Chapter 4 - Algebraic Identities

Exercise : 4.1

Q.1) Evaluate each of the following using identities.

i) $(2x - 1/x)^2$

→ we know that, $(a-b)^2 = a^2 - 2ab + b^2$

$\Rightarrow (2x - 1/x)^2 = (2x)^2 - 2(2x)(1/x) + (1/x)^2$

$= 4x^2 - 4 + 1/x^2$

$\boxed{(2x - 1/x)^2 = 4x^2 + 1/x^2 - 4}$

ii) $(2x + y)(2x - y)$

→ Given that, $(2x+y)(2x-y)$

we know that, $(a+b)(a-b) = (a^2 - b^2)$

$\Rightarrow (2x+y)(2x-y) = (2x)^2 - (y)^2$

$\boxed{(2x+y)(2x-y) = 4x^2 - y^2}$

iii) $(a^2b - b^2a)^2$

We know that, $(a-b)^2 = a^2 - 2ab + b^2$

$\Rightarrow (a^2b - b^2a)^2 = [ab(a-b)]^2$

$= a^2b^2 (a-b)^2$

$= a^2b^2 (a^2 - 2ab + b^2)$

$\boxed{(a^2b - b^2a^2)^2 = a^4b^2 - 2a^3b^3 + a^2b^4}$

iv) $(a-0.1)(a+0.1)$

→ we know that, $(a+b)(a-b) = a^2 - b^2$

$\Rightarrow (a-0.1)(a+0.1) = (a)^2 - (0.1)^2$

$= a^2 - 0.01$

v) $(1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2)$

→ We know that, $(a-b)(a+b) = a^2 - b^2$

Here, $(1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2)$

$= (1.5x^2)^2 - (0.3y^2)^2$

$= 2.25x^2 - 0.9y^4$

Q.2) Evaluate each of the following using identities:

i) $(399)^2$

ii) $(0.98)^2$

iii) 991×1009

iv) 117×83

→ i) $(399)^2 = (400-1)^2$

$= (400)^2 - 2(400)(1) + 1$ $\quad \because (a-b)^2 = a^2 - 2ab + b^2$

$= 1{,}60{,}000 - 800 + 1$

$= 159200 + 1$

$= 159201$

$\boxed{(399)^2 = 1{,}59{,}201}$

ii) $(0.98)^2 = ~~(1~~$

$(0.98)^2 = (1-0.02)^2$ $\quad \because \begin{array}{r} 1.00 \\ -\ 0.02 \\ \hline 0.98 \end{array}$

$= (1)^2 - 2(1)(0.02) + (0.02)^2$ $\quad \because (a-b)^2 = a^2 - 2ab + b^2$

$= 1 - 0.04 + 0.0004$

$= 1.004 - 0.04$

$= 0.9604$

$\Rightarrow \boxed{(0.98)^2 = 0.9604}$

iii) 991×1009

$\rightarrow$ $991 \times 1009 = (1000-9) \times (1000+9)$

$= (1000)^2 - (9)^2$ $\quad \because (a-b)(a+b) = a^2 - b^2$

$= 10{,}00{,}000 - 81$

$= 9{,}99{,}919$

$\boxed{991 \times 1009 = 999919}$

iv) 117×83

$\rightarrow$ $(117) \times 83 = (100+17)(100-17)$

$= (100)^2 - (17)^2$ $\quad \because (a-b)(a+b) = a^2 - b^2$

$= 10000 - 289$

$= 9711$

$\boxed{117 \times 83 = 9711}$

Q.3.) Simplify each of the following:

i) $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25$

$\rightarrow$ Here,

$175 \times 175 + 2 \times 175 \times 25 + 25 \times 25$

$= (175)^2 + 2(175) \times (25) + (25)^2$

$= (175+25)^2 \qquad \because (a^2+2ab+b^2) = (a+b)^2$

$= (200)^2$

$= 40000$

Thus, $\boxed{175 \times 175 + 2 \times 175 \times 25 + 25 \times 25 = 40000}$

ii) $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$

$\rightarrow$ Here, $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$

$= (322)^2 - 2(322)(22) + (22)^2$

$= (322-22)^2 \qquad \because (a^2-2ab+b^2) = (a-b)^2$

$= (300)^2$

$= 90000$

Thus, $\boxed{(322 \times 322 - 2 \times 322 \times 22 + 22 \times 22) = 90000}$

iii) $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$

$\rightarrow$ Here, $0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$

$= (0.76)^2 + 2(0.76)(0.24) + (0.24)^2$

$= (0.76+0.24)^2 \qquad \because (a^2+2ab+b^2) = (a+b)^2$

$= (1.00)^2 = 1$

$\boxed{0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24 = 1}$

iv) $\frac{7.83\times7.83-1.17\times1.17}{6.66}$

→ Here, $\frac{7.83\times7.83-1.17\times1.17}{6.66}$

$= \frac{(7.83)^2-(1.17)^2}{6.66}$

$= \frac{(7.83+1.17)(7.83-1.17)}{6.66}$ $\quad \because a^2-b^2=(a+b)(a-b)$

$= \frac{(9.00)(6.66)}{(6.66)}$

$= 9$

$$\boxed{\frac{7.83\times7.83-1.17\times1.17}{6.66}=9}$$

Q.4) If $x+1/x=11$, find the value of x^2+1/x^2.

Soln:- Given that, $\left(x+\frac{1}{x}\right)=11$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = x^2+2(x)\left(\frac{1}{x}\right)+\left(\frac{1}{x}\right)^2$

$(11)^2 = x^2+2+\frac{1}{x^2}$

$121-2 = x^2+\frac{1}{x^2}$

$\Rightarrow \boxed{x^2+\frac{1}{x^2}=119}$

Q.5.) If $x - 1/x = -1$, find the value of $(x^2 + 1/x^2)$.

