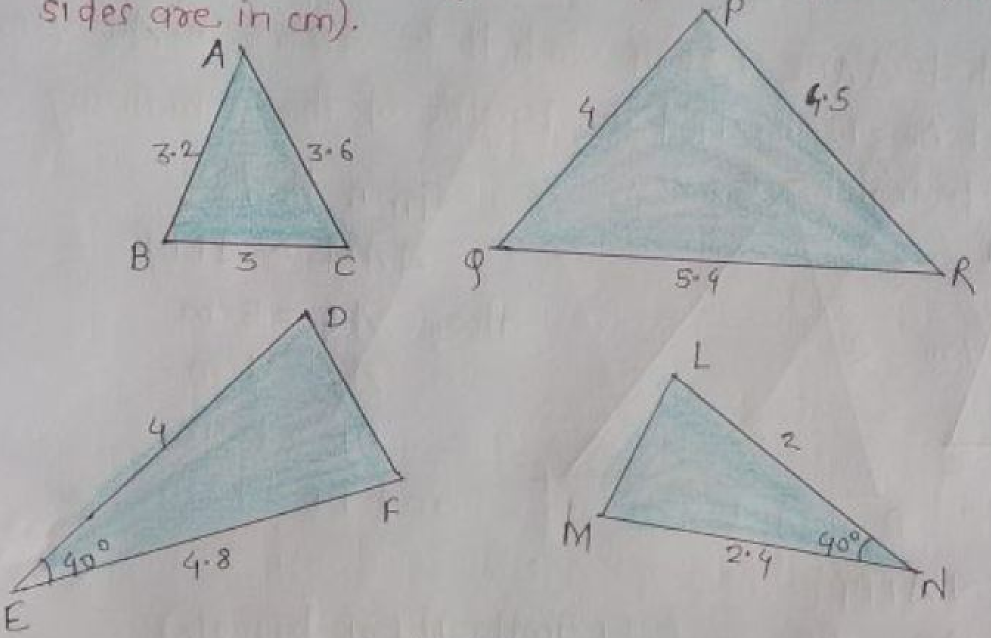
Chapter 13.

Similarity

Exercise 13.1

1) State which pairs of triangles in the figure given below are similar. Write the similarity rule used and also write the pairs of similar triangles in symbolic form (all lengths of sides are in cm).



→ • In ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{3.2}{4} \neq, \quad \frac{AC}{PR} = \frac{3.6}{4.5} \text{ and } \frac{BC}{QR} = \frac{3}{5.4}$$

• In ΔDEF and ΔLMN,

$$\angle E = \angle N = 90^\circ$$

$$\frac{DE}{LN} = \frac{4}{2} = \frac{2}{1} \text{ and } \frac{EF}{MN} = \frac{4.8}{2.4} = \frac{2}{1}$$

Thus, $\boxed{\Delta DEF \sim \Delta LMN}$ $\because$ By SAS axiom

2.) If in two right triangles, one of the acute angle of one triangle is equal to an acute angle of other triangle, can you say that the two triangles are simillar? why?

→ Given that,

In two right triangles,

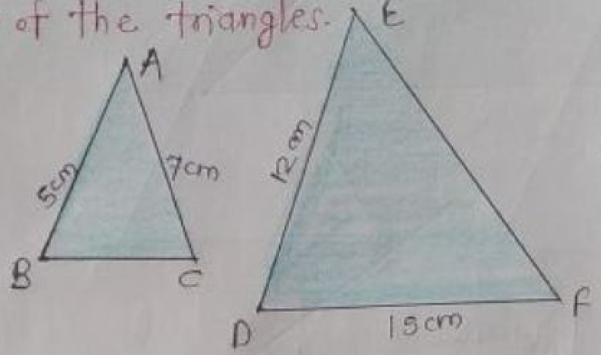
One acute angle of one triangle is equal to one of the acute angle of another triangle.

⇒ The given two right triangles are simillar.

(By AAA axiom).

3.) It is given that $\Delta ABC \sim \Delta EDF$ such that $AB = 5cm$, $AC = 7cm$, $DF = 15cm$ and $DE = 12cm$. Find the lengths of the remaining sides of the triangles.

→



Given that,

$\Delta ABC \sim \Delta EDF$

also, $AB = 5cm$

$AC = 7cm$

$DF = 15cm$

and $DE = 12cm$.

