Chapter 6.
Factorisation

Exercise

1.) Find the remainder (without division) on dividing $f(x)$ by $(x-2)$ where i) $f(x) = 5x^2 - 7x + 4$
ii) $f(x) = 2x^3 - 7x^2 + 3$

→ Given that, $x - 2 = 0$
$\boxed{x = 2}$

ii) put $x = 2$ in $f(x) = 2x^3 - 7x^2 + 3$
$f(2) = 2(2)^3 - 7(2)^2 + 3$
$= 16 - 28 + 3$
$= 19 - 28$
$\boxed{f(-2) = -9}$ is the required remainder

i) put $x = 2$ in $f(x) = 5x^2 - 7x + 4$
$f(2) = 5(2)^2 - 7(2) + 4$
$= 20 - 14 + 4$
$= 24 - 14$
$\boxed{f(2) = 10}$ is the required remainder.

2.) Using the remainder theorem, find the remainder on dividing $f(x)$ by $(x+3)$ where
i) $f(x) = 2x^2 - 5x + 1$
ii) $f(x) = 3x^3 + 7x^2 - 5x + 1$

→ Given divider is $x + 3 = 0$
$\boxed{x = -3}$

i) put $x=-3$ in $f(x) = 2x^2 - 5x + 1$

$$f(-3) = 2(-3)^2 - 5(-3) + 1$$
$$= 18 + 15 + 1$$

$\boxed{f(-3) = 34}$ is the required remainder.

ii) put $x=-3$ in $f(x) = 3x^3 + 7x^2 - 5x + 1$

$$f(-3) = 3(-3)^3 + 7(-3)^2 - 5(-3) + 1$$
$$= -81 + 63 + 15 + 1$$
$$= -81 + 79$$

$\boxed{f(-3) = -2}$ is the required remainder.

3.) find the remainder (without division) on dividing $f(x)$ by $(2x+1)$ where i) $f(x) = 4x^2 + 5x + 3$
ii) $f(x) = 3x^3 - 7x^2 + 4x + 11$

→ Given divider is $2x + 1 = 0$

$$\boxed{x = -1/2}$$

i) put $x = -1/2$ in $f(x) = 4x^2 + 5x + 3$

$$f(-1/2) = 4(-1/2)^2 + 5(-1/2) + 3$$
$$= 1 - 5/2 + 3 = 4 - 5/2 = 3/2$$

$\boxed{f(-1/2) = 3/2}$ is the required remainder.

ii) put $x = -1/2$ in $f(x) = 3x^3 - 7x^2 + 4x + 11$

$$f(-1/2) = 3(-1/2)^3 - 7(-1/2)^2 + 4(-1/2) + 11$$
$$= -3/8 + 7/4 - 2 + 11$$
$$= -3/8 - 7/4 + 9$$
$$= \frac{-3-14}{8} + 9 = \frac{-17}{8} + 9$$
$$= \frac{-17+72}{8} = \frac{55}{8}$$

$\boxed{f(-1/2) = 55/8}$ is the required remainder

4.) Using remainder theorem, find the value of k if on dividing $(2x^3+3x^2-kx+5)$ by $(x-2)$, leaves a remainder 7.

→ Given polynomial is $f(x) = 2x^3+3x^2-kx+5$

and $g(x) = x-2$

If $g(x)=0 \Rightarrow x-2=0 \Rightarrow \boxed{x=2}$

put $x=2$ in $f(x) = 2x^3+3x^2-kx+5$

$f(2) = 2(2)^3+3(2)^2-k(2)+5$

$= 2(8)+3(4)-2k+5$

$= 16+12-2k+5$

$\boxed{f(2) = 33-2K}$ is the required remainder.

But, given that 7 is the required remainder.

$\Rightarrow \quad 33-2k=7$

$33-7=2k$

$26=2k$

$\boxed{K=13}$ is the required value of k

5.) Using remainder theorem, find the value of 'a' if the division of (x^3+5x^2-ax+6) by $(x-1)$ leaves the remainder 2a.

→ Given polynomial is $x^3+5x^2-ax+6 = f(x)$

and $g(x) = x-1$

If $g(x)=0 \Rightarrow x-1=0 \Rightarrow \boxed{x=1}$

put $x=1$ in $f(x) = x^3+5x^2-ax+6$

$f(1) = 1^3+5(1)^2-a+6$

$= 1+5-a+6$

$\boxed{f(1) = 12-a}$ is the required remainder.