Soln:- Given that, $(x - 1/x) = -1$

Thus, $(x - 1/x)^2 = x^2 - 2(x)(1/x) + (1/x)^2$

$\because (a-b)^2 = a^2 - 2ab + b^2$

$(-1)^2 = x^2 - 2 + 1/x^2$

$1 + 2 = x^2 + 1/x^2$

$\Rightarrow \boxed{x^2 + 1/x^2 = 3}$

Exercise 4.2

Q.1.) Write the following in the expanded form:

i) $(a + 2b + c)^2$

ii) $(2a - 3b - c)^2$

Soln:- i) $(a + 2b + c)^2$

we know that, $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$(a+2b+c)^2 = a^2 + 4b^2 + c^2 + 2(a)(2b) + 2(2b)(c) + 2ac$

$= a^2 + 4b^2 + c^2 + 4ab + 4bc + 2ac$

ii) $(2a - 3b - c)^2$

→ we have, $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

$(2a-3b-c)^2 = (2a)^2 + (-3b)^2 + (-c)^2 + 2(2a)(-3b) + 2(2a)(-c) + 2(-3b)(-c)$

$= 4a^2 + 9b^2 + c^2 - 12ab - 4ac + 6bc$

iii) $(-3x+y+z)^2$

$\rightarrow$ we have, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$(-3x+y+z)^2 = (-3x)^2+(y)^2+(z)^2+2(-3x)(y)+2(yz)+2(-3x)(z)$

$= 9x^2+y^2+z^2-6xy+2yz-6xz$

iv) $(m+2n-5p)^2$

$\rightarrow$ we have, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$(m+2n-5p)^2 = (m)^2+(2n)^2+(-5p)^2+2(m)(2n)+2(2n)(-5p)$
$+2(m)(-5p)$

$= m^2+4n^2+25p^2+4mn-20np-10mp$

v) $(2+x-2y)^2$

$\rightarrow$ we have, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$(2+x-2y)^2 = 4+x^2+4y^2+2(2)(x)+2(x)(-2y)+2(2)(-2y)$

$= 4+x^2+4y^2+4x-4xy-8y$

vi) $(a^2+b^2+c^2)^2$

$\rightarrow$ we have, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$(a^2+b^2+c^2)^2 = (a^2)^2+(b^2)^2+(c^2)^2+2(a^2)(b^2)+2(b^2)(c^2)+2(a^2)(c^2)$

$= a^4+b^4+c^4+2(ab)^2+2(bc)^2+2(ac)^2$

$= a^4+b^4+c^4+2a^2b^2+2b^2c^2+2a^2c^2$

vii) $(ab+bc+ca)^2$

$\rightarrow$ we have, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$$(ab+bc+ca)^2 = (ab)^2+(bc)^2+(ca)^2+2(ab)(bc)+2(bc)(ca)+2(ab)(ca)$$

$$= a^2b^2+b^2c^2+c^2a^2+2ab^2c+2abc^2+2a^2bc$$

viii) $(x/y+y/z+z/x)^2$

$\rightarrow$ we have, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$$\Rightarrow (x/y+y/z+z/x)^2 = \left(\frac{x}{y}\right)^2+(y/z)^2+(z/x)^2+2\left(\frac{x}{y}\right)(y/z)+2\left(\frac{y}{z}\right)(z/x)+2\left(\frac{x}{y}\right)(z/x)$$

$$= \frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}+2x/z+2y/x+2z/y$$

ix) $(a/bc+b/ac+c/ab)^2$

$\rightarrow$ we have, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$$(a/bc+b/ac+c/ab)^2 = \left(\frac{a}{bc}\right)^2+\left(\frac{b}{ac}\right)^2+\left(\frac{c}{ab}\right)^2+2\left(\frac{a}{bc}\right)\left(\frac{b}{ac}\right)+2\left(\frac{b}{ac}\right)\left(\frac{c}{ab}\right)+2\left(\frac{a}{bc}\right)\left(\frac{c}{ab}\right)$$

$$= \frac{a^2}{b^2c^2}+\frac{b^2}{a^2c^2}+\frac{c^2}{a^2b^2}+2/c^2+2/a^2+2/b^2$$

x) $(x+2y+4z)^2$

$\rightarrow$ we have, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$$(x+2y+4z)^2 = x^2+4y^2+16z^2+2(x)(2y)+2(2y)(4z)+2(x)(4z)$$

$$= x^2+4y^2+16z^2+4xy+8yz+8xz$$

xi) $(2x-y+z)^2$

$\rightarrow$ we have, $(2x-y+z)^2$

But, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$(2x-y+z)^2 = (2x)^2+(-y)^2+z^2+2(2x)(-y)+2(-y)z+2(2x)(z)$

$= 4x^2+y^2+z^2-4xy-2yz+4xz$

xii) $(-2x+3y+2z)^2$

$\rightarrow$ we have, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc$

$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+2(-2x)(3y)+2(3y)(2z)$
$+2(-2x)(2z)$

$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

Q.2) Simplify: i) $(a+b+c)^2+(a-b+c)^2$

$\rightarrow$ we allready know that, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$\Rightarrow (a+b+c)^2+(a-b+c)^2 = a^2+b^2+c^2+a^2+b^2+c^2+2ab+2bc+2ac$
$+2a(-b)+2(-b)c+2ac$

$= 2a^2+2b^2+2c^2+4ac$

ii) $(a+b+c)^2-(a-b+c)^2$

$\rightarrow$ we know that, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$(a+b+c)^2-(a-b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac-[a^2+b^2+c^2$
$-2ab-2bc+2ac]$

$$(a+b+c)^2-(a-b+c)^2 = 4ab+4bc$$

$$\boxed{(a+b+c)^2-(a-b+c)^2 = 4b(a+c)}$$

iii) $(a+b+c)^2+(a-b+c)^2+(a+b-c)^2$

$\rightarrow$ we allready know that, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$$(a+b+c)^2+(a-b+c)^2+(a+b-c)^2$$

$$=(a^2+b^2+c^2+2ab+2bc+2ac)+(a^2+b^2+c^2-2ab-2bc+2ac)+$$
$$(a^2+b^2+c^2+2ab-2bc-2ac)$$

$$=3a^2+3b^2+3c^2+2ab-2bc+2ac$$

iv) $(2x+p-c)^2-(2x-p+c)^2$

$\rightarrow$ we know that, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$$(2x+p-c)^2-(2x-p+c)^2 = 4x^2+p^2+c^2+4xp-2pc-4xc$$
$$-(4x^2+p^2+c^2-4xp-2pc+4xc)$$