. As $\Delta ABC \sim \Delta EDF$

$\Rightarrow \frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$ ($\because$ by similarity of triangle)

Let us consider, $\frac{AB}{ED} = \frac{AC}{EF}$

$\frac{5}{12} = \frac{7}{EF} \Rightarrow EF = \frac{7 \times 12}{5} = 16.8cm$

Now, Let us consider, $\frac{AB}{ED} = \frac{BC}{DF}$

$\frac{5}{12} = \frac{BC}{15} \Rightarrow BC = \frac{5 \times 15}{12} = 6.25cm$

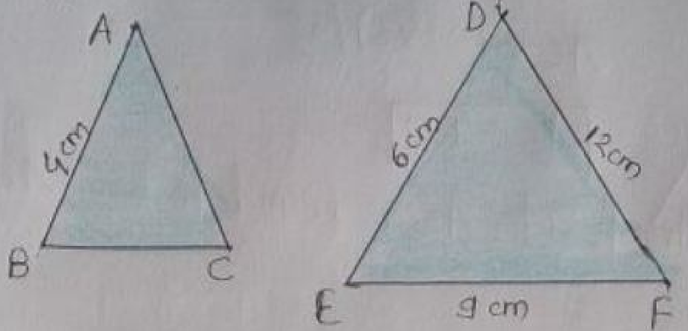
Thus, the lengths of the remaining sides of a triangle is found to be

$\boxed{EF = 16.8cm}$ & $\boxed{BC = 6.25cm}$

4.) a) If $\Delta ABC \sim \Delta DEF$, $AB = 4cm$, $DE = 6cm$, $EF = 9cm$ and $FD = 12cm$, then find the perimeter of ΔABC.

b) If $\Delta ABC \sim \Delta PQR$, Perimeter of $\Delta ABC = 32$ cm, perimeter of $\Delta PQR = 48$ cm and $PR = 6$cm, then find the length of AC.

→



a) $\Delta ABC \sim \Delta DEF$ (Given)

And, $AB = 4cm$, $DE = 6cm$, $EF = 9$cm and $FD = 12$cm

As $\boxed{\Delta ABC \sim \Delta DEF}$

Let us consider, $\frac{AB}{DE} = \frac{AC}{DE}$ ∵ by similarity of triangles

$$\frac{4}{6} = \frac{AC}{12}$$

$$\Rightarrow AC = \frac{4 \times 12}{6} = 8 \text{ cm} \qquad \boxed{AC = 8cm}$$

Now, consider, $\frac{AB}{DE} = \frac{BC}{EF}$ ∵ By similarity of triangles

$$\frac{4}{6} = \frac{BC}{9} \qquad \boxed{BC = 6cm}$$

$$BC = 36/6 = 6 \text{ cm}$$

Thus, perimeter of triangle ABC is found to be

$P(\Delta ABC) = AB + BC + AC = 4 + 6 + 8 = 18$cm

Thus, the perimeter of required triangle is found to be 18cm.

b) Given that, $\Delta ABC \sim \Delta PQR$

Perimeter of $\Delta ABC = 32$cm, $\Delta PQR = 48$ cm, $PR = 6$cm

Perimeter of $\Delta PQR = 48$ cm

Then, by similarity of triangles,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$$

$$\Rightarrow \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \frac{AC}{PR}$$

$$\frac{32}{48} = \frac{AC}{6}$$

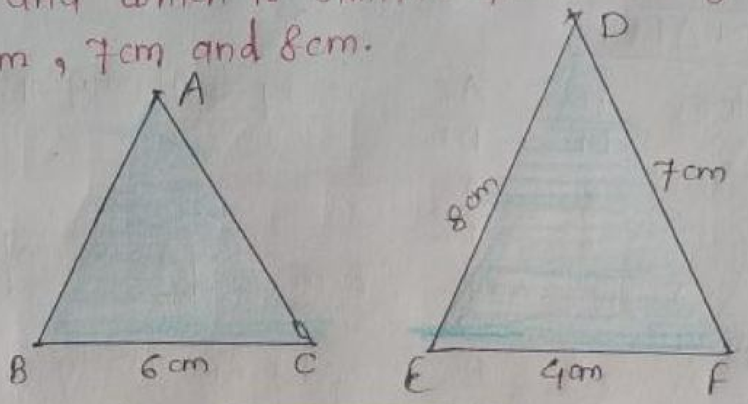
$$AC = (32 \times 6)/48$$

$$\boxed{AC = 4cm}$$

Thus, the length of side AC is found to be $AC = 4$ cm.

