Chapter 15: Areas Related to Circles

Exercise 15.1

1. Find the circumference and area of a circle of radius 4.2 cm

→ Here, given that

Radius of circle $= r = 4.2$ cm

we know that, circumference of a circle $= 2\pi r$

$= 2\left(\frac{22}{7}\right)(4.2)$

$= 26.4$ cm

Area of circle $= \pi r^2$

$= \left(\frac{22}{7}\right)(4.2)^2$

$= 22 \times 0.6 \times 4.2$

$= 55.44 \text{ cm}^2$

Thus, here circumference of a circle is 26.4 cm.
and area of a circle is 55.44 cm^2.

2) Find the circumference of a circle whose area is 301.84 cm^2.

→ Given that, Area of circle $= 301.84 \text{ cm}^2$

we know that, Area of a circle $= \pi r^2$

$301.84 = \left(\frac{22}{7}\right) r^2$

$13.72 \times 7 = r^2$

$r^2 = 96.04$

$r = \sqrt{96.04}$

$\boxed{r = 9.8}$

Thus, circumference of a circle $= 2\pi r = 2\left(\frac{22}{7}\right)(9.8)$

$= 61.6$ cm

3) Find the area of a circle whose circumference is 44 cm.

→ Given that, Circumference of a circle = 44 cm

we have, Circumference of a circle = $2\pi r$

$$44 = 2\left(\frac{22}{7}\right)r$$

$$22 = \left(\frac{22}{7}\right)r$$

$$\boxed{r = 7 \text{ cm}}$$

Now, Area of circle $= \pi r^2$

$$= \left(\frac{22}{7}\right)(7)^2$$

$$= 22 \times 7$$

$$= 154 \text{ cm}^2$$

Thus, Area of circle is found to be 154 cm^2.

4) The circumference of a circle exceeds the diameter by 16.8 cm. Find the circumference of the circle.

→ Let us consider 'r' be the radius of a circle.

Circumference of a circle $= 2\pi r$

from given condition, the circumference of the circle exceeds its diameter by 16.8 cm.

$$C = d + 16.8$$

$$2\pi r = 2r + 16.8$$

$$2\pi r - 2r = 16.8$$

$$2r(\pi - 1) = 16.8$$

$$2r(3.14 - 1) = 16.8$$

$$\boxed{r = 3.92 \text{ cm}}$$

Thus, circumference of a circle $= 2\pi r$

$$= 2 \times 3.14 \times 3.92$$

$$C = 24.64 \text{ cm}$$

Thus, the circumference of a circle is 24.64 cm.

5) A horse is tied to a pole with 28m long string. find the area where the horse can graze.

→ Given that,
the length of the string = 28 m

The area where the horse can graze is the area of the circle whose radius is 28 m.

Thus, Area of the circle $= \pi r^2$

$$= \left(\frac{22}{7}\right)(28)(28)$$

$$= 2464 \text{ m}^2$$

Hence, the area where the horse can graze is 2464 cm^2.

6) A steel wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent in the form of a circle, find the area of the circle.

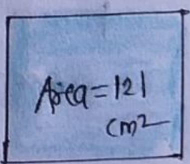
→ Given that, Area of the square $= 121 \text{ cm}^2$

$$\Rightarrow a^2 = 121$$

$$\boxed{a = 11 \text{ cm}}$$

we have, Area of circle $= \pi r^2$

Area of square $= a^2$



Then, each side of square is 11 cm.

Perimeter of square $= 4a = 4(11) = 44$ cm

From given condition,

Perimeter of square = circumference of the circle

$$4a = 2\pi r$$

$$44 = 2\left(\frac{22}{7}\right) r$$

$$\boxed{r = 7 \text{ cm}}$$

Now, Area of circle $= \pi r^2$

$$= \left(\frac{22}{7}\right)(7)(7) = 154 \text{ cm}^2$$

Hence, the area of the circle is 154 cm^2.

7) The circumference of two circles are in the ratio 2:3. Find the ratio of their areas.

→ Let us consider r_1 & r_2 be the radius of circles C_1 and C_2 respectively.

Now, Circumference of a circle $= 2\pi r$

$C_1 \rightarrow 2\pi r_1$ and $C_2 \rightarrow 2\pi r_2$

Given that, $\frac{C_1}{C_2} = \frac{2}{3}$.

$$\frac{2\pi r_1}{2\pi r_2} = \frac{2}{3}$$

$$\boxed{\frac{r_1}{r_2} = \frac{2}{3}}$$

Now, the ratio of areas of circles $= \frac{\pi r_1^2}{\pi r_2^2}$

$$= \frac{r_1^2}{r_2^2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Hence, ratio of their areas $= 4:9$

8) Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of radii 15cm and 18 cm.

→ Let us consider, radius of circle $C_1 \Rightarrow r_1 = 15$cm

Radius of circle $C_2 \Rightarrow r_2 = 18$cm

Now, Circumference of $C_1 = 2\pi r_1$.

Circumference of $C_2 = 2\pi r_2$

Let 'r' be the radius of a circle with circumference C.

from given condition,

$$C = C_1 + C_2$$

$$2\pi r = 2\pi r_1 + 2\pi r_2$$

$$r = r_1 + r_2$$

$$r = 15 + 18 = 33\text{cm}$$

Hence, the radius of circle is 33 cm.

10) The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having its area equal to the sum of the areas of two circles.

→ Given that, Radii of two circles are 8 cm & 6 cm respe.

Area of circle with radius 8 cm $= \pi(8)^2 = 64\pi \text{ cm}^2$

Area of circle with radius 6 cm $= \pi(6)^2 = 36\pi \text{ cm}^2$

Then, sum of the areas $= 64\pi + 36\pi = 100\pi \text{ cm}^2$

Let us consider 'x' be the radius of the circle.

from given condition,

$$\pi x^2 = 100\pi$$
$$x = \sqrt{100}$$
$$\boxed{x = 10 \text{ cm}}$$

Thus, the radius of the circle is found to be 10 cm.