$$= 8xp+\cancel{4pc}-8xc$$

$$= 8xp-8xc$$

$$= 8(xp-xc)$$

$$= 8x(p-c)$$

v) $(x^2+y^2-z^2)^2-(x^2-y^2+z^2)^2$

→ we know that, $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$$(x^2+y^2-z^2)^2-(x^2-y^2+z^2)^2 = (x^4+y^4+z^4+2x^2y^2-2y^2z^2-2x^2z^2)$$
$$-(x^4+y^4+z^4-2x^2y^2-2y^2z^2+2x^2z^2)$$

$$= 4x^2y^2-4x^2z^2$$
$$= 4(x^2y^2-x^2z^2)$$

$$\boxed{(x^2+y^2-z^2)^2-(x^2-y^2+z^2)^2 = 4x^2(y^2-z^2)}$$

Q.3) If $a+b+c=0$ and $a^2+b^2+c^2=16$, find the value of $(ab+bc+ca)$.

Soln:- Given, $a+b+c=0$ and $a^2+b^2+c^2=16$

$(a+b+c)=0$

on squaring both sides, we get

$(a+b+c)^2=0$

$a^2+b^2+c^2+2ab+2bc+2ac=0$

But $a^2+b^2+c^2=16$

$16+2(ab+bc+ac)=0$

$2(ab+bc+ac)=-16$

$$\boxed{(ab+bc+ac)=-8}$$

Exercise : 4.3

Q.1) Find the cube of each of the following binomial expressions:

i) $(1/x + y/3)$

$\rightarrow$ we know that, $(a+b)^3 = a^3+b^3+3ab(a+b)$

$$(1/x+y/3)^3 = \left(\frac{1}{x}\right)^3 + (y/3)^3 + 3(1/x)(y/3)(1/x+y/3)$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + 3\frac{y}{x}\left(\frac{1}{x}+\frac{y}{3}\right)$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + \left(\frac{y}{x}\times\frac{y}{x}\right) + \left(\frac{y}{x}\times\frac{y}{3}\right)$$

$$\boxed{(1/x+y/3)^3 = \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x}}$$

ii) $(3/x - 2/x^2)$

$\rightarrow$ we all know that, $(a+b)^3 = a^3+b^3+3ab(a+b)$

$(a-b)^3 = a^3-b^3-3ab(a-b)$

Here, $\left(\frac{3}{x}-\frac{2}{x^2}\right)^3 = \left(\frac{3}{x}\right)^3 - \left(\frac{2}{x^2}\right)^3 - 3\left(\frac{3}{x}\right)\left(\frac{2}{x^2}\right)\left(\frac{3}{x}-\frac{2}{x^2}\right)$

$$= \frac{27}{x^3} - \frac{8}{x^6} - \frac{18}{x^3}\left(\frac{3}{x}-\frac{2}{x^2}\right)$$

$$\boxed{\left(\frac{3}{x}-\frac{2}{x^2}\right) = \frac{27}{x^3} - \frac{8}{x^6} - \frac{54}{x^4} + \frac{36}{x^5}}$$

iii) $\left(2x+\frac{3}{x}\right)^3$

→ we know that, $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

Here, $\left(2x+\frac{3}{x}\right)^3 = (2x)^3 + \left(\frac{3}{x}\right)^3 + 3(2x)\left(\frac{3}{x}\right)\left(2x+3/x\right)$

$= 8x^3 + \frac{27}{x^3} + 18\left(2x+3/x\right)$

$$\boxed{\left(2x+3/x\right)^3 = 8x^3 + \frac{27}{x^3} + 36x + \frac{54}{x}}$$

iv) $\left(4-\frac{1}{3x}\right)^3$

→ we know that, $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

Here, $(4-1/3x)^3 = (4)^3 - \left(\frac{1}{3x}\right)^3 - 3(4)\left(\frac{1}{3x}\right)\left(4-1/3x\right)$

$= 64 - \frac{1}{27x^3} - \frac{4}{x}\left(4-1/3x\right)$

$$\boxed{\left(4-\frac{1}{3x}\right)^3 = 64 - \frac{1}{27x^3} - \frac{16}{x} + \frac{4}{3x^2}}$$

Q.2.) Simplify the following:

i) $(x+3)^3 + (x-3)^3$

→ we know that, $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

Here, $(x+3)^3 = x^3 + 27 + 9x(x+3)$

$= x^3 + 27 + 9x^2 + 27x$

$(x+3)^3 = x^3 + 9x^2 + 27x + 27$

and $(x-3)^3 = x^3 - 27 - 9x(x-3)$

$= x^3 - 27 - 9x^2 + 27x$

$(x-3)^3 = x^3 - 9x^2 + 27x - 27$

Now, $(x+3)^3 + (x-3)^3 = x^3 + 9x^2 + 27x + 27 + (x^3 - 9x^2 + 27x - 27)$

$= 2x^3 + 54x$

Thus, $\boxed{(x+3)^3 + (x-3)^3 = 2x^3 + 54x}$.

ii) $(x/2 + y/3)^3 - (x/2 - y/3)^3$

→ we know that, $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

Here, $(x/2 + y/3)^3 = \frac{x^3}{8} + \frac{y^3}{27} + 3\left(\frac{x}{2}\right)\left(\frac{y}{3}\right)\left(\frac{x}{2} + y/3\right)$

$= \frac{x^3}{8} + \frac{y^3}{27} + \frac{xy}{2}\left(\frac{x}{2} + \frac{y}{3}\right)$

$(x/2 + y/3)^3 = \frac{x^3}{8} + \frac{y^3}{27} + \frac{x^2y}{4} + \frac{xy^2}{6}$

and $(x/2 - y/3)^3 = \frac{x^3}{8} - \frac{y^3}{27} - 3\left(\frac{x}{2}\right)\left(\frac{y}{3}\right)\left(\frac{x}{2} - y/3\right)$