But, given that 2a is the required remainder.

$\Rightarrow \quad 12-a=2a$

$12=3a$

$\boxed{a=4}$ is the required value of a.

6) What number must be subtracted from $(2x^2-5x)$ so that the resulting polynomial leaves the remainder 2. when divided by $(2x+1)$?

$\rightarrow$ Let us consider, 'p' be subtracted from $(2x^2-5x)$.

Now, $f(x) = 2x^2-5x$ and $g(x) = 2x+1$

Then
$$
\begin{array}{r|l}
 & x-3 \\
(2x+1) & 2x^2-5x-p \\
 & \underline{2x^2+x} \\
 & \quad -6x-p \\
 & \quad \underline{-6x-3} \\
 & \quad +3-p \longrightarrow \text{Remainder}
\end{array}
$$

But given that remainder $= 2$

$\Rightarrow 3-p=2$

$\boxed{p=1}$ is the required number.

7.) i) When divided by $(x-3)$ the polynomials x^3-px^2+x+6 & $2x^3-x^2-(p+3)x-6$ leaves the same remainder. find p.

$\rightarrow$ Given that, $f(x) = x^3-px^2+x+6$
and $g(x) = x-3$

If $g(x)=0 \Rightarrow x-3=0$

$\boxed{x=3}$

put $x=3$ in $f(x)$.

$\Rightarrow f(3) = (3)^3-p(3)^2+3+6$

$= 27-9p+9$

$\boxed{f(3) = 36-9p}$ is the required remainder

$f(x) = 2x^3-x^2-(p+3)x-6$
and $g(x) = x-3$

If $g(x)=0 \Rightarrow x-3=0$

$\boxed{x=3}$

put $x=3$ in $f(x) = 2x^3-x^2-(p+3)x-6$

$f(3) = 2(3)^3-(3)^2-(p+3)3-6$

$= 54-9-3p-9-6$

$= 54-24-3p$

$\boxed{f(3) = 30-3p}$ is the required remainder

But, from given condition,

$36-9p = 30-3p$

$36-30 = 9p-3p$

$6 = 6p$

$\boxed{p=1}$ is the required value of p.

ii) Find 'a' if the two polynomials ax^3+3x^2-9 and $2x^3+4x+a$, leaves the same remainder when divided by $(x+3)$.

→ Given that,

$f(x) = ax^3+3x^2-9$

$g(x) = x+3$

If $g(x)=0 \Rightarrow x+3=0$

$\boxed{x=-3}$

put $x=-3$ in $f(x)$.

$\Rightarrow f(-3) = a(-3)^3+3(-3)^2-9$

$= -27a+27-9$

$\boxed{f(-3) = -27a+18}$

is the required remainder

and

Given that

$f(x) = 2x^3+4x+a$

$g(x) = x+3$

If $g(x)=0 \Rightarrow x+3=0$

$\boxed{x=-3}$

put $x=-3$ in $f(x)$.

$\Rightarrow f(-3) = 2(-3)^3+4(-3)+a$

$= -54-12+a$

$\boxed{f(-3) = -66+a}$

is the required remainder.

But, from given condition,

$-27a+18 = -66+a$

$18+66 = 28a$

$84 = 28a$

$a = \frac{12}{4} = 3$

$\boxed{a=3}$ is the required value of 'a'!

8.) Using remainder theorem find the remainder obtained when $x^3+(kx+8)x+k$, is divided by $(x+1)$ and $(x-2)$. Hence, find k if the sum of the two remainders is 1.

→ Given polynomial is $f(x) = x^3+(kx+8)x+k$

and $g(x) = x+1$

If $g(x)=0 \Rightarrow x+1=0 \Rightarrow \boxed{x=-1}$

put $x=-1$ in $f(x)$.

$\Rightarrow f(-1) = (-1)^3+(-k+8)(-1)+k$

$= -1+k-8+k$

$\boxed{f(-1) = -9+2k}$ is the required remainder.

If $g(x) = x-2 = 0$

$\Rightarrow \boxed{x=2}$ put in $f(x)$

$f(2) = 2^3 + (2k+8)2 + k$

$f(2) = 8 + 4k + 16 + k$

$\boxed{f(2) = 24+5k}$ is the required remainder.

But, from given condition, we can write

$(-9+2k)+(24+5k) = 1$

$15+7k = 1$

$7k = -14$

$\boxed{k=-2}$ is the required value of k.