5.) Calculate the other side of a triangle whose shortest side is 6cm and which is similar to a triangle whose sides are 4cm, 7cm and 8cm.

→



Let us consider, $\boxed{\Delta ABC \sim \Delta DEF}$

And the shortest side of ΔABC is found to be $BC = 6$ cm.

And, In ΔDEF

$DE = 8$ cm, $EF = 4$ cm and $DF = 7$ cm

$$\Rightarrow \boxed{\Delta ABC \sim \Delta DEF}$$

Thus, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ∵ by similarity of triangles.

Let us consider, $\frac{AB}{DE} = \frac{BC}{EF}$

$$\frac{AB}{8} = \frac{6}{4} \Rightarrow AB = 48/4$$

$$\boxed{AB = 12cm}$$

Let us consider, $\frac{BC}{EF} = \frac{AC}{DF}$

$$\frac{6}{4} = \frac{AC}{7}$$

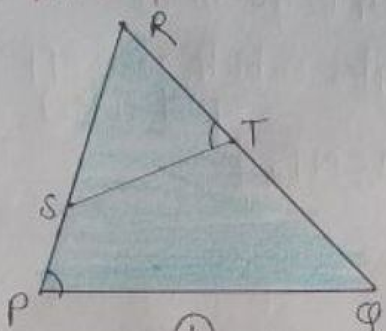
$$\Rightarrow AC = (6 \times 7)/4 = 42/4 = 21/2$$

$$\boxed{AC = 10.5\text{cm}}$$

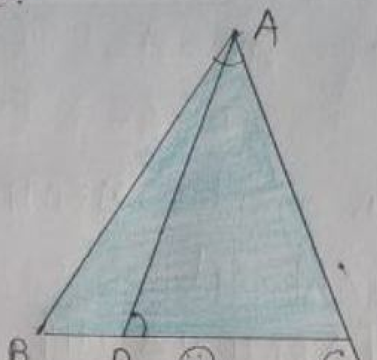
7) a) In the figure i) Given below, $\angle P = \angle RTS$ then prove that $\Delta RPQ \sim \Delta RTS$.

b) In the fig. ii) given below, $\angle ADC = \angle BAC$.

Prove that, $CA^2 = DC \times BC$.



(i)



(ii)

a) In fig.(i), $\angle P = \angle RTS$

In ΔRPQ and ΔRTS,

$\angle R = \angle R$ ($\because$ common angle)

$\angle P = \angle RTS$ ($\because$ given)

$\boxed{\Delta RPQ \sim \Delta RTS}$ ($\because$ by AA axiom)

b) In fig.(ii), $\angle ADC = \angle BAC$

Now, In ΔABC and ΔADC,

$\angle C = \angle C$ ($\because$ common angle)

$\angle BAC = \angle ADC$ ($\because$ given)

$\Rightarrow \boxed{\Delta ABC \sim \Delta ADC}$

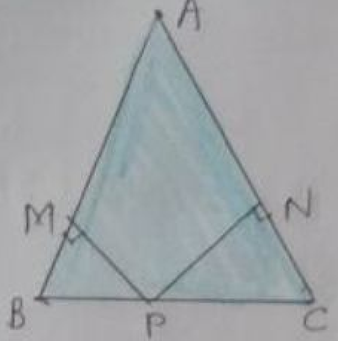
Thus, by similarity of two triangles, $\frac{CA}{DC} = \frac{BC}{CA}$

$\Rightarrow \boxed{(CA)^2 = DC \times BC}$

Hence proved.

9) In the given fig, ABC is a triangle in which $AB = AC$. P is a point on the side BC such that $PM \perp AB$ and $PN \perp AC$. Prove that, $BM \times NP = CN \times MP$.

→



Given that,
In ΔABC, $AB = AC$
P is a point on the side BC so that
PM $\perp^{er}$ AB and PN $\perp^{er}$ AC.