$= \frac{x^3}{8} - \frac{y^3}{27} - \frac{xy}{2}\left(\frac{x}{2} - y/3\right)$

$= \frac{x^3}{8} - \frac{y^3}{27} - \frac{x^2y}{4} + \frac{xy^2}{6}$

Now, $(x/2 + y/3)^3 - (x/2 - y/3)^3 = x^3/8 + y^3/27 + x^2y/4 + \frac{xy^2}{6}$

$- \left[\frac{x^3}{8} - \frac{y^3}{27} - \frac{x^2y}{4} + \frac{xy^2}{6}\right]$

$= \frac{2y^3}{27} + \frac{x^2y}{2}$

$$\boxed{(x/2 + y/3)^3 - (x/2 - y/3)^3 = \frac{x^2y}{2} + \frac{2y^3}{27}}$$

iii) $(x+2/x)^3+(x-2/x)^3$

$\rightarrow$ we know that, $(a+b)^3 = a^3+b^3+3ab(a+b)$.

$(a-b)^3 = a^3-b^3-3ab(a-b)$

Here, $(x+2/x)^3 = x^3+\frac{8}{x^3}+3x\left(\frac{2}{x}\right)(x+2/x)$

$= x^3+\frac{8}{x^3}+6(x+2/x)$.

$= x^3+8/x^3+6x+12/x$

And $(x-2/x)^3 = x^3-8/x^3-3(x)\left(\frac{2}{x}\right)\left(x-\frac{2}{x}\right)$

$= x^3-8/x^3-6(x-2/x)$

$= x^3-8/x^3-6x+12/x$

Now, $(x+2/x)^3+(x-2/x)^3 = x^3+8/x^3+6(x+12/x)$

$+(x^3-8/x^3-6x+12/x)$

$(x+2/x)^3+(x-2/x)^3 = 2x^3+\frac{24}{x}$

iv) $(2x-5y)^3-(2x+5y)^3$

$\rightarrow$ we know that, $(a+b)^3 = a^3+b^3+3ab(a+b)$.

$(a-b)^3 = a^3-b^3-3ab(a-b)$

Now, $(2x-5y)^3 = 8x^3+25y^3-3(2x)(5y)(2x-5y)$

$= 8x^3-25y^3-30xy(2x-5y)$

$= 8x^3-125y^3-60x^2y+150xy^2$

And $(2x+5y)^3 = 8x^3+125y^3 + 3(2x)(5y)(2x+5y)$

$= 8x^3+125y^3+30xy(2x+5y)$

$= 8x^3+125y^3+60x^2y+150xy^2$

Now, $(2x-5y)^3-(2x+5y)^3 = (8x^3-125y^3-60x^2y+150xy^2)$
$-(8x^3+125y^3+60x^2y+150xy^2)$

$(2x-5y)^3-(2x+5y)^3 = -250y^3-120x^2y$

Q.3.) If $a+b=10$ & $ab=21$, find the value of (a^3+b^3).

Soln: we know that, $(a+b)^3 = a^3+b^3+3ab(a+b)$

Given that, $a+b=10$ and $ab=21$

$\Rightarrow (a+b)^3 = a^3+b^3+3ab(a+b)$

$(10)^3 = (a^3+b^3)+3(21)(10)$

$1000 = (a^3+b^3)+30\times21$

$1000 = a^3+b^3+630$

$a^3+b^3 = 1000-630$

$a^3+b^3 = 370$

Q.4.) If $a-b=4$ and $ab=21$, find the value of a^3-b^3.

Soln:- we know that, $(a-b)^3 = a^3-b^3-3ab(a-b)$

Given that, $(a-b)=4$ and $ab=21$

Now, $(a-b)^3 = a^3-b^3-3ab(a-b)$

$(4)^3 = (a^3-b^3) - 3(21)(4)$

$64 = (a^3-b^3) - 12\times21$

$\Rightarrow (a^3-b^3) = 64+12\times21 = 64+252$

$a^3-b^3 = 316$

Q.5.) If $x+1/x=5$, find the value of x^3+1/x^3.

Soln:- We have, $(a+b)^3 = a^3+b^3+3ab(a+b)$

Given that, $(x+1/x)=5$

Now, $(x+1/x)^3 = x^3+\frac{1}{x^3}+3x\left(\frac{1}{x}\right)(x+1/x)$

$5^3 = x^3+\frac{1}{x^3}+3(5)$

$125 = x^3+\frac{1}{x^3}+15$

$x^3+\frac{1}{x^3} = 125-15$

$\Rightarrow x^3+\frac{1}{x^3} = 110$

Q.6.) If $x - 1/x = 7$, find the value of $x^3 - 1/x^3$.

Soln:- Given that, $x - \frac{1}{x} = 7$ and $x^3 - \frac{1}{x^3} = ?$

we have, $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

and $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$.

Now, $(x - 1/x)^3 = x^3 - \frac{1}{x^3} - 3x\left(\frac{1}{x}\right)(x - 1/x)$

$$7^3 = \left(x^3 - \frac{1}{x^3}\right) - 3(7)$$

$$343 = (x^3 - 1/x^3) - 21$$

$$\left(x^3 + \frac{1}{x^3}\right) = 343 + 21$$

$$\left(x^3 - \frac{1}{x^3}\right) = 364$$

Q.7.) If $x - 1/x = 5$, find the value of $x^3 - 1/x^3$.

Soln:- Given that, $(x - 1/x) = 5$

we have, $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

Now, $(x - 1/x)^3 = x^3 - \frac{1}{x^3} - 3x\left(\frac{1}{x}\right)(x - 1/x)$

$$5^3 = x^3 - \frac{1}{x^3} - 3(5)$$

$$125 = x^3 - 1/x^3 - 15$$

$$125 + 15 = x^3 - 1/x^3$$

$$x^3 - 1/x^3 = 140$$

Q.8) If $(x^2 + 1/x^2) = 51$, find the value of $x^3 - 1/x^3$.