9.) By factor theorem, show that $(x+3)$ and $(2x-1)$ are factors of $(2x^2+5x-3)$.

$\rightarrow$ ~~Let~~ Given that $f(x) = 2x^2+5x-3$

Let us consider $x+3 = 0 \Rightarrow \boxed{x=-3}$

put $x=-3$ in $f(x) \Rightarrow f(-3) = 2(-3)^2 + 5(-3) - 3$

$= 18-15-3$

$= 18-18$

$\boxed{f(-3) = 0} \longrightarrow$ Remainder

Hence, $(x+3)$ is the factor of $f(x)$.

Now, $2x-1 = 0 \Rightarrow \boxed{x = 1/2}$

put $x = 1/2$ in $f(x) \Rightarrow f(1/2) = 2(1/2)^2 + 5(1/2) - 3$

$= 1/2 + 5/2 - 3$

$= 3-3$

$\boxed{f(1/2) = 0} \longrightarrow$ Remainder

Thus, $(2x-1)$ is also the factor of $f(x)$.

10) Without actual division, prove that $x^4+2x^3-2x^2+2x-3$ is exactly divisible by (x^2+2x-3).

$\rightarrow$ Given that, $f(x) = x^4+2x^3-2x^2+2x-3$

And $g(x) = x^2+2x-3$

$= x^2+3x-x-3$

$= x(x+3)-1(x+3)$

$\boxed{g(x) = (x-1)(x+3)} \longrightarrow$ divider

When $(x-1)=0 \Rightarrow \boxed{x=1}$ put in $f(x)$

$\Rightarrow f(1) = 1^4+2(1)^3-2(1)^2+2(1)-3$

$= 1+2-2+2-3$

$\boxed{f(1)=0} \longrightarrow$ Remainder

And When $x+3=0 \Rightarrow \boxed{x=-3}$ put in $f(x)$.

$f(-3) = (-3)^4+2(-3)^3-2(-3)^2+2(-3)-3$

$= 81-54-18-6-3$

$\boxed{f(-3)=0} \longrightarrow$ Remainder

Thus, $(x-1)$ & $(x+3)$ both are factors of $f(x)$.

Hence $g(x) = (x-1)(x+3)$ is also the factor of $f(x)$.

12.) Using the factor theorem show that $(x-2)$ is a factor of (x^3+x^2-4x-4). Hence factorise the polynomial completely.

$\rightarrow$ Given polynomial is $f(x) = x^3+x^2-4x-4$.

And $g(x) = x-2$

If $g(x)=0 \Rightarrow x-2=0 \Rightarrow \boxed{x=2}$

put $x=2$ in $f(x) \Rightarrow f(2) = 2^3+2^2-8-4$

$= 8+4-8-4$

$\boxed{f(2)=0} \rightarrow$ Remainder

Hence, $(x-2)$ is a factor of $f(x)$.

Now, $x^3+x^2-4x+4 = x^2(x+1)-4(x+1)$

$= (x+1)(x^2-4)$

$\boxed{f(x) = (x+1)(x+2)(x-2)}$

13.) Show that $(2x+7)$ is a factor of $(2x^3+5x^2-11x-14)$. Hence factorise the given expression completely, using factor theorem.

→ Given polynomial is $f(x) = 2x^3+5x^2-11x-14$

And $g(x) = 2x+7$

If $g(x) = 2x+7 = 0 \quad \Rightarrow 2x+7=0$

$\Rightarrow \boxed{x = -7/2}$ put in $f(x)$

$$f(-7/2) = 2(-7/2)^3 + 5(-7/2)^2 - 11(-7/2) - 14$$
$$= -343/4 + 245/4 - 77/2 - 14$$
$$= (-343+245+154-56)/4$$
$$= (-399+399)/4$$

$\boxed{f(-7/2) = 0}$ → Remainder

Hence, $(2x+7)$ is the factor of $f(x)$.

Now,

$$\begin{array}{r l}
 & x^2 - x - 2 \\
(2x+7) & \overline{)\, 2x^3+5x^2-11x-14} \\
 & \underline{\,2x^3+7x^2} \\
 & \;-\;\; - \\
 & -2x^2-11x-14 \\
 & \underline{-2x^2-7x} \\
 & \;\; + \quad\; + \\
 & -4x-14 \\
 & \underline{-4x-14} \\
 & \;\; + \quad + \\
 & 0
\end{array}$$

Thus, $f(x) = 2x^3+5x^2-11x-14$

$$f(x) = (2x+7)(x^2-x-2)$$
$$= (2x+7)(x^2-2x+x-2)$$
$$= (2x+7)[x(x-2)+1(x-2)]$$
$$\boxed{f(x) = (2x+7)(x-2)(x+1)}$$

Thus, $(2x+7)$, $(x-2)$ & $(x+1)$ are factors of $f(x)$.