In ΔABC, $AB = AC$ (∵ given in question)
$\angle B = \angle C$ (∵ angles which are opposite to equal sides)

Now, Let us consider ΔBMP & ΔCNP,
$\angle M = \angle N$
$\Rightarrow \boxed{\Delta BMP \sim \Delta CNP}$

Then, by similarity of triangles $\frac{BM}{CN} = \frac{MP}{NP}$
$\Rightarrow \boxed{BM \times NP = CN \times MP}$

Hence proved.

10.) Prove that the ratio of the perimeters of two simillar triangles is the same as the ratio of their corresponding sides.

→ Given that, $\Delta ABC \sim \Delta PQR$ or $\Delta MNO \sim \Delta XYZ$

We have, if two triangles are simillar then their corresponding angles are equal and their corresponding sides are proportional.

$$\Rightarrow \frac{MN}{XY} = \frac{NO}{YZ} = \frac{MO}{XZ}$$

Perimeter of $\Delta MNO = MN + NO + MO$

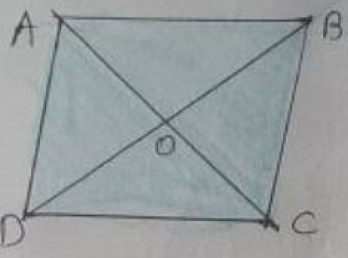
And perimeter of $\Delta XYZ = XY + YZ + XZ$

Thus, $$\frac{MN}{XY} = \frac{NO}{YZ} = \frac{MO}{XZ} = \left(\frac{MN}{XY} + \frac{NO}{YZ} + \frac{MO}{XZ}\right)$$
$$= \frac{\text{Perimeter of } \Delta MNO}{\text{Perimeter of } \Delta XYZ}$$

11.) In the fig., ABCD is a trapezium in which AB ∥ DC. The diagonals AC and BD intersect at O. Prove that $\frac{AO}{OC} = \frac{BO}{OD}$.

→ Using the above result, find the value of x if $OA = 3x-19$, $OB = x-4$, $OC = x-3$ and $OD = 4$.

→



Given that,

ABCD is a trapezium with AB ∥ DC.

And diagonals AC & BD of trapezium ABCD intersects each other at point 'O'.

Now, In ΔAOB and ΔCOD,

$\angle AOB = \angle COD$ ($\because$ vertically opposite angles are always equal.)

$\angle OAB = \angle OCD$

$\Rightarrow$ $\boxed{\Delta AOB \sim \Delta COD}$

Then, by similarity of triangles, $\frac{OA}{OC} = \frac{OB}{OD}$

Given that, $OA = 3x-19$, $OB = x-4$, $OC = x-3$ and $OD = 4$

Then, $\frac{OA}{OC} = \frac{OB}{OD}$

$$\frac{(3x-19)}{(x-3)} = \frac{(x-4)}{4}$$

$$\Rightarrow (x-3)(x-4) = 4(3x-19)$$

$$x^2 - 7x + 12 - 12x + 76 = 0$$

$$x^2 - 19x + 88 = 0$$

$$x^2 - 8x - 11x + 88 = 0$$

$$x(x-8) - 11(x-8) = 0$$

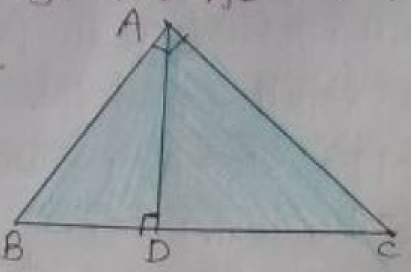
$$(x-8)(x-11) = 0$$

$$x-8 = 0 \quad \text{or} \quad x-11 = 0$$

$$\boxed{x = 8} \quad \text{or} \quad \boxed{x = 11}$$

Thus, the required values of x are found to be 8 & 11.

13) In the given fig. $\angle A = 90°$ and $AD \perp BC$ if $BD = 2cm$ and $CD = 8cm$, find AD.