Soln:- we have, $(x-y)^2 = x^2 + y^2 - 2xy$

In above equn, we replace 'y' by '1/x',

we get, $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2x \cdot \left(\frac{1}{x}\right)$

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

Given that, $\left(x^2 + \frac{1}{x^2}\right) = 51$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 51 - 2 = 49$$

$$\boxed{\therefore \left(x - \frac{1}{x}\right) = \pm 7}$$

Now, To find $\left(x^3 - \frac{1}{x^3}\right)$:

we have, $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x\left(\frac{1}{x}\right)(x - 1/x)$$

$$7^3 = \left(x^3 - \frac{1}{x^3}\right) - 3(7)$$

$$343 = (x^3 - 1/x^3) - 21$$

$$\boxed{(x^3 - 1/x^3) = 364}$$

Q.9.) If $(x^2+1/x^2)=98$, find the value of x^3+1/x^3.

Soln:- we have, $(x+y)^2 = x^2+y^2+2xy$

In above eqn, we replace 'y' by '1/x'.

we get, $(x+1/x)^2 = x^2+1/x^2+2x\left(\frac{1}{x}\right)$

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

But, Given that, $\left(x^2+\frac{1}{x^2}\right)=98$

$$\left(x+\frac{1}{x}\right)^2 = 98+2 = 100$$

$$\boxed{\left(x+\frac{1}{x}\right) = \pm 10}$$

To find (x^3+1/x^3):

we have, $(a+b)^3 = a^3+b^3+3ab(a+b)$

$$(x+1/x)^3 = x^3+\frac{1}{x^3}+3x\left(\frac{1}{x}\right)(x+1/x)$$

$$10^3 = \left(x^3+\frac{1}{x^3}\right)+3\times 10$$

$$1000 = \left(x^3+\frac{1}{x^3}\right)+30$$

$$\left(x^3+\frac{1}{x^3}\right) = 1000-30$$

$$\boxed{\left(x^3+\frac{1}{x^3}\right) = 970}$$

Q.10.) If $2x+3y=13$ and $xy=6$, find the value of

$8x^3+27y^3$.

Soln:- Given that, $2x+3y=13$ and $xy=6$

we have, $(a+b)^3 = a^3+b^3+3ab(a+b)$

Now, $(2x+3y)=13$

Taking cube on both sides,

we get, $(2x+3y)^3 = 13^3$

$8x^3+27y^3+3(2x)(3y)(2x+3y) = 2197$

$8x^3+27y^3+18(6)(13) = 2197$

$8x^3+27y^3+1404 = 2197$

$8x^3+27y^3 = 2197-1404$

$\boxed{8x^3+27y^3 = 793}$

Q.11.) If $3x-2y=11$ and $xy=12$, find the value of $27x^3-8y^3$.

Soln: Given that, $3x-2y=11$ & $xy=12$

we have, $(a-b)^3 = a^3-b^3-3ab(a-b)$

Now, $(3x-2y)=11$

Taking cube on both sides,

we get $(3x-2y)^3 = 11^3$

$27x^3+8y^3-3(3x)(2y)(3x-2y) = 1331$

$27x^3-8y^3-18(12)(11) = 1331$

$27x^3-8y^3-2376 = 1331$

$27x^3-8y^3 = 1331+2376$

$\boxed{27x^3-8y^3 = 3707}$

Exercise 4.4

Q.1) Find the following products:

i) $(3x+2y)(9x^2-6xy+4y^2)$

→ Given, $(3x+2y)(9x^2-6xy+4y^2)$

$= (3x+2y)[(3x)^2-2(3x)y+(2y)^2]$

But, we know that, $(a^3+b^3) = (a+b)(a^2+b^2-ab)$.

$\Rightarrow (3x+2y)(9x^2-6xy+4y^2) = (3x)^3+(2y)^3$

$$\boxed{(3x+2y)(9x^2-6xy+4y^2) = 27x^3+8y^3}$$

ii) $(4x-5y)(16x^2+20xy+25y^2)$

→ we have, $(4x-5y)(16x^2+20xy+25y^2)$

$= (4x-5y)[(4x)^2+(4x)(5y)+(5y)^2]$

we know that, $a^3-b^3 = (a-b)(a^2+b^2+ab)$

$\Rightarrow (4x-5y)(16x^2+20xy+25y^2) = (4x)^3-(5y)^3$

$$\boxed{(4x-5y)(16x^2+20xy+25y^2) = 64x^3-125y^3}$$

iii) $(7p^4+q)(49p^8-7p^4q+q^2)$

$\rightarrow$ Given, $(7p^4+q)(49p^8-7p^4q+q^2)$

$= (7p^4+q)[(7p^4)^2-(7p^4)q+(q)^2]$

But, we know that, $(a^3+b^3) = (a+b)(a^2-ab+b^2)$

$\Rightarrow (7p^4+q)(49p^8-7p^4q+q^2) = (7p^4)^3+(q)^3$

$$\boxed{(7p^4+q)(49p^8-7p^4q+q^2) = 343p^{12}+q^3}$$

iv) $(x/2+2y)(x^2/4-xy+4y^2)$

$\rightarrow$ Given that,

$(\frac{x}{2}+2y)(x^2/4-xy+4y^2) = (\frac{x}{2}+2y)[(\frac{x}{2})^2-(\frac{x}{2})(2y)+(2y)^2]$

But, we know that, $(a^3+b^3)=(a+b)(a^2-ab+b^2)$

$\Rightarrow (\frac{x}{2}+2y)(x^2/4-xy+4y^2) = (\frac{x}{2})^3+(2y)^3$

$$\boxed{(\frac{x}{2}+2y)(\frac{x^2}{4}-xy+4y^2) = \frac{x^3}{8}+8y^3}$$

v) $(3/x-5/y)(9/x^2+25/y^2+15/xy)$

$\rightarrow$ Given that, $(3/x-5/y)(9/x^2+25/y^2+15/xy)$

$= (3/x-5/y)[(3/x)^2+(3/x)(\frac{5}{y})+(\frac{5}{y})^2]$

we know that, $(a^3-b^3) = (a-b)(a^2+ab+b^2)$

Here, $(\frac{3}{x}-\frac{5}{y})(9/x^2+25/y^2+15/xy) = (\frac{3}{x})^3-(5/y)^3$

$= \frac{27}{x^3}-\frac{125}{y^3}$

$$\boxed{(\frac{3}{x}-\frac{5}{y})(9/x^2+25/y^2+15/xy) = 27/x^3-125/y^3}$$

vi) $(3+5/x)(9-15/x+25/x^2)$

$\rightarrow$ Given that, $(3+5/x)(9-15/x+25/x^2)$

$= (3+5/x)\left[(3)^2-(3)\left(\frac{5}{x}\right)+\left(\frac{5}{x}\right)^2\right]$