14.) Use factor theorem to factorise the following polynomial completely.

i) x^3+2x^2-5x-6

Given that $f(x)= x^3+2x^2-5x-6$

Let $\boxed{x=-1}$ put in $f(x)$

$f(-1) = (-1)^3+2(-1)^2-5(-1)-6$

$= -1+2+5-6$

$= 7-7$

$\boxed{f(-1)=0}$ → Remainder

Hence, $(x+1)$ is the factor of $f(x)$.

Now, by Actual division method,

$$
\begin{array}{r|l}
 & x^2+x-6 \\
\hline
(x+1) & x^3+2x^2-5x-6 \\
 & \underline{\overset{-}{x^3} \overset{-}{+x^2}} \\
 & \quad x^2-5x-6 \\
 & \quad \underline{\overset{-}{x^2} \overset{-}{+x}} \\
 & \qquad -6x-6 \\
 & \qquad \underline{\overset{+}{-6x} \overset{+}{-6}} \\
 & \qquad\quad 0
\end{array}
$$

→ Remainder

Thus, $f(x) = x^3+2x^2-5x-6$

$f(x) = (x+1)(x^2+x-6)$

$= (x+1)[x^2+3x-2x-6]$

$= (x+1)[x(x+3)-2(x+3)]$

$= (x+1)[(x-2)(x+3)]$

$\boxed{f(x) = (x+1)(x-2)(x+3)}$

Thus, $(x+1)$, $(x-2)$ & $(x+3)$ are the factors of $f(x)$.

15.) Use the Remainder theorem to factorise the following expression i) $2x^3 + x^2 - 13x + 6$

$\rightarrow$ Given that, $f(x) = 2x^3 + x^2 - 13x + 6$

Let $\boxed{x=2}$ put in $f(x)$.

$\Rightarrow f(2) = 2(2)^3 + (2)^2 - 13(2) + 6$

$= 16 + 4 - 26 + 6$

$= 26 - 26$

$\boxed{f(2) = 0} \rightarrow$ Remainder

Thus, $(x-2)$ is the factor of $f(x)$.

Now, By actual division method,

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ \hline (x-2) & 2x^3 + x^2 - 13x + 6 \\ & 2x^3 - 4x^2 \\ & - \quad + \\ \hline & 5x^2 - 13x + 6 \\ & 5x^2 - 10x \\ & - \quad + \\ \hline & -3x + 6 \\ & -3x + 6 \\ & + \quad - \\ \hline & 0 \end{array}$$

$\rightarrow$ Remainder

Thus, $f(x) = 2x^3 + x^2 - 13x + 6$

$= (x-2)(2x^2 + 5x - 3)$

$= (x-2)[2x^2 + 6x - x - 3]$

$= (x-2)[2x(x+3) - 1(x+3)]$

$\boxed{f(x) = (x-2)(x+3)(2x-1)}$

Thus, $(x-2)$, $(x+3)$ & $(2x-1)$ are the factors of $f(x)$

ii) $2x^3+3x^2-9x-10$

$\rightarrow$ Given polynomial is $f(x) = 2x^3+3x^2-9x-10$

Let $\boxed{x=2}$ put in $f(x)$

$$f(2) = 2(2)^3+3(2)^2-9(2)-10$$
$$= 16+12-18-10$$
$$= 28-28$$
$$\boxed{f(2)=0} \rightarrow \text{Remainder}$$

Hence, $(x-2)$ is the factor of $f(x)$.

Now, By actual division method,

$$\begin{array}{r|l} & 2x^2+7x+5 \\ \hline (x-2) & 2x^3+3x^2-9x-10 \\ & 2x^3-4x^2 \\ & - \quad + \\ \hline & \quad 7x^2-9x-10 \\ & \quad 7x^2-14x \\ & \quad - \quad + \\ \hline & \qquad 5x-10 \\ & \qquad 5x-10 \\ & \qquad + \quad + \\ \hline & \qquad 0 \rightarrow \text{Remainder} \end{array}$$

Thus, $f(x) = 2x^3+3x^2-9x-10$

$$f(x) = (x-2)[2x^2+7x+5]$$
$$= (x-2)[2x^2+2x+5x+5]$$
$$= (x-2)[2x(x+1)+5(x+1)]$$
$$\boxed{f(x) = (x-2)(2x+5)(x+1)}$$

Hence, $(x-2)$, $(2x+5)$ & $(x+1)$ are the factors of $f(x)$.