11) The radii of two circles are 19 cm and 9 cm respectively, Find the radius and area of the circle which has circumference is equal to sum of the circumference of two circles.

→ Given that, radius of circle $C_1 \Rightarrow r_1 = 19$ cm

Radius of circle $C_2 \Rightarrow r_2 = 9$ cm

But, Circumference of circle $= 2\pi r$

$$C_1 = 2\pi r_1 \text{ and } C_2 = 2\pi r_2$$

Let 'r' be the radius of the circle 'c'.

Then, from given condition,

$$C = C_1 + C_2$$
$$2\pi r = 2\pi r_1 + 2\pi r_2$$
$$r = r_1 + r_2$$
$$r = 19 + 9$$
$$\boxed{r = 28 \text{ cm}}$$

Area of required circle $= \pi r^2 = \left(\frac{22}{7}\right)(28)^2 = 2464 \text{ cm}^2$.

12) The area of a circular playground is $22176\ cm^2$. Find the cost of fencing this ground at the rate of ₹50 per meter.

→ Given that, Area of circular playground $= 22176\ m^2$

And cost of fencing per meter = ₹50

Let us suppose, 'r' be the radius of the playground.

Then, Area $= \pi r^2$

$$\pi r^2 = 22176$$

$$r^2 = 22176\left(\frac{7}{22}\right)$$

$$r^2 = 7056$$

$$\boxed{r = 84m}$$

The total area of fencing means only circumference of the circle.

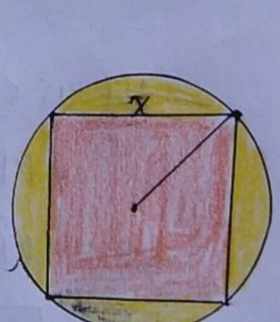
Circumference of ground $= 2\pi r = 2\left(\frac{22}{7}\right)(84) = 528\ m$

the cost of fencing 528m = ₹50 × 528 = ₹ 26400

Hence, the total ground fencing cost is ₹ 26400.

13) If a square is inscribed in a circle, find the ratio of areas of the circle & the square.

→ Let 'x' be the side of the square which is inscribed in a circle.



Given that,

Radius of circle $= \frac{1}{2}$ (diagonal of square)

$$= \frac{1}{2}(x\sqrt{2})$$

$$\boxed{r = x/\sqrt{2}}$$

Area of square $= x^2$

Area of circle $= \pi r^2$

$$= \pi\left(\frac{x}{\sqrt{2}}\right)^2 = \frac{\pi x^2}{2}$$

Now, $\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi}{2} = \pi : 2$

Thus, the ratio of areas of the circle & the square is found to be $\pi : 2$.

15) The area of circle inscribed in a equilateral triangle is $154\,cm^2$. Find the perimeter of the triangle.

→ Let us suppose, a circle with centre 'o' & radius 'r' is inscribed in the equilateral triangle.

Area of circle $= \pi r^2$

From given, $\pi r^2 = \left(\frac{22}{7}\right) r^2 = 154$

$r^2 = \frac{154 \times 7}{22}$

$r^2 = 7 \times 7$

$\boxed{r = 7\,cm}$

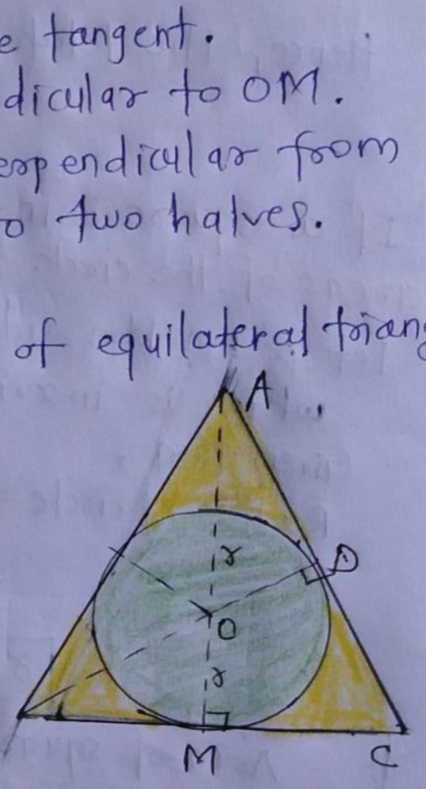
From fig., at point M, BC side is the tangent.

And at point M, BM is perpendicular to OM.

But, in an equilateral triangle the perpendicular from ~~for~~ vertex divides the side into two halves.

$BM = \frac{1}{2} \times BC$

Let us consider 'x' be the side of equilateral triangle.



$BM = \frac{1}{2} x = \frac{x}{2}$

$OB^2 = BM^2 + MO^2$

$OB = \sqrt{r^2 + \frac{x^2}{4}} = \sqrt{49 + \frac{x^2}{4}}$

$BD = \frac{\sqrt{3}}{2}\,\text{side} = \frac{\sqrt{3}}{2} x = OB + OD$

$\frac{\sqrt{3}}{2} x - r = \sqrt{49 + \frac{x^2}{4}}$, $r = 7$

$\Rightarrow \boxed{x = 14\sqrt{3}\,cm}$

Hence, perimeter $= 3x = 3\times14\sqrt{3} = 42\sqrt{3}$ cm
$= 42(1.73) = 72.7$ cm
Thus, the perimeter of the triangle is found to be 72.7 cm.

$\boxed{\text{Exercise 15.2}}$

1) Find the in terms of π, the length of the arc that subtends an angle of $30°$ at the centre of a circle of radius 4 cm.

→ Given that, Radius $= r = 4$ cm
Angle subtended at the centre 'θ' $= 30°$
But we have,
Length of arc $= \frac{\theta}{360} \times 2\pi r$
Length of arc $= \frac{30}{360} \times 2\pi(4) = \frac{2\pi}{3}$ cm

Hence, the length of the arc which subtends an angle of $30°$ is $\frac{2\pi}{3}$ cm.