But, we know that, $(a^3+b^3) = (a+b)(a^2-ab+b^2)$

$\Rightarrow \left(\frac{3}{\cancel{x}}+\frac{5}{x}\right)\left(9-15/x+\frac{25}{x^2}\right) = (3)^3+\left(\frac{5}{x}\right)^3$

$\boxed{\left(x+\frac{5}{x}\right)\left(9-\frac{15}{x}+\frac{25}{x^2}\right) = 27+\frac{125}{x^3}}$

vii) $\left(\frac{2}{x}+3x\right)(4/x^2+9x^2-6)$

$\rightarrow$ Given that, $\left(\frac{2}{x}+3x\right)\left(\frac{4}{x^2}+9x^2-6\right)$

$= \left(\frac{2}{x}+3x\right)\left[\left(\frac{2}{x}\right)^2-\left(\frac{2}{x}\right)(3x)+(3x)^2\right]$

But, we know that, $(a^3+b^3) = (a+b)(a^2-ab+b^2)$

$\Rightarrow \left(\frac{2}{x}+3x\right)\left(\frac{4}{x^2}+9x^2-6\right) = \left(\frac{2}{x}\right)^3+(3x)^3$

$\boxed{\left(\frac{2}{x}+3x\right)\left(\frac{4}{x^2}+9x^2-6\right) = \frac{8}{x^3}+27x^3}$

viii) $(3/x-2x^2)(9/x^2+4x^4-6x)$

$\rightarrow$ Given that, $(3/x-2x^2)(9/x^2+4x^4-6x)$

$= (3/x-2x^2)\left[\left(\frac{3}{x}\right)^2+(2x^2)^2-(2x^2)(3/x)\right]$

But, we know that, $(a^3-b^3) = (a-b)(a^2+ab+b^2)$

$\Rightarrow \left(\frac{3}{x}-2x^2\right)\left(\frac{9}{x^2}+4x^4-6x\right) = \left(\frac{3}{x}\right)^3-(2x^2)^3$

$\boxed{\left(\frac{3}{x}-2x^2\right)\left(\frac{9}{x^2}+4x^4-6x\right) = \frac{27}{x^3}-8x^6}$

ix) $(1-x)(1+x+x^2)$

$\rightarrow$ Here, $(1-x)(1+x+x^2) = (1-x)[(1)^2+1(x)+(x)^2]$

we know that, $(a^3-b^3) = (a-b)(a^2+ab+b^2)$

$\Rightarrow (1-x)(1+x+x^2) = (1)^3-(x)^3$

$$\boxed{(1-x)(1+x+x^2) = 1-x^3}$$

x) $(1+x)(1-x+x^2)$

$\rightarrow$ Here, $(1+x)(1-x+x^2) = (1+x)[(1)^2-1(x)+(x)^2]$

But, we know that, $(a^3+b^3) = (a+b)(a^2-ab+b^2)$

$\Rightarrow (1+x)(1-x+x^2) = 1^3+x^3$

$$\boxed{(1+x)(1-x+x^2) = 1+x^3}$$

xi) $(x^2-1)(x^4+x^2+1)$

$\rightarrow$ Here, $(x^2-1)(x^4+x^2+1) = (x^2-1)[(x^2)^2+(x^2)(1)+(1)^2]$

But, we know that, $(a^3-b^3) = (a-b)(a^2+ab+b^2)$

$\Rightarrow (x^2-1)(x^4+x^2+1) = (x^2)^3-(1)^3$

$$\boxed{(x^2-1)(x^4+x^2+1) = x^6-1}$$

xii) $(x^3+1)(x^6-x^3+1)$

$\rightarrow$ Here, $(x^3+1)(x^6-x^3+1) = (x^3+1)[(x^3)^2-(x^3)(1)+(1)^2]$

But, we know that, $(a^3+b^3) = (a+b)(a^2-ab+b^2)$.

$\Rightarrow (x^3+1)(x^6-x^3+1) = (x^3)^3+(1)^3$

$$\boxed{(x^3+1)(x^6-x^3+1) = x^9+1}$$

Q.2) If $x=3$ and $y=-1$, find the values of each of the following using an identity:

i) $(9y^2-4x^2)(81y^4+36x^2y^2+16x^4)$

$\rightarrow = (9y^2-4x^2)[81y^4+36x^2y^2+16x^4]$

$= (9y^2-4x^2)[(9y^2)^2+(9y^2)(4x^2)+(4x^2)^2]$

But, we have $(a^3-b^3) = (a-b)(a^2+ab+b^2)$

$\Rightarrow (9y^2-4x^2)(81y^4+36x^2y^2+16x^4) = (9y^2)^3-(4x^2)^3$

$= 729y^6-64x^6$.

by putting $x=3$ & $y=-1$,

we get, $(9y^2-4x^2)(81y^4+36x^2y^2+16x^4) = 729(-1)^6-64(3)^6$

$= 729-46656$

$$\boxed{(9y^2-4x^2)(81y^4+36x^2y^2+16x^4) = -45927}$$

ii) $(3/x - x/3)(x^2/9 + 9/x^2 + 1)$

$\rightarrow$ Here, $(3/x - x/3)(x^2/9 + 9/x^2 + 1)$

$= (3/x - x/3)\left[\left(\frac{x}{3}\right)^2 + \left(\frac{3}{x}\right)^2 + \left(\frac{3}{x}\right)\left(\frac{x}{3}\right)\right]$

But, we know that, $(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

$\Rightarrow \left(\frac{3}{x} - \frac{x}{3}\right)\left(\frac{x^2}{9} + \frac{9}{x^2} + 1\right) = \left(\frac{3}{x}\right)^3 - \left(\frac{x}{3}\right)^3$