16) If $(2x+1)$ is a factor of $(6x^3+5x^2+ax-2)$, find the value of a.

→ Given polynomial is $f(x) = 6x^3+5x^2+ax-2$

and $g(x) = 2x+1$ is a factor of $f(x)$

Then, $g(x)=0 \Rightarrow 2x+1=0$

$\boxed{x=-1/2}$ put in $f(x)$.

$\Rightarrow f(-1/2) = 0$

$6(-1/2)^3+5(-1/2)^2+a(-1/2)-2=0$

$-\frac{6}{8}+\frac{5}{4}-a/2-2=0$

$-6+10-4a-16=0$

$10-22=4a$

$-12=4a$

$\boxed{a=-3}$ is the required value of a.

17) If $(3x-2)$ is a factor of $3x^3-kx^2+21x-10$, find k.

→ Given polynomial is $3x^3-kx^2+21x-10 = f(x)$

And $g(x) = 3x-2$ is the factor of $f(x)$.

Then, $g(x)=0 \Rightarrow 3x-2=0$

$\boxed{x=2/3}$ put in $f(x)$

$f(2/3)=0$

$3(2/3)^3-k(2/3)^2+21(2/3)-10=0$

$\frac{8}{9}-k\left(\frac{4}{9}\right)+14-10=0$

$\frac{8}{9}-\frac{4k}{9}+4=0$

$8-4k+36=0$

$-4k=-44$

$\boxed{k=11}$ is the required value of k.

18.) If $(x-2)$ is a factor of $2x^3-x^2+px-2$, then
i) find the value of p
ii) With this value of p, factorise the above expression completely.

→ Given polynomial is $f(x) = 2x^3 - x^2 + px - 2$
$g(x) = x-2$ is the factor of $f(x)$.

i) Then $g(x) = 0 \Rightarrow x-2=0 \Rightarrow \boxed{x=2}$ put in $f(x)$

$f(2) = 2(2)^3 - 2^2 + 2p - 2 = 0$

$16 - 4 + 2p - 2 = 0$

$8 + 2p = 0$

$2p = -8$

$\boxed{p=-4}$ is the required value of p.

ii) put $p=-4$ in $f(x) = 2x^3 - x^2 - 4x - 2$

$f(x) = (x-2)(2x^2+3x+1)$

$= (x-2)(2x^2+2x+x+1)$

$= (x-2)[2x(x+1)+1(x+1)]$

$\boxed{f(x) = (x-2)(2x+1)(x+1)}$

Thus, $(x-2)$, $(2x+1)$ & $(x+1)$ are the factors of $f(x)$

19.) What number should be subtracted from $2x^3-5x^2+5x$ so that the resulting polynomial has $(2x-3)$ as a factor.

→ Given polynomial is $f(x) = 2x^3 - 5x^2 + 5x$
Let p be the no. subtracted from $f(x)$.
given that, $g(x) = 2x-3 = 0$

$\boxed{x = 3/2}$ put in $f(x)$

$f(3/2) = 2(3/2)^3 - 5(3/2)^2 + 5(3/2)$

$0 = 2(27/8) - 5(9/4) + 15/2$

$\frac{27}{4} - \frac{45}{4} + \frac{15}{2} - p = 0$

$27 - 45 + 30 - 4p = 0$

$57 - 45 - 4p = 0$

$12 - 4p = 0$

$\boxed{p=3}$ is the required value of p.

20.) Find the value of the constants a and b, if $(x-2)$ & $(x+3)$ are both factors of the expression $x^3 + ax^2 + bx - 12$.

→ Given polynomial is $f(x) = x^3 + ax^2 + bx - 12$

And $x - 2 = 0 \Rightarrow \boxed{x=2}$ put in $f(x)$

$\Rightarrow f(2) = 2^3 + a(2)^2 + b(2) - 12$

$= 8 + 4a + 2b - 12$

$\boxed{f(2) = 4a + 2b - 4} \longrightarrow$ Remainder

But $(x-2)$ is a factor of $f(x)$.