2) Find the angle subtended at the centre of a circle of radius 5 cm by an arc of length $\frac{5\pi}{3}$ cm.

→ Given that, Radius $\Rightarrow r = 5$ cm
Length of an arc $= \frac{5\pi}{3}$ cm
we have, Length of an arc $= \frac{\theta}{360} \times 2\pi r$

$$\frac{5\pi}{3} = \frac{\theta}{360} \times 2\pi r$$

$$\frac{5}{3} = \frac{\theta}{360} \times 2\times5$$

$$\frac{5\times360}{3\times10} = \theta$$

$$\frac{120}{2} = \theta$$

$$\Rightarrow \boxed{\theta = 60°}$$

Hence, the angle subtended at the centre of circle is $60°$.

3) An arc of length 20π cm subtends an angle of $144°$ at the centre of a circle. find the radius of the circle.

→ Given that, Length of arc $= 20\pi$ cm

Angle subtended at the centre $= 144°$

we have, Length of an arc $= \frac{\theta}{360} \times 2\pi r$

$$\frac{\theta}{360} \times 2\pi r = \frac{144}{360} \times 2\pi r$$

$$= \frac{4\pi}{5} \times r$$

from given condition,

$$20\pi \text{ cm} = \frac{4\pi}{5} \times r \text{ cm}$$

$$\boxed{r = 25 \text{ cm}}$$

Hence, the radius of the circle is found to be 25 cm.

4) An arc of length 15 cm subtends an angle of $45°$ at the centre of a circle. find in terms of π, the radius of the circle.

→ Let, length of an arc $= 15$ cm

Angle subtended at centre $= \theta = 45°$

we have, Length of arc $= \frac{\theta}{360} \times 2\pi r$

$$= \frac{45}{360} \times 2\pi r$$

from given condition,

$$15 \text{ cm} = \frac{45}{360} \times 2\pi r$$

$$15 = \frac{\pi r}{4}$$

$$r = \frac{15 \times 4}{\pi} = \frac{60}{\pi} \text{ cm} \quad \boxed{r = \frac{60}{\pi} \text{ cm}}$$

Thus, the radius of the circle is found to be $\frac{60}{\pi}$ cm.

5) find the angle subtended at the centre of a circle of radius 'a' cm by an arc of length $\left(\frac{a\pi}{4}\right)$ cm.

→ Given that, Radius = a cm

Length of an arc = $\frac{a\pi}{4}$ cm

we have, Length of an arc = $\frac{\theta}{360} \times 2\pi r$

$$\frac{a\pi}{4} = \frac{\theta}{360} \times 2\pi a$$

$$\boxed{\theta = 45^\circ}$$

Thus, the angle subtended at the centre of circle is 45°.

6) A sector of a circle of radius 4 cm subtends an angle of 30°. Find the area of the sector.

→ Given that, Radius = 4 cm

Angle subtended at centre = $\theta = 30^\circ$

we have, Area of sector = $\frac{\theta}{360} \times \pi r^2$

$$= \frac{30}{360} \times \pi (4)^2$$

$$= \frac{1}{12} \times \pi \times 16$$

$$= \frac{4\pi}{3} \text{ cm}^2$$

Thus, the area of the sector of a circle is $\frac{4\pi}{3}$ cm^2.

7) A sector of a circle of radius 8 cm contains an angle of 135°. Find the area of sector.

→ Given that, Radius = 8 cm

Angle subtended at the centre = $\theta = 135^\circ$

we have, Area of sector = $\frac{\theta}{360} \times \pi r^2$

$$= \frac{135}{360} \times \pi (8)^2$$

$$= 24\pi \text{ cm}^2$$

Thus, the area of sector is found to be 24π cm^2.

8) The area of a sector of a circle of radius 2 cm is $\pi\ cm^2$. Find the angle contained by the sector.

→ Given that, radius of circle = 2cm

Area of sector of circle = $\pi\ cm^2$

we have, Area of sector $= \frac{\theta}{360} \times \pi r^2$

$$= \frac{\theta}{360} \times \pi (2)^2$$

$$= \frac{\pi\theta}{90}$$

from given condition, $\pi = \frac{\pi\theta}{90}$

$$\Rightarrow \boxed{\theta = 90^\circ}$$

Hence, the angle subtended at the centre of a circle is 90°.

9) The area of a sector of a circle of radius 5cm is $5\pi\ cm^2$. Find the angle contained by the sector.

→ Given that, radius of circle = r = 5cm

Area of sector of circle = $5\pi\ cm^2$

we have, Area of sector $= \frac{\theta}{360} \times \pi r^2$

$$= \frac{\theta}{360} \times \pi (5)^2$$

$$= \frac{5\pi\theta}{72}$$

from given condition,

$$5\pi = \frac{5\pi\theta}{72}$$

$$\Rightarrow \boxed{\theta = 72^\circ}$$

Hence, the angle subtended at the centre is 72°.

10) Find the area of the sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.

→ Given that, Radius of circle $= r = 5\,cm$

Length of an arc $= 3.5\,cm$

Length of an arc $= \frac{\theta}{360} \times 2\pi r$

$= \frac{\theta}{360} \times 2\pi(5)$

From given condition,

$3.5 = \frac{\theta}{360} \times 2\pi(5)$

$3.5 = 10\pi\left(\frac{\theta}{360}\right)$

$\theta = 360 \times \frac{3.5}{10\pi}$

$\boxed{\theta = \frac{126}{\pi}}$

The area of the sector $= \frac{\theta}{360} \times \pi r^2$

$= \left(\frac{126}{\pi}\right) \times \frac{1}{360} \times \pi(5)^2$

$= \frac{126 \times 25}{360} = 8.75\,cm^2$

Hence, the area of the sector is found to be $8.75\,cm^2$.

11) In a circle of radius 35 cm, an arc subtends an angle of 72° at the centre. Find the length of the arc and area of the sector.