$= \frac{27}{x^3} - \frac{x^3}{27}$

put $x = 3$ & $y = -1$

$\Rightarrow \left(\frac{3}{x} - \frac{x}{3}\right)\left(\frac{x^2}{9} + \frac{9}{x^2} + 1\right) = \frac{27}{27} - \frac{27}{27} = 0$

$\boxed{\left(\frac{3}{x} - \frac{x}{3}\right)(x^2/9 + 9/x^2 + 1) = 0}$

iii) $(x/7 + y/3)(x^2/49 + y^2/9 - xy/21)$

$\rightarrow$ Here, $(x/7 + y/3)(x^2/49 + y^2/9 - xy/21)$

$= (x/7 + y/3)\left[\left(\frac{x}{7}\right)^2 + \left(\frac{y}{3}\right)^2 - \left(\frac{x}{7}\right)\left(\frac{y}{3}\right)\right]$

But, we have $(a^3 + b^3) = (a+b)(a^2 + b^2 - ab)$

$\Rightarrow \left(\frac{x}{7} + \frac{y}{3}\right)\left(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21}\right) = \left(\frac{x}{7}\right)^3 + \left(\frac{y}{3}\right)^3$

$= \frac{x^3}{343} + \frac{y^3}{27}$

put $x = 3$ & $y = -1$

$\Rightarrow \left(\frac{x}{7} + \frac{y}{3}\right)\left(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21}\right) = \frac{27}{343} - \frac{1}{27}$

iv) $(x/4 - y/3)(x^2/16 + xy/12 + y^2/9)$

$\rightarrow$ Here, $(x/4 - y/3)(x^2/16 + xy/12 + y^2/9)$

$= (x/4 - y/3)\left[\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 + \left(\frac{x}{4}\right)\left(\frac{y}{3}\right)\right]$

But, we have $(a^3 - b^3) = (a-b)(a^2 + b^2 + ab)$

$\Rightarrow \left(\frac{x}{4} - \frac{y}{3}\right)\left(\frac{x^2}{16} + \frac{xy}{12} + \frac{y^2}{9}\right) = \left(\frac{x}{4}\right)^3 - \left(\frac{y}{3}\right)^3$

$= \frac{x^3}{64} - \frac{y^3}{27}$

put $x = 3$ & $y = -1$

$\Rightarrow \left(\frac{x}{4} - \frac{y}{3}\right)\left(\frac{x^2}{16} + \frac{xy}{12} + \frac{y^2}{9}\right) = \frac{27}{64} + \frac{1}{27}$

$$\boxed{\left(\frac{x}{4} - \frac{y}{3}\right)\left(\frac{x^2}{16} + \frac{xy}{12} + \frac{y^2}{9}\right) = \frac{793}{1728}}$$

v) $(5/x + 5x)(25/x^2 - 25 + 25x^2)$

$\rightarrow$ Here, $(5/x + 5x)(25/x^2 - 25 + 25x^2)$

$= (5/x + 5x)\left[(5/x)^2 - \left(\frac{5}{x}\right)(5x) + (5x)^2\right]$

But, we have, $(a^3 + b^3) = (a+b)(a^2 + b^2 - ab)$

$\Rightarrow (5/x + 5x)(25/x^2 - 25 + 25x^2) = \left(\frac{5}{x}\right)^3 + (5x)^3$

$= \frac{125}{x^3} + 125x^3$

put $x = 3$ & $y = -1$

$\Rightarrow \left(\frac{5}{x} + 5x\right)(25/x^2 - 25 + 25x^2) = \frac{125}{27} + 125 \times 27$

$= \frac{125}{27} + 3375$

$$\boxed{\left(\frac{5}{x} + 5x\right)(25/x^2 - 25 + 25x^2) = \frac{91250}{27}}$$

vii) Q.3.) If $(a+b) = 10$ and $ab = 16$, find the value of $a^2 - ab + b^2$ and $a^2 + ab + b^2$.

Sol^n:- Given that, $a+b = 10$, $ab = 16$

$a+b = 10$

squaring on both sides,

$(a+b)^2 = 100$

$a^2 + 2ab + b^2 = 100$ $\quad \because (a+b)^2 = a^2 + 2ab + b^2$

$a^2 + 2\times16 + b^2 = 100$ $\quad \because ab = 16$

$a^2 + b^2 + 32 = 100$

$a^2 + b^2 = 100 - 32$

$\boxed{a^2 + b^2 = 68}$

Again, we know that,

$(a-b)^2 = a^2 - 2ab + b^2$

$= (a^2 + b^2) - 2ab$

$= 68 - 2\times16$ $\quad \because a^2 + b^2 = 68,\ ab = 16$

$(a-b)^2 = 68 - 32 = 36$

$=$

Now, $a^2 - ab + b^2 = (a^2 + b^2) - ab$

$= 68 - 16$

$\boxed{a^2 - ab + b^2 = 52}$

And $a^2 + ab + b^2 = (a^2 + b^2) + ab$

$= 68 + 16$

$\boxed{a^2 + ab + b^2 = 84}$

Q.4) If $a+b=8$ & $ab=6$, find the value of a^3+b^3.