$\Rightarrow f(2) = 0$

$\Rightarrow 4a + 2b - 4 = 0$

$4a + 2b = 4$

$2a + b = 2$ ——— ①

Now, $x + 3 = 0 \Rightarrow \boxed{x=-3}$ put in $f(x)$

$f(-3) = (-3)^3 + a(-3)^2 + b(-3) - 12$

$= -27 + 9a - 3b - 12$

$\boxed{f(-3) = -39 + 9a - 3b} \longrightarrow$ Remainder

But $(x+3)$ is a factor of $f(x)$.

$\Rightarrow f(-3) = 0$

$-39 + 9a - 3b = 0$

$9a - 3b = 39$

$3a - b = 13$ ——— ②

① + ② $\Rightarrow$ $5a = 15$

$\boxed{a=3}$ $\Rightarrow$ $6 + b = 2$

$\boxed{b=-4}$

$a=3$ & $b=-4$ are the required values.

22.) $(x-2)$ is a factor of the expression x^3+ax^2+bx+6. When this expression is divided by $(x-3)$, it leaves the remainder 3. Find the values of a and b.

$\rightarrow$ Given polynomial is $f(x) = x^3+ax^2+bx+6$

$(x-2)$ is the factor of $f(x)$.

$x-2=0 \Rightarrow \boxed{x=2}$ put in $f(x)$.

$$f(2) = 2^3+a2^2+b2+6$$
$$= 8+4a+2b+6$$
$$\boxed{f(2) = 14+4a+2b} \rightarrow \text{Remainder}$$

$\Rightarrow$ $f(2)=0$

$$14+4a+2b=0$$
$$4a+2b=-14$$
$$2a+b=-7 \text{ — } ①$$

When $f(x)$ is divided by $(x-3)$ remainder is 3.

$\Rightarrow x-3=0$

$\Rightarrow \boxed{x=3}$ put in $f(x)$

$$f(3) = 3^3+a(3)^2+b(3)+6$$
$$= 27+9a+3b+6$$
$$\boxed{f(3) = 33+9a+3b} \rightarrow \text{Remainder}$$

$$3 = 33+9a+3b$$
$$9a+3b=-30$$
$$3a+b=-10 \text{ — } ②$$

$① - ② \Rightarrow (2a+b)-(3a+b) = -7+10$

$$-a=3$$

$\boxed{a=-3}$ put in ① $\Rightarrow -6+b=-7$

$$\boxed{b=-1}$$

Thus, $a=-3$ & $b=-1$ are the required values of a & b respectively.

23.) If $(x-2)$ is a factor of the expression $2x^3+ax^2+bx-14$ and when the expression is divided by $(x-3)$, it leaves remainder 52, find the values of a and b.

$\rightarrow$ Given polynomial is $f(x) = 2x^3+ax^2+bx-14$

$(x-2)$ is the factor of $f(x)$.

Then, $x-2=0 \Rightarrow \boxed{x=2}$ put in $f(x)$

$$f(2) = 2(2)^3 + a(2)^2 + b(2) - 14$$
$$= 16 + 4a + 2b - 14$$
$$\boxed{f(2) = 2 + 4a + 2b} \rightarrow \text{Remainder}$$

$\Rightarrow f(2) = 0$

$4a + 2b = -2$

$2a + b = -1$ —— ①

Now, when $f(x)$ is divided by $(x-3)$ remainder is 52.

$\rightarrow x-3=0 \Rightarrow \boxed{x=3}$ put in $f(x)$

$$f(3) = 2(3)^3 + a(3)^2 + b(3) - 14$$
$$= 54 + 9a + 3b - 14$$

$52 = 40 + 9a + 3b$

$9a + 3b = 52 - 40$

$9a + 3b = 12$

$3a + b = 4$ —— ②

① - ② $\Rightarrow$ $(2a+b) - (3a+b) = -1-4$

$-a = -5$

$\boxed{a=5}$ put in ① $10 + b = -1$

$\boxed{b=-11}$

Thus, $a=5$ & $b=-11$ are the values of a and b respectively.

24) If ax^3+3x^2+bx-3 has a factor $(2x+3)$ & leaves remainder (-3) when divided by $(x+2)$, find the values of a & b. With these values of a & b, factorise the given expression.

→ Given polynomial is $f(x) = ax^3+3x^2+bx-3$

$(2x+3)$ is a factor of $f(x)$.