→ Given that, radius of circle $= 35\,cm$

Angle subtended at centre $= 72°$

we have, Length of arc $= \frac{\theta}{360} \times 2\pi r$

$= \frac{72}{360} \times 2\pi(35)$

$= 14\pi$

$= 44\,cm$

Area of sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{72}{360} \times \pi(35)^2$

$= (0.2)\left(\frac{22}{7}\right)(35)(35)$

$= 0.2 \times 22 \times 5 \times 35$

Area of sector $= (35 \times 22) = 770\ cm^2$

Hence, the length of arc $= 44$ cm and

the area of the sector $= 770\ cm^2$.

13) The perimeter of a certain sector of a circle of radius is 5.6m and 27.2 m. Find the area of the sector.

→ Given that, Radius of the circle = 5.6 m

Here, $OA = OB = r = 5.6$ cm

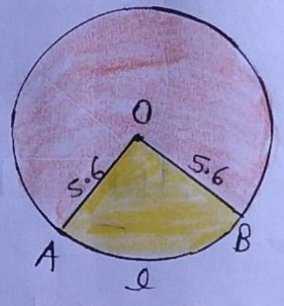
Perimeter of the sector = OA + OB + (AB arc length)

$= 27.2$

Let 'θ' be the angle subtended at centre 'O'.

Length of arc $= \frac{\theta}{360} \times 2\pi r$

$\Rightarrow \frac{\theta}{360} \times 2\pi r + OA + OB = 27.2$ m



$\frac{\theta}{360} \times 2\pi r + 5.6 + 5.6 = 27.2$ m

$\Rightarrow \boxed{\theta = 163.64°}$

Area of sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{163.64}{360} \times \pi (5.6)^2$

$= 44.8$

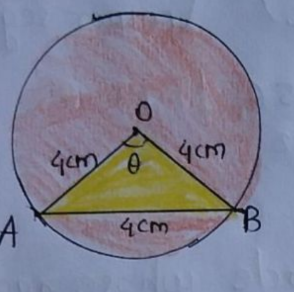
Thus, the area of the sector is $44.8\ m^2$.

Exercise 15.3

1) AB is a chord of a circle with centre 'O' and radius 4cm. AB is of length 4cm and divides the circle into two segments. Find the area of the minor segment.

→ Given that, Radius of circle $= r = 4$ cm

$\therefore OA = OB = 4$ cm



Length of the chord $AB = 4$cm

As, ΔOAB is the equilateral triangle and angle $AOB = 60°$.

An angle subtended at centre $= \theta = 60°$

Now, Area of minor segment $=$ (Area of sector) $-$ (Area of ΔAOB)

$$= \frac{\theta}{360} \times \pi r^2 - \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{60}{360} \times \pi(4)^2 - \frac{\sqrt{3}}{4}(4)^2$$

$$= \left(\frac{8\pi}{3} - 4\sqrt{3}\right)$$

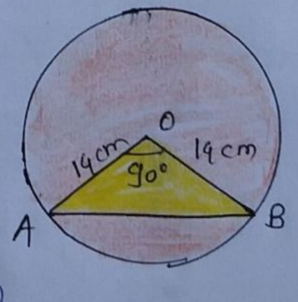
$$= 8.37 - 6.92 = 1.45 \text{ cm}^2$$

Hence, the required area of the segment is 1.45 cm^2.

2) A chord of a circle of radius 14cm makes a right angle at the centre. Find the areas of the minor and major segments of the circle.

→ Given that, Radius of circle $= 14$cm

Angle subtended by chord AB at the centre of circle 'O' $= \theta = 90°$



Now,

Area of minor segment $= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times r^2 \sin\theta$

$$= \frac{90}{360} \times \pi(14)^2 - \frac{1}{2} \times (14)^2 \sin(90)$$

$= \frac{1}{4} \times \left(\frac{22}{7}\right) (14)^2 - 7 \times 14$

$= 56 \text{ cm}^2$

Area of circle $= \pi r^2$

$= \frac{22}{7} \times (14)^2 = 616 \text{ cm}^2$

Hence, Area of major segment $= \left(\begin{matrix}\text{Area of} \\ \text{circle}\end{matrix}\right) - \left(\begin{matrix}\text{area of} \\ \text{minor segment}\end{matrix}\right)$

$= 616 - 56$

$= 560 \text{ cm}^2$

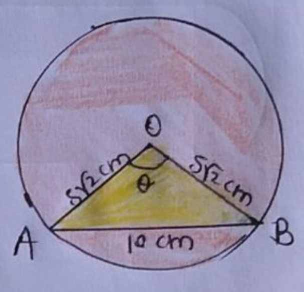
4) A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find the area of both the segments.

$\rightarrow$ Given that, Radius of circle $= r = 5\sqrt{2}$ cm

$\therefore OA = OB = 5\sqrt{2}$ cm

Length of the chord $= AB = 10$ cm

In ΔOAB, $OA^2 + OB^2$



$AB^2 = (5\sqrt{2})^2 + (5\sqrt{2})^2$

$AB^2 = 50 + 50$

$AB^2 = 100$

$\Rightarrow$ Pythagoras theorem is satisfied.

Hence, ΔOAB is a right angle triangle.

Angle subtended by chord AB at centre 'O' $\Rightarrow \theta = 90°$

Area of minor segment $= \left(\begin{matrix}\text{area of} \\ \text{sector}\end{matrix}\right) - \left(\begin{matrix}\text{area of} \\ \text{triangle}\end{matrix}\right)$

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times r^2 \sin\theta$

$= \frac{90}{360} \times 3.14 (5\sqrt{2})^2 - \frac{1}{2} (5\sqrt{2})^2 \sin 90$

$= \left[\frac{1}{4} \times 3.14 \times 25 \times 2\right] - \left[\frac{1}{2} \times 25 \times 2 \times 1\right]$

$= 25 (1.57 - 1)$

$= 14.25 \text{ cm}^2$

Now, ~~Area~~ Area of ~~circle~~ circle $= \pi r^2$

$= 3.14 \times (5\sqrt{2})^2$

$= 3.14 \times 50$

$\doteq 157\ cm^2$

Thus, Area of major segment $= \begin{pmatrix}\text{Area of}\\ \text{circle}\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{minor seg.}\end{pmatrix}$

$= 157 - 14.25$

$= 142.75\ cm^2$

5) A chord AB of circle of radius 14 cm makes an angle of $60°$, at the centre of a circle. Find the area of the minor segment of the circle.