Given that,

$\angle A = 90°$ and

$AD \perp^{er} BC$

Also, $BD = 2cm$ and $CD = 8cm$.

from fig., $\angle DCA + \angle DAC = 90°$ ——— ①

and $\angle BAD + \angle DAC = 90°$ ——— ②

from ① & ②, $\angle BAD + \angle DAC = \angle DCA + \angle DAC$

$\Rightarrow \boxed{\angle BAD = \angle DCA}$ ——— ③

Now, In ΔBDA and ΔADC,

$\angle BDA = \angle ADC$ ($\because$ each angle is of $90°$)

$\angle BAD = \angle DCA$ ($\because$ from ③)

$\Rightarrow \boxed{\Delta BDA \sim \Delta ADC}$

Then, By similarity of triangles,

$$\frac{BD}{AD} = \frac{AD}{DC} = \frac{AB}{AC}$$

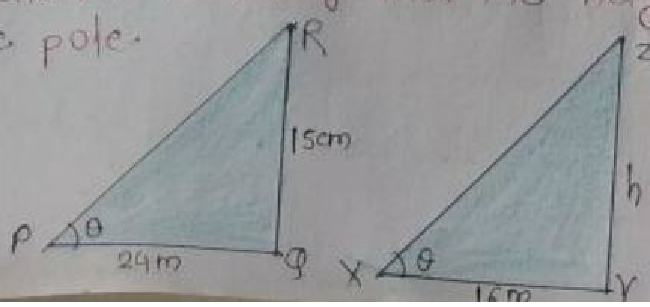
$$\Rightarrow \frac{BA}{AD} = \frac{AD}{DC} \Rightarrow (AD)^2 = BD \times CD$$

$$(AD)^2 = 2 \times 8 = 16$$

$\boxed{AD = 4cm}$ is the required answer.

14) A 15m high tower casts a shadow of 24 meters long at a certain time and at the same time, a telephone pole casts a shadow 16m long. Find the height of the telephone pole.

→



Given that, height of tower $AB = 15m$

and shadow of $BC = 24m$

At the same time & position, the height of a telephone pole is found to be $\boxed{DE = xm}$

And shadow of pole $EF = 16m$

$\Rightarrow \Delta PQR \sim \Delta MNO$

Then, By similarity of triangles,

$$\frac{PQ}{MN} = \frac{ON}{RQ}$$

$$\Rightarrow \frac{15}{x} = \frac{24}{16} \Rightarrow x = \frac{15 \times 16}{24} = \frac{240}{24} = 10$$

Thus, the height of telephone pole is found to be 10m.

15.) A street light bulb is fixed on a pole 6m above the level of street. If a woman of height casts a shadow of 3m. find the how far she is away from the base of the pole?

→ Given that,

A street light bulb is fixed on a pole 6m above the level of street.

Heigh t of pole $= AB = 6m$

And height of woman $= 1.5m$.

And shadow of woman $EF = 3m$.

As woman and pole are standing on the same line.

$PM \parallel MR \Rightarrow \Delta PRQ \sim \Delta MNR$

$$\Rightarrow \frac{RQ}{RN} = \frac{PQ}{MN} \quad \because \text{by simillarity of triangles}$$

$$\frac{3+x}{3} = \frac{6}{1.5}$$

$$\frac{3+x}{3} = 4$$

$$3 + x = 12$$

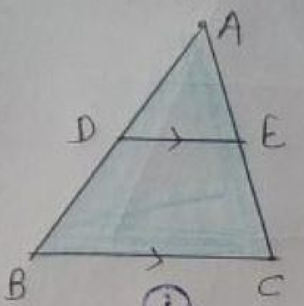
$$\boxed{x = 9m}$$

Thus, the woman is 9m far away from the base of the pole.

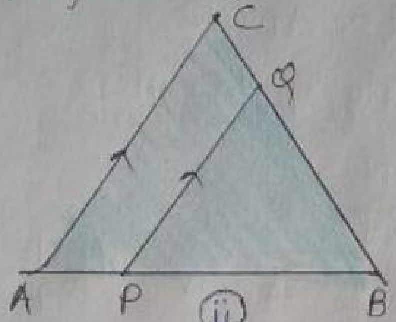
Exercise 13.2

1) a) In fig. (i) given below if $DE \parallel BG$, $AD = 3$ cm, $BD = 4$ cm and $BC = 5$ cm. Find i) $AE : EC$ ii) DE

b) In the fig. (ii) given below, $PQ \parallel AC$, $AP = 4$ cm, $PB = 6$ cm and $BC = 8$ cm. Find CQ and BQ.