Then, $2x+3=0 \Rightarrow \boxed{x=-3/2}$ put in $f(x)$

$$f(-3/2) = a(-3/2)^3 + 3(-3/2)^2 + b(-3/2) - 3$$

$$= -\frac{27}{8}a + \frac{27}{4} - \frac{3b}{2} - 3$$

$$0 = -\frac{27a}{8} + \frac{27}{4} - \frac{3b}{2} - 3$$

$$\Rightarrow -27a + 54 - 12b - 24 = 0$$

$$-27a - 12b = -30$$

$$9a + 4b = 10 \quad \text{—— ①}$$

When $f(x)$ is divided by $(x+2)$ the remainder is -3.

$x+2=0 \Rightarrow \boxed{x=-2}$ put in $f(x)$

$$f(-2) = a(-2)^3 + 3(-2)^2 + b(-2) - 3$$

$$= -8a + 12 - 2b - 3$$

$$f(-2) = -8a - 2b + 9$$

$$-3 = -8a - 2b + 9$$

$$-8a - 2b = -12$$

$$4a + b = 6 \quad \text{—— ②}$$

$$① \Rightarrow \quad 9a + 4b = 10$$

$$② \times 4 \quad 16a + 4b = 24$$

$$-7a = -14$$

$\boxed{a=2}$ put in ① ⇒

$$9a + 4b = 10$$

$$18 + 4b = 10$$

$$4b = -8$$

$$\boxed{b=-2}$$

Thus, $a=2$ & $b=-2$ are the required values.

Now, $f(x) = ax^3 + 3x^2 + bx - 3$

$f(x) = 2x^3 + 3x^2 - 2x - 3$

Given that, $(2x+3)$ is also the factor of $f(x)$.

Then, By actual division method,

$$\begin{array}{r|l} & x^2 - 1 \\ (2x+3) & 2x^3 + 3x^2 - 2x - 3 \\ & \underline{2x^3 + 3x^2} \\ & - 2x - 3 \\ & - 2x - 3 \\ & \underline{+ \quad +} \\ & 0 \end{array} \longrightarrow \text{Remainder}$$

Thus, $f(x) = 2x^3 + 3x^2 - 2x - 3$

$f(x) = (2x+3)(x^2 - 1)$

$\boxed{f(x) = (2x+3)(x-1)(x+1)}$

Thus, $(x+1)$, $(x-1)$ & $(2x+3)$ are the factors of $f(x)$.

25.) Given $f(x) = ax^2 + bx + 2$ and $g(x) = bx^2 + ax + 1$. If $(x-2)$ is a factor of $f(x)$ but leaves the remainder -15 when it divides $g(x)$, find the values of a and b. With these values of a and b, factorise the expression $f(x) + g(x) + 4x^2 + 7x$.

$\longrightarrow$ Given polynomial is $f(x) = ax^2 + bx + 2$

and $g(x) = bx^2 + ax + 1$

$(x-2)$ is the factor of $f(x)$.

Then $x - 2 = 0 \Rightarrow \boxed{x = 2}$ put in $f(x)$

$f(2) = a(2)^2 + b(2) + 2 = 4a + 2b + 2$

$0 = 4a + 2b + 2$

$4a + 2b = -2$

$2a + b = -1$ —— ①

When $(x-2)$ divides $g(x)$ remainder is -15.

$\Rightarrow x - 2 = 0 \Rightarrow \boxed{x = 2}$ put in $g(x)$

$g(2) = 4b + 2a + 1$

$-15 = 4b + 2a + 1$

$4b+2a+1=-15$

$4b+2a=-16$

$a+2b=-8$ —— (2) $\qquad 2a+b=-1$ —— (1)

$(a+2b)-(4a+2b)=-8+2$

$-3a=-6$

$\boxed{a=2}$ put in (2) $\qquad 2+2b=-8$

$2b=-10$

$\boxed{b=-5}$

Now, $f(x)=ax^2+bx+2=2x^2-5x+2$

and $g(x)=bx^2+ax+1=-5x^2+2x+1$

Now, $f(x)+g(x)+4x^2+7x$

$=2x^2-5x+2-5x^2+2x+1+4x^2+7x$

$=x^2+4x+3$

$=x^2+3x+x+3$

$=x(x+3)+1(x+3)$

$\Rightarrow \boxed{(x+1)(x+3)=f(x)+g(x)+4x^2+7x}$

This is the required answer