→ Given that, Radius of a circle $= r = 14cm$

$\therefore OA = OB = 14cm$

Angle subtended by the chord at centre 'O' ⇒ $\theta = 60°$

In ΔAOB, $\angle A = \angle B$ ∵ angle opposite to equal sides are equal.

By angle sum property,

$\angle A + \angle B + \angle O = 180°$

$x + x + 60° = 180°$

$2x + 60° = 180°$

$\boxed{x = 60°}$

As all angles of ΔAOB are $60°$ & hence ΔOAB is equilateral triangle.

$\therefore OA = OB = AB$

Area of minor segment = Area of sector − Area of ΔOAB

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times (14)^2 \cdot \sin 60$

$= \frac{60}{360} \times \left(\frac{22}{7}\right)(14)^2 - \frac{\sqrt{3}}{4} \times (14)^2$

$= 14^2 \left[\frac{1}{6} \times \frac{22}{7}\right] - 0.4330$

$= 14^2 \left[\left(\frac{22}{42}\right)\right] - 0.4330$

$= 14^2 \left[(11 - 9.093)/21\right]$

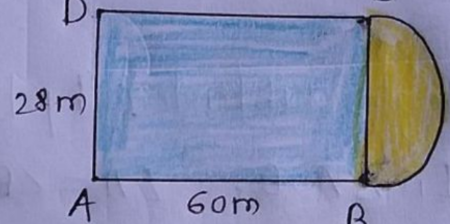
$= 14^2 (0.09080)$

$= 17.76$

Hence, the area of the minor segment $= 17.79 \text{ cm}^2$.

Exercise 15.4

1) A plot is in the form of rectangle ABCD having semicircle on BC as shown in fig. If AB = 60m and BC = 28m, find the area of the plot.



→ From fig, ABCD is a rectangle.

Thus, AB = CD = 60m

BC = AD = 28m

The radius of semicircle $= \frac{BC}{2} = \frac{28}{2} = 14\text{m}$

Now, Area of plot = (Area of rectangle ABCD) + (Area of semicircle)

$= (l \times b) + \frac{1}{2}\pi r^2$

$= (60 \times 28) + \frac{1}{2}\left(\frac{22}{7}\right)(14)^2$

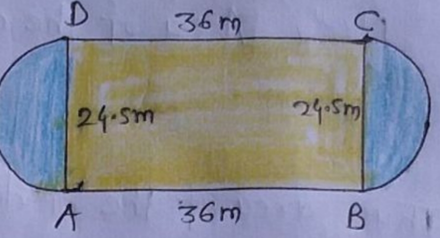
$= 1680 + 308$

$= 1988 \text{ cm}^2$

Thus, the area of the plot is found to be 1988 cm^2.

2) A playground has the shape of a rectangle, with two semi-circles on its smaller sides as diameters, added to its outside. If the sides of the rectangle are 36m & 24.5m, find the area of the playground.

→ Here, given that



Length of rectangle = 36m

Breadth of rectangle = 24.5 m

Radius of the semi-circle $= \frac{\text{breadth}}{2}$

$= \frac{24.5}{2}$

$= 12.25\text{m}$

Now,

Area of the playground = (Area of rectangle) + 2 × (Area of semicircle)

$= (l \times b) + 2 \times \frac{1}{2} (\pi r^2)$

$= (36 \times 24.5) + \left(\frac{22}{7}\right)(12.25)^2$

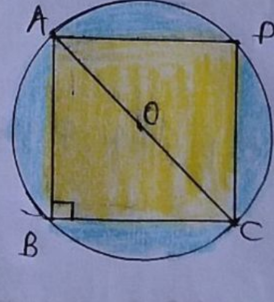
$= 882 + 471.625$

$= 1353.625$

Hence, the area of playground is found to be 1353.625m^2.

3) Find the area of the circle in which a square of area 64cm^2 is inscribed.

→ Given that,



Area of the square inscribed in a circle $= 64\text{ cm}^2$

$\text{Side}^2 = 64$

$\Rightarrow \text{side} = 8\text{ cm}$

Thus, AB = BC = CD = DA = 8cm

Now, In △ABC, By Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 8^2 + 8^2 = 64 + 64 = 128$

$AC = \sqrt{128} = 8\sqrt{2}$ cm

Now, $\angle B = 90^\circ$ & AC is the diameter of the circle.

$$\text{Radius} = \frac{AC}{2} = \frac{8\sqrt{2}}{2} = 4\sqrt{2} \text{ cm}$$

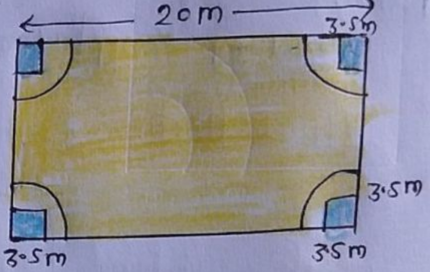
Now, Area of the circle $= \pi r^2$

$$= 3.14\,(4\sqrt{2})^2$$

$$= 100.48 \text{ cm}^2$$

Thus, area of the circle is found to be 100.48 cm^2.

4) A rectangular piece is 20m long & 15m wide. From its four corners, quadrants of radii 3.5m have been cut. Find the area of the remaining part.