(i)



(ii)

a) In fig. (i) $DE \parallel BG$,

$AD = 3$ cm, $BD = 4$ cm

and $BC = 5$ cm.

i) $\frac{AD}{BD} = \frac{AE}{EC}$

and $\frac{AE}{EC} = \frac{AD}{BD}$

$\frac{AE}{EC} = 3/4 \Rightarrow \boxed{AE : EC = 3 : 4}$

ii) In ΔADE and ΔABC,

$\angle D = \angle B$, $\angle E = \angle C$

$\Rightarrow \boxed{\Delta ADE \sim \Delta ABC}$

Then, By similarity of triangles

$\frac{DE}{BC} = \frac{AD}{AB}$

$\frac{DE}{5} = \frac{3}{(3+4)}$

$\boxed{DE = 15/7}$

b) In fig (ii) $PQ \parallel AG$, $AP = 4$ cm, $PB = 6$ cm and $BC = 8$ cm.

And $\angle BQP = \angle BCA$ ($\because$ alternate angles are always equal.)

$\angle B = \angle B$ ($\because$ common angle)

$\Rightarrow \boxed{\Delta ABC \sim \Delta BPQ}$

Then, By similarity of triangles, $\frac{BQ}{BC} = \frac{BP}{AB} = \frac{PQ}{AC}$

$\frac{BQ}{BC} = \frac{6}{(6+4)} = \frac{PQ}{AC}$

$\Rightarrow \frac{BQ}{BC} = \frac{6}{10} = \frac{PQ}{AC}$

$\frac{BQ}{8} = \frac{6}{10} = \frac{PQ}{AC}$

$\Rightarrow \frac{BQ}{8} = \frac{6}{10}$

$\Rightarrow \boxed{BQ = 4.8\,cm}$

Again, $CQ = BC - BQ$

$CQ = (8 - 4.8)$

$\boxed{CQ = 3.2\,cm}$

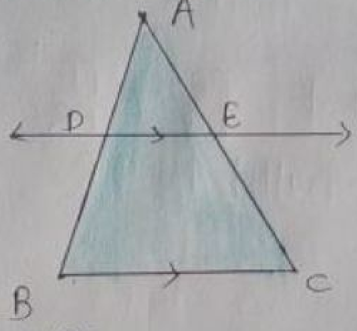
Thus, the required answer is
$BQ = 4.8\,cm$ and $CQ = 3.2\,cm$.

2.) In given fig., $DE \parallel BC$

i) If $AD = x$, $DB = x-2$, $AE = x+2$ and $EC = x-1$, find the value of x.

ii) If $DB = x-3$, $AB = 2x$, $EC = x-2$ and $AC = 2x+3$, find the value of x.

→



Given that, $DE \parallel BC$

i) Let us consider ΔABC,

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{x}{x-2} = \frac{(x+2)}{(x-1)}$

$\Rightarrow x(x-1) = (x-2)(x+2)$

$x^2 - x = x^2 - 4$

$-x = -4$

$\boxed{x = 4}$ is the required value of x.

ii) Given that, $DB = x-3$, $AB = 2x$, $EC = x-2$ and $AC = 2x+3$.
Let us consider ΔABC,

Then, $\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{2x}{(x-2)} = \frac{(2x+3)}{(x-3)}$

$2x(x-3) = (2x+3)(x-2)$

$2x^2 - 4x - 2x^2 + 6x - 3x = -9$

$-7x + 6x = -9$

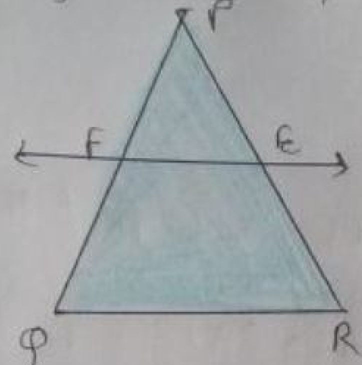
$\boxed{x = 9}$ is the required value of x.

3) E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether $EF \parallel QR$.

i) $PE = 3.9\text{cm}$, $EQ = 3\text{cm}$, $PF = 8\text{cm}$ and $RF = 9\text{cm}$

ii) $PQ = 1.28\text{cm}$, $PR = 2.56\text{cm}$, $PE = 0.18\text{cm}$ and $PF = 0.36\text{cm}$

→



i) Given that,

- E and F are points on the sides PQ & PR respectively of ΔPQR.

$$\frac{PE}{EQ} = \frac{3.9}{3} = \frac{39}{30} = \frac{13}{10}$$

Then, $\frac{PF}{FR} = 8/9$

$$\Rightarrow \frac{13}{10} \neq \frac{8}{9}$$

Thus, we conclude that $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Hence, EF is not parallel to QR.