Soln:- Given that, $a+b=8$ & $ab=6$

$(a+b)=8$

Taking cube on both sides,

$(a+b)^3 = 8^3$

$a^3+b^3+3ab(a+b) = 512$ $\quad \because (a+b)^3 = a^3+b^3+3ab(a+b)$

$\quad \because 8^3 = 512$

$a^3+b^3+3\times6\times8 = 512$ $\quad \because a+b=8,\ ab=6$

$a^3+b^3+18\times8 = 512$

$a^3+b^3 = 512-144$

$\boxed{a^3+b^3 = 368}$

Exercise 4.5

Q.1.) Find the following products.

i) $(3x+2y+2z)(9x^2+4y^2+4z^2-6xy-4yz-6zx)$

Soln:- $(3x+2y+2z)(9x^2+4y^2+4z^2-6xy-4yz-6zx)$

$= (3x+2y+2z)\left[(3x)^2+(2y)^2+(2z)^2-(3x)(2y)-(2y)(2z)-(2z)(3x)\right]$

$= (3x)^3+(2y)^3+(2z)^3-3(3x)(2y)(2z)$

$= 27x^3+8y^3+8z^3-36xyz$

ii) $(4x-3y+2z)(16x^2+9y^2+4z^2+12xy+6yz-8zx)$

Soln:- $(4x-3y+2z)(16x^2+9y^2+4z^2+12xy+6yz-8zx)$

Here, we rewrite the above equn as

$= (4x-3y+2z)\left[(4x)^2+(-3y)^2+(2z)^2-4x(-3y)-(-3y)(2z)-(2z)(4x)\right]$

$= (4x)^3+(-3y)^3+(2z)^3-3(4x)(-3y)(2z)$

$= 64x^3-27y^3+8z^3+72xyz$

iii) $(2a-3b-2c)(4a^2+9b^2+4c^2+6ab-6bc+4ca)$

Soln:- $(2a-3b-2c)(4a^2+9b^2+4c^2+6ab-6bc+4ca)$

Here, we can rewrite the above eqn as,

$$= (2a-3b-2c)[(2a)^2+(-3b)^2+(-2c)^2-2a(-3b)-(-3b)(-2c) - (-2c)(2a)]$$

$$= (2a)^3+(-3b)^3+(-2c)^3-3(2a)(-3b)(-2c)$$

$\because (a^3+b^3+c^3-3abc) = (a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

$$= 8a^3-21b^3-8c^3-36abc$$

iv) $(3x-4y+5z)(9x^2+16y^2+25z^2+12xy-15zx+20yz)$

Soln:- $(3x-4y+5z)(9x^2+16y^2+25z^2+12xy-15zx+20yz)$

Here, we can rewrite the above eqn as,

$$= (3x-4y+5z)[(3x)^2+(-4y)^2+(5z)^2-3(-4y)-(-4y)(5z)-(5z)(3x)]$$

$$= (3x)^3+(-4y)^3+(5z)^3-3x(3x)(-4y)(5z)$$

$\because (a^3+b^3+c^3-3abc) = (a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

$$= 27x^3-64y^3+125z^3+180xyz$$

Q.2) If $x+y+z=8$ and $xy+yz+zx=20$, find the value of $x^3+y^3+z^3-3xyz$.

Soln:- Given that, $(x+y+z)=8$ & $xy+yz+zx=20$

$(x+y+z)=8$

squaring on both sides,

$(x+y+z)^2 = 8^2$

$x^2+y^2+z^2+2(xy+yz+zx)=64$

$x^2+y^2+z^2+2\times20=64$ $\quad \because xy+yz+zx=20$

$x^2+y^2+z^2+40=64$

$$x^2+y^2+z^2+40=64$$
$$x^2+y^2+z^2=64-40$$
$$\boxed{x^2+y^2+z^2=24}$$

Now, we have,

$$(x^3+y^3+z^3-3xyz)=(x+y+z)\,(x^2+y^2+z^2-xy-yz-zx)$$
$$=8\,(24-20)$$
$$=8\times 4$$
$$\boxed{x^3+y^3+z^3-3xyz=32}$$

Q.3) If $a+b+c=9$ and $ab+bc+ac=26$, find the value of $a^3+b^3+c^3-3abc$.

Soln:- Given that, $(a+b+c)=9$ & $ab+bc+ac=26$

$$a+b+c=9$$

squaring on both sides,

$$(a+b+c)^2=9^2$$
$$a^2+b^2+c^2+2(ab+bc+ca)=81$$
$$a^2+b^2+c^2+2(26)=81$$
$$a^2+b^2+c^2+52=81$$
$$a^2+b^2+c^2=81-52$$
$$\boxed{a^2+b^2+c^2=29}$$

Now, $a^3+b^3+c^3-3abc = (a+b+c)\,[a^2+b^2+c^2-(ab+bc+ca)]$

$$= 9\,(29-26)$$

$$= 9\times 3$$

$$= 27$$

$$\Rightarrow \boxed{a^3+b^3+c^3-3abc = 27}$$

Exercise VSAQs:

Q.1) If $x+1/x=3$, then find the value of x^2+1/x^2.

Soln:- Given that, $(x+1/x)=3$

squaring on both sides,

$$(x+1/x)^2 = 3^2$$

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=9$$

$$x^2+1/x^2+2=9$$

$$x^2+1/x^2 = 9-2 = 7$$

$$\boxed{x^2+1/x^2 = 7}$$

Q.2) If $x+1/x=3$, then find the value of x^6+1/x^6.

Soln:- Given that, $(x+1/x)=3$

squaring on both sides, we get

$$(x+1/x)^2=3^2$$

$$x^2+\frac{1}{x^2}+2=9$$

$$(x^2+1/x^2)=7$$

Now, $(x^2+1/x^2)=7$

Taking cube on both sides,

$$(x^2+1/x^2)^3=7^3$$

$$x^6+\frac{1}{x^6}+3\left(x^2+\frac{1}{x^2}\right)=343$$

$$x^6+\frac{1}{x^6}+3\times 7=343 \quad \because (x^2+1/x^2=7)$$

$$x^6+\frac{1}{x^6}=343-21$$

$$\boxed{x^6+1/x^6=322}$$

Q.3) If $(a+b)=7$ & $ab=12$ find the value of (a^2+b^2)

Soln:- $a+b=7$ & $ab=12$

squaring on both sides,

$$(a+b)^2=7^2$$

$$a^2+b^2+2ab=49$$

$$a^2+b^2+2\times 12=49$$

$$a^2+b^2+24=49$$

$$\boxed{a^2+b^2=25}$$

Q.4) If $(a-b)=5$ & $ab=12$ find the value of a^2+b^2.

Soln: - Given that, $a-b=5$ & $ab=12$

$a-b=5$

squaring on both sides,

$(a-b)^2=5^2$

$a^2-2ab+b^2=25$

$a^2+b^2-2\times12=25$

$a^2+b^2=25+24$

$\boxed{a^2+b^2=49}$