→ Given that,

Length of the rectangle = 20m

Breadth of the rectangle = 15m

Radius of quadrant = 3.5m

Now, Area of remaining part = (Area of rectangle) − (4 × Area of one quadrant)

$$= (l \times b) - 4 \times \frac{1}{4}(\pi r^2)$$

$$= (l \times b) - \pi r^2$$

$$= (20 \times 15) - \left(\frac{22}{7}\right) \times (3.5)^2$$

$$= 300 - 38.5$$

$$= 261.5 \text{ m}^2$$

Thus, the area of the remaining part is 261.5 m^2.

5) In fig, PQRS is square of side 4cm. Find the area of the shaded square.

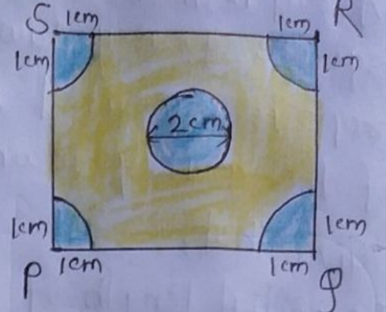
→ We have, each quadrant is a sector of 90° in a circle of radius 1cm.

That means, $\frac{1}{4}$th of a circle.

Now, Area of quadrant $= \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \left(\frac{22}{7}\right)(1)^2$

$= \frac{22}{28}\ cm^2$



Area of square $= (\text{side})^2$

$= 4^2 = 16\ cm^2$

Area of circle $= \pi r^2 = \pi (1)^2 = \frac{22}{7}\ cm^2$

Now, Area of shaded region = (area of the square) − (area of circle) − 4 × (area of quadrant).

$= 16 - \frac{22}{7} - 4 \times \frac{22}{28}$

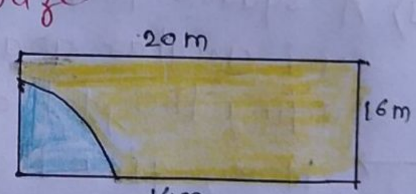
$= 16 - \frac{22}{7} - \frac{22}{7}$

$= 16 - 44/7 = \frac{68}{7}\ cm^2$

Hence, Area of shaded region is $\frac{68}{7}\ cm^2$.

7) A cow is tied with a rope of length 14m at the corner of a rectangle field of dimensions 20m × 16m, find the area of the field in which the cow can graze.

→ The blue colored portion indicates the area over which cow can graze.



Now, the shaded area is the area of a quadrant of a circle of radius equal to the length of the rope.

Hence, The required area $= \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \left(\frac{22}{7}\right) \times 14 \times 14$

$= 154\ cm^2$

Hence, the area of the field over which cow can graze is $154\ cm^2$.

8) A calf is tied with a rope of length 6m at the corner of a square grassy lawn of side 20m. If the length of the rope is increased by 5.5m, find the increase in area of the grassy lawn in which the calf can graze.

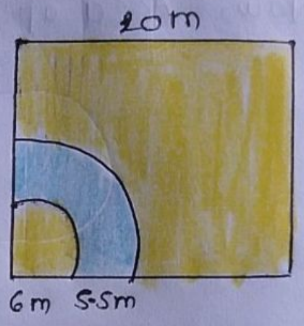
→ Given that,

The initial length of rope = 6m

Then the length of the rope is increased by 5.5m.

Then increased length of rope = 6 + 5.5 = 11.5m

But, here the corner of the lawn is nothing but the quadrant of the circle.



$$\therefore \begin{pmatrix}\text{The required}\\ \text{area}\end{pmatrix} = \frac{1}{4}(\pi r^2) - \frac{1}{4}\pi r^2$$

$$= \frac{1}{4} \times \left(\frac{22}{7}\right)\left[(11.5)^2 - 6^2\right]$$

$$= \frac{1}{4} \times \frac{22}{7}(132.25 - 36)$$

$$= \frac{1}{4} \times \frac{22}{7} \times 96.25$$

$$= 75.625 \text{ cm}^2$$

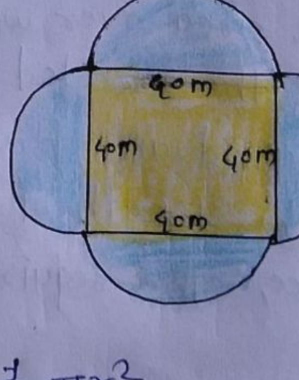
Hence, the area of required grassy lawn is 75.625 cm^2.

9) A square tank has its side equal to 40m. There are four semi-circular grassy plots all around it. Find the cost of turfing the plot at Rs. 1.25 per square meter.

→ Here, given that

Side of the square tank = 40m

$$\begin{pmatrix}\text{The diameter of semi-circular}\\ \text{grassy plot}\end{pmatrix} = 40\text{m}$$



Radius of the grassy plot $= \frac{40}{2} = 20$m

$$\text{Now, } \begin{pmatrix}\text{Area of the semi-circular}\\ \text{grassy plots}\end{pmatrix} = 4 \times \frac{1}{2}\pi r^2$$

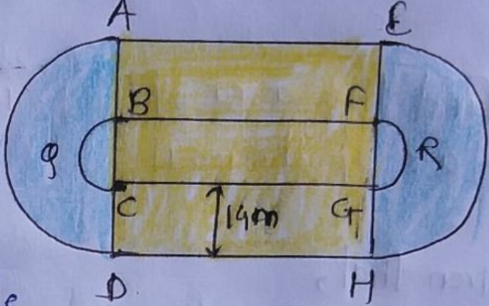
$$= 4 \times \frac{1}{2}(3.14)(20)^2$$

$$= 2512 \text{ m}^2$$

Now, Rate of turfing the plot = Rs. 1.25 per m^2
Total cost of turfing the plot of area 2512 m^2 is
$1.25 \times 2512 =$ Rs. 3140.

11.) The inner perimeter of a running track as shown in fig. is 400m. The length of each of the straight portion is 90m and the ends are semi-circles. If the track is everywhere 14m wide, find the area of the track. Also, find the length of the outer running track.