ii) Let us consider, ΔPQR

Here, $\frac{PQ}{PE} = \frac{1.28}{0.18} = \frac{128}{18} = \frac{64}{9}$

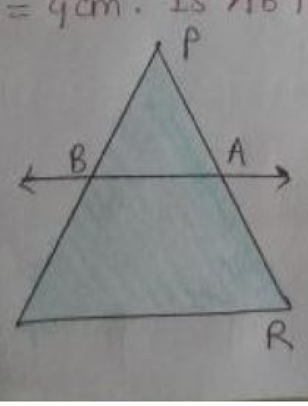
and $\frac{PR}{PF} = \frac{2.56}{0.36} = \frac{256}{36} = \frac{64}{9}$ $\Rightarrow \frac{64}{9} = \frac{64}{9}$

Thus, we conclude that $\frac{PQ}{PE} = \frac{PR}{PF}$

Hence, EF is parallel to QR.

4) A and B are respectively the points on the sides PQ and PR of a triangle PQR so that $PQ = 12.5\text{cm}$, $PA = 5\text{cm}$, $BR = 6\text{cm}$, $PB = 4\text{cm}$. Is $AB \parallel QR$? Give reason for your answer.

→



Given that,

A & B are respectively the points on the sides PQ and PR of a triangle PQR.

And $PQ = 12.5\text{cm}$, $PA = 5\text{cm}$, $BR = 6\text{cm}$, $PB = 4\text{cm}$.

Here, $\frac{PQ}{PA} = \frac{12.5}{5} = \frac{2.5}{1}$ —— ①

and $\frac{PR}{PB} = \frac{(PB+BR)}{PB} = \frac{(4+6)}{4} = \frac{10}{4} = 2.5$ —— ②

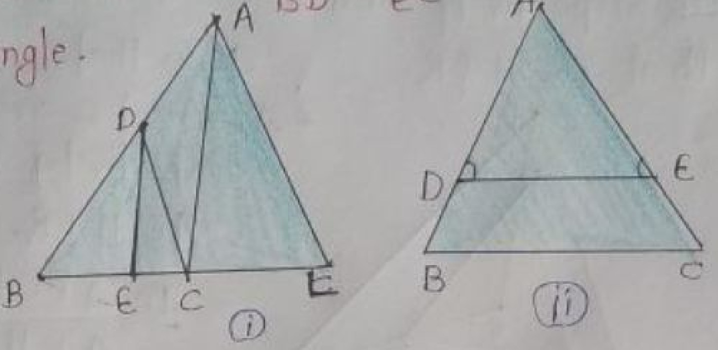
from ① & ② $\Rightarrow \frac{PQ}{PA} = \frac{PR}{PB}$

Thus, AB is parallel to QR.

Hence proved.

5.) a) In the fig ① given below, $CD \parallel LA$ and $DE \parallel AC$. Find the length of CL if BE = 4 cm and EC = 2 cm.

b) In the given fig. $\angle D = \angle E$ and $\frac{AD}{BD} = \frac{AE}{EC}$. Prove that BAC is an isosceles triangle.



→

a) From fig. ①

$CD \parallel LA$ and

$DE \parallel AC$

In $\triangle BCA$,

$\frac{BE}{BC} = \frac{BD}{BA} \Rightarrow \frac{BE}{(BE+EC)} = \frac{BD}{AB}$

$\frac{4}{(4+2)} = \frac{BD}{AB}$ —— ①

In $\triangle BLA$, $\frac{BC}{BL} = \frac{BD}{AB} \Rightarrow \frac{6}{(6+CL)} = \frac{BD}{AB}$ —— ②

from ① & ② $\Rightarrow \frac{6}{(6+CL)} = \frac{4}{6}$

$\Rightarrow 36 = 24 + 4CL$

$CL = 12/4$

$\boxed{CL = 3cm}$ is the required answer

b) In fig. $\angle D = \angle E$ and $\frac{AD}{BD} = \frac{AE}{EC}$

In $\triangle ADE$, $\angle D = \angle E$

$AD = AE$ ($\because$ sides opposite to equal angles)

