

15.) Given $x = \dfrac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}$. Use componendo & dividendo to prove $b^2 = \dfrac{2a^2x}{(x^2+1)}$.

$\rightarrow$ Given that, $x = \dfrac{(\sqrt{a^2+b^2}+\sqrt{a^2-b^2})}{(\sqrt{a^2+b^2}-\sqrt{a^2-b^2})}$

By componendo & dividendo,

$$\frac{(x+1)}{(x-1)} = \frac{(\sqrt{a^2+b^2}+\sqrt{a^2-b^2}+\sqrt{a^2+b^2}-\sqrt{a^2-b^2})}{(\sqrt{a^2+b^2}+\sqrt{a^2-b^2}-\sqrt{a^2+b^2}+\sqrt{a^2+b^2})}$$

$$\Rightarrow \frac{(x+1)}{(x-1)} = \frac{2\sqrt{a^2+b^2}}{2\sqrt{a^2-b^2}} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2-b^2}}$$

$$\Rightarrow \frac{(x+1)}{(x-1)} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2-b^2}}$$ squaring on both sides,

$$\frac{(x+1)^2}{(x-1)^2} = \frac{(a^2+b^2)}{(a^2-b^2)}$$

$$\Rightarrow \frac{(x^2+1+2x)}{(x^2+1-2x)} = \frac{(a^2+b^2)}{(a^2-b^2)}$$ By componendo & dividendo

$$\Rightarrow \frac{(x^2+1+2x+x^2+1)}{(x^2+1+2x-x^2-1+2x)} = \frac{(a^2+b^2+a^2-b^2)}{(a^2+b^2-a^2+b^2)}$$

$$\Rightarrow \frac{(2x^2+2)}{4x} = \frac{2a^2}{2b^2}$$

$$\Rightarrow \frac{(x^2+1)}{2x} = \frac{a^2}{b^2} \quad \Rightarrow \boxed{b^2 = \frac{2a^2x}{(x^2+1)}}$$

Hence proved.

16.) Given that : $\dfrac{(a^3+3ab^2)}{(b^3+3a^2b)} = \dfrac{63}{62}$, Using componendo & dividendo find $a:b$.

$\rightarrow$ Given that, $\dfrac{(a^3+3ab^2)}{(b^3+3a^2b)} = \dfrac{63}{62}$ By componendo & dividendo

$$\frac{(a^3+3ab^2+b^3+3a^2b)}{(a^3+3ab^2-b^3-3a^2b)} = \frac{(63+62)}{(63-62)} = \frac{125}{1}$$

$$\Rightarrow \frac{(a+b)^3}{(a-b)^3} = \left(\frac{5}{1}\right)^3$$

$$\Rightarrow (a+b) = 5a-5b$$

$$5a-a-5b-b=0$$

$$4a-6b=0$$
$$4a=6b$$
$$\frac{a}{b}=\frac{6}{4}=\frac{3}{2} \quad \text{Thus } \boxed{a:b=3:2}$$

17.) Given $\frac{(x^3+12x)}{(6x^2+8)}=\frac{(y^3+27y)}{(9y^2+27)}$, Using componendo & dividendo find $x:y$.

$\rightarrow$ Given that, $\frac{(x^3+12x)}{(6x^2+8)}=\frac{(y^3+27y)}{(9y^2+27)}$ By componendo & dividendo,

$$\frac{(x^3+12x+6x^2+8)}{(x^3+12x-6x^2-8)}=\frac{(y^3+27y+9y^2+27)}{(y^3+27y-9y^2-27)}$$

$$\frac{(x+2)^3}{(x-2)^3}=\frac{(y+3)^3}{(9y-3)^3}$$

$$\Rightarrow \frac{(x+2)}{(x-1)^3}=\frac{(y+3)}{(y-3)^3}$$

$$\frac{(x+2)}{(x-2)}=\frac{(y+3)}{(y-3)} \quad \text{By componendo \& dividendo}$$

$$\Rightarrow \frac{(x+2+x-2)}{(x+2-x+2)}=\frac{(y+3+y-3)}{(y+3-y+3)}$$

$$\frac{2x}{4}=\frac{2y}{3} \Rightarrow \frac{x}{2}=\frac{y}{3} \Rightarrow \frac{x}{y}=\frac{2}{3}$$

Thus, $\boxed{x:y=2:3}$

18.) Using the properties of proportion, solve the following equation for x; Given $\Rightarrow \frac{x^3+3x}{3x^2+1}=\frac{341}{91}$

$\rightarrow$ Given that, $\frac{(x^3+3x)}{(3x^2+1)}=\frac{341}{91}$

By componendo & dividendo,

$$\frac{(x^3+3x+3x^2+1)}{(x^3+3x-3x^2-1)}=\frac{(341+91)}{(341-91)}$$

$$\frac{(x^3+3x^2+3x+1)}{(x^3-3x^2+3x-1)}=\frac{432}{250}=\frac{216}{125}$$

$$\frac{(x+1)^3}{(x-1)^3}=\frac{216}{125}=\left(\frac{6}{5}\right)^3$$

$\therefore \frac{(x+1)}{(x-1)} = \frac{6}{5}$

$6x - 6 = 5x + 5$

$\Rightarrow 6x - 5x = 5 + 6$

$\boxed{x = 11}$ This is the required value of x.

19.) If $\frac{(x+y)}{(ax+by)} = \frac{(y+z)}{(ay+bz)} = \frac{(z+x)}{(az+bx)}$, prove that each of these ratio is equal to $\frac{2}{(a+b)}$ unless $x+y+z=0$.

$\longrightarrow$ Given that, $\frac{(x+y)}{(ax+by)} = \frac{(y+z)}{(ay+bz)} = \frac{(z+x)}{(az+bx)}$

$$= \frac{(x+y+y+z+z+x)}{(ax+by+ay+bz+az+bx)}$$

$$= \frac{2(x+y+z)}{x(a+b)+y(a+b)+z(a+b)}$$

$$= \frac{2(x+y+z)}{(a+b)(x+y+z)} = \frac{2}{(a+b)} \quad \text{if } x+y+z \neq 0$$

Hence proved.