→ Given that,
Length of straight portion = 90m
width of the track = 14m
The inner perimeter of the track = 400m
Let 'r' be the radius of inner semi-circle.
And 'R' be the radius of outer semi-circle.



Then, inner perimeter of the track

$\Rightarrow BF + FRG + GC + CQB = 400$

$90 + \pi r + 90 + \pi r = 400$

$2 \times \frac{22}{7} \times r = 220$

$\boxed{r = 35m}$

Then, the radius of outer semi-circle $= 35 + 14 = 49$ m

Now, $\begin{pmatrix}\text{Area of the}\\ \text{track}\end{pmatrix} = 2\left\{\begin{pmatrix}\text{Area of the}\\ \text{rectangle AEFB}\end{pmatrix} + \begin{pmatrix}\text{Area of}\\ \text{semi-circle APD}\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{semi-circle}\\ \text{BQC}\end{pmatrix}\right\}$

$= 2\left\{(90 \times 14) + \frac{1}{2}\left(\frac{22}{7}\right)(49)^2 - \left(\frac{1}{2}\right)\left(\frac{22}{7}\right)(35)^2\right\}$

$= 2\,[1260 + 11 \times 7 \times 49 - 11 \times 5 \times 35]$

$= 2 \times 3108$

$= 6216\ m^2$

Now, the length of outer running track = AE + APD + DH + HSE

$= 90 + \pi R + 90 + \pi R$

$= 180 + 2\pi R$

$= 180 + 2\left(\frac{22}{7}\right)(49)$

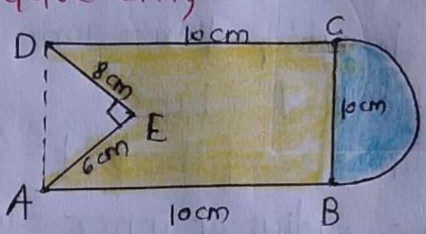
$= 180 + 308$

$= 488\,m$

Hence, the area of track is $6216\,m^2$.

And length of outer running track is 488 m.

12) Find the area of fig. in square cm, correct to one place of decimal.



→

Given that,

Radius of semi-circle $= \frac{10}{2} = 5\,cm$

Area of figure = (Area of square) + (Area of semi-circle) − (Area of triangle AED)

$= 10 \times 10 + \frac{1}{2}\pi r^2 - \frac{1}{2} \times 6 \times 8$

$= 100 + \frac{1}{2}\left(\frac{22}{7}\right)(5)^2 - 24$

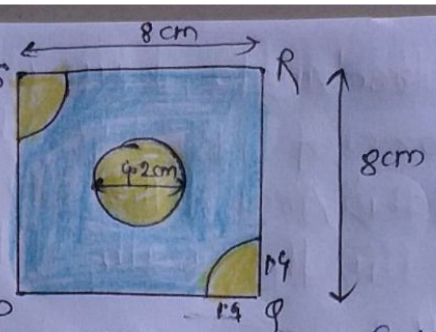
$= (700 + 275 - 168)/7$

$= (807)/7 = 115.3\,cm^2$

13) From each of the two opposite corners of a square of side 8 cm, a quadrant of a circle of radius 1.4 cm is cut. Another circle of radius 4.2 cm is also cut from the centre as shown in fig. Find the area of the remaining shaded portion of the square. (Use $\pi = \frac{22}{7}$).

→

Given that,
Side of square = 8cm
Radius of circle = 4.2 cm
Radius of quadrant = 1.4cm



Then,

$$\begin{pmatrix}\text{Area of shaded}\\ \text{portion}\end{pmatrix} = \begin{pmatrix}\text{Area of}\\ \text{square}\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{circle}\end{pmatrix} - 2\begin{pmatrix}\text{Area of the}\\ \text{quadrant}\end{pmatrix}$$

$= (\text{side})^2 - \pi r^2 - 2 \times \frac{1}{2}\pi r^2$

$= 8^2 - \pi (4.2)^2 - 2 \times \frac{1}{2} \pi (1.4)^2$

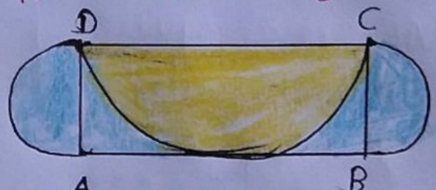
$= 64 - \frac{22}{7}(4.2 \times 4.2) - \left(\frac{22}{7}\right)(1.4 \times 1.4)$

$= 64 - 388.08/7 - 21.56/7$

$= 5.48\ cm^2$

Hence, the area of the shaded portion is $5.48\ cm^2$.

15) ABCD is a rectangle with AB = 14cm and BC = 7cm. Taking DC, BC and AD as diameters, three semi-circles are drawn as shown in fig. Find the area of the shaded region.



→

Given that,
ABCD is a rectangle with AB = 14cm and BC = 7cm

Here,

$$\begin{pmatrix}\text{The area of}\\ \text{shaded region}\end{pmatrix} = \begin{pmatrix}\text{Area of rectangle}\\ \text{ABCD}\end{pmatrix} + 2\begin{pmatrix}\text{Area of semi-circle with}\\ \text{AD \& BC as diameters}\end{pmatrix} - \begin{pmatrix}\text{Area of the semi-circle with}\\ \text{DC as diameter}\end{pmatrix}$$

$= 14 \times 7 + 2 \times \frac{1}{2}\pi\left(\frac{7}{2}\right)^2 - \frac{1}{2}\pi(7)^2$

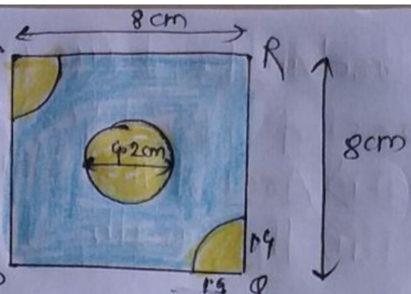
$= 98 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2 - \frac{1}{2}\left(\frac{22}{7}\right)(7)^2$

$= 98 + \frac{77}{2} - 77$

$= 59.5\ cm^2$

Thus, the area of the shaded region is $59.5\ cm^2$.