In ΔABC, $\frac{AD}{DB} = \frac{AE}{EC}$

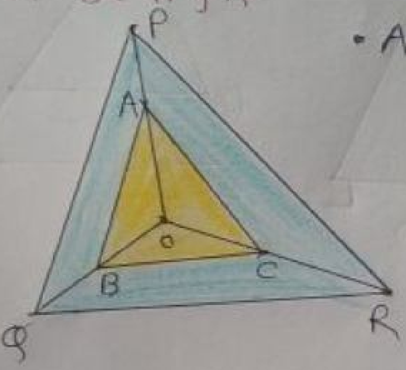
$\Rightarrow$ DE parallel to BC. $\because DB = EC$

$AD + DB = AE + EC$

$\boxed{AB = AC}$

Thus, ΔABC is an isosceles triangle. Hence proved.

6.) In the fig. given below, A, B and C are points on OP, OQ & OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.



Given that,

- A, B and C are points on OP, OQ & OR respectively so that $AB \parallel PQ$ and $AC \parallel PR$.

In ΔPOQ,

$AB \parallel PQ$

$\Rightarrow \frac{OA}{AP} = \frac{OB}{BQ}$ —— ①

In ΔOPR, $AC \parallel PR$

Then, $\frac{OA}{AP} = \frac{OC}{CR}$ —— ②

from ① & ② $\Rightarrow$ $\frac{OB}{BQ} = \frac{OC}{CR}$

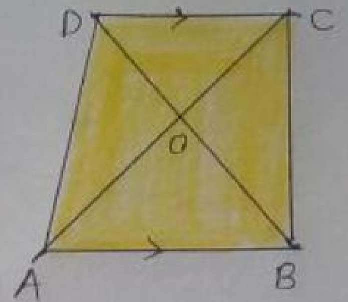
Now, In ΔOQR $\boxed{\frac{OB}{BQ} = \frac{OC}{CR}}$

$\Rightarrow$ $BC \parallel QR$ Hence proved.

7) ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersects each other at O. Using basic proportionality theorem prove that $\frac{AO}{BO} = \frac{CO}{DO}$.

→

· Given that, ABCD is a trapezium in which $AB \parallel DC$ & its diagonals intersects each other at O.



In $\triangle OAB$ and $\triangle OCD$,

$\angle AOB = \angle COD$
($\because$ vertically opposite angles are always equal)

$\angle OBA = \angle ODC$
($\because$ alternate angles are always equal.)

$\angle OAB = \angle OCD$
($\because$ alternate angles are always equal.)

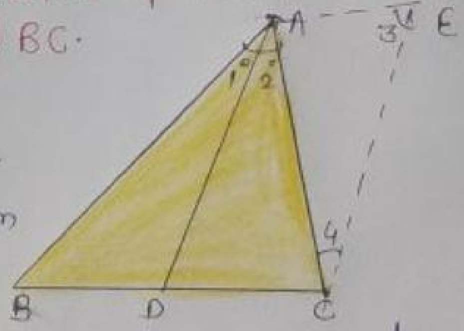
$\Rightarrow \boxed{\triangle OAB \sim \triangle OCD}$

Then, By simillarity of two triangles,

$\frac{OA}{OC} = \frac{OB}{OD} \Rightarrow \boxed{\frac{AO}{BO} = \frac{CO}{DO}}$

Hence proved.

8) In the fig. below AD is bisector of $\angle BAC$. If $AB = 6cm$, $AC = 4cm$ and $BD = 3cm$. find BC.



→ Given that,
In fig. AD is bisector of $\angle BAC$.
$AB = 6cm$, $AC = 4cm$ & $BD = 3cm$

Here, we drawn a straight line from point C i.e. CE which is parallel to DA and joined AE.

$\angle 1 = \angle 2$ —— ①

$CE \parallel DE$

$\Rightarrow \angle 2 = \angle 4$ ($\because$ alternate angles are always equal)
$\angle 1 = \angle 3$ ($\because$ corresponding angles are always equal)

$\Rightarrow \boxed{\angle 3 = \angle 4}$

and $AC = AE$

In $\triangle BCE$, $CE \parallel DE$

$\frac{BD}{DC} = \frac{AB}{AC} \Rightarrow \frac{3}{DC} = \frac{6}{4}$

$DC = 12/6$

$\boxed{DC = 2cm}$

Thus, $BC = BD + DC$
$= 3 + 2$
$\boxed{BC = 5cm}$