Given that,
Side of square = 8cm
Radius of circle = 4.2 cm
Radius of quadrant = 1.4cm



Then,

$$\begin{pmatrix}\text{Area of shaded}\\ \text{portion}\end{pmatrix} = \begin{pmatrix}\text{Area of}\\ \text{square}\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{circle}\end{pmatrix} - 2\begin{pmatrix}\text{Area of the}\\ \text{quadrant}\end{pmatrix}$$

$$= (\text{side})^2 - \pi r^2 - 2 \times \frac{1}{2}\pi r^2$$

$$= 8^2 - \pi(4.2)^2 - 2 \times \frac{1}{2}\pi(1.4)^2$$

$$= 64 - \frac{22}{7}(4.2 \times 4.2) - \left(\frac{22}{7}\right)(1.4 \times 1.4)$$

$$= 64 - 388.08/7 - 21.56/7$$

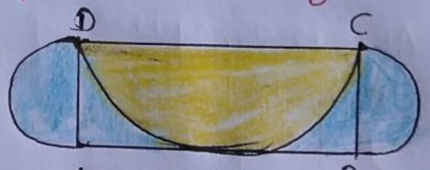
$$= 5.48\ cm^2$$

Hence, the area of the shaded portion is 5.48 cm^2.

15) ABCD is a rectangle with AB = 14cm and BC = 7cm. Taking DC, BC and AD as diameters, three semi-circles are drawn as shown in fig. Find the area of the shaded region.

→

Given that,
ABCD is a rectangle with AB = 14cm and BC = 7cm



Here,

$$\begin{pmatrix}\text{The area of}\\ \text{shaded region}\end{pmatrix} = \begin{pmatrix}\text{Area of rectangle}\\ ABCD\end{pmatrix} + 2\begin{pmatrix}\text{Area of semi-circle with}\\ AD \& BC \text{ as diameters}\end{pmatrix} - \begin{pmatrix}\text{Area of the semi-circle with}\\ DC \text{ as diameter}\end{pmatrix}$$

$$= 14 \times 7 + 2 \times \frac{1}{2}\pi\left(\frac{7}{2}\right)^2 - \frac{1}{2}\pi(7)^2$$

$$= 98 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2 - \frac{1}{2}\left(\frac{22}{7}\right)(7)^2$$

$$= 98 + \frac{77}{2} - 77$$

$$= 59.5\ cm^2$$

Thus, the area of the shaded region is 59.5 cm^2.

16) ABCD is rectangle, having AB = 20cm and BC = 14cm. Two sectors of 180° have been cut off. Calculate: i) Area of the shaded region
ii) The length of the boundary of the shaded region.

→ Given that,
Length of rectangle = AB = 20cm
Breadth of rectangle = BC = 14cm

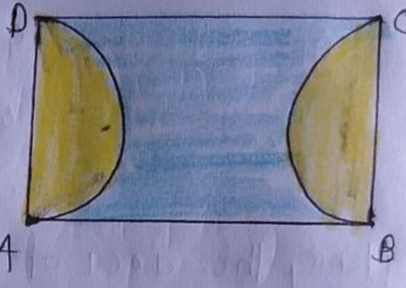
i) $\begin{pmatrix}\text{Area of shaded} \\ \text{region}\end{pmatrix} = \begin{pmatrix}\text{Area of} \\ \text{rectangle}\end{pmatrix} - 2\begin{pmatrix}\text{Area of the} \\ \text{semi-circle}\end{pmatrix}$

$= (l \times b) - 2\left(\frac{1}{2}\pi r^2\right)$

$= (20 \times 40) - \cdot\left(\frac{22}{7}\right)(7)^2$

$= 280 - 154$

$= 126\ cm^2$



ii) $\begin{pmatrix}\text{Length of the boundary} \\ \text{of the shaded region}\end{pmatrix} = 2 \times AB + 2\begin{pmatrix}\text{Circumference of a} \\ \text{semi-circle}\end{pmatrix}$

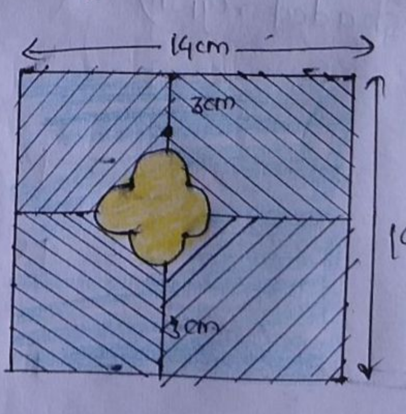
$= 2 \times 20 + 2(\pi r)$

$= 40 + 2\left(\frac{22}{7}\right)(7)$

$= 40 + 44$

$= 84\ cm$

17) In fig. find the area of shaded region. (Use $\pi = 3.14$)



→ Given that,
Side of square = 14cm
Area of square = $(14)^2 = 196\ cm^2$
Let 'r' be the radius of each semi-circle formed here.

Then, $r + 2r + r = 14 - 3 - 3$

$4r = 8$

$\boxed{r = 2}$

Hence, the radius of semi-circle is 2 cm.

Now, Area of 4 semi-circles $= (4 \times \frac{1}{2} \times 3.14 \times 2 \times 2)$

$= 25.12 \text{ cm}^2$

And $\left(\begin{array}{c}\text{length of the side of the} \\ \text{smaller square}\end{array}\right) = 2r = 2 \times 2 = 4 \text{ cm}$

And hence, Area of smaller square $= (4)^2 = 16 \text{ cm}^2$

Now, $\left(\begin{array}{c}\text{Area of unshaded} \\ \text{region}\end{array}\right) = \left(\begin{array}{c}\text{Area of 4} \\ \text{semi-circles}\end{array}\right) + \left(\begin{array}{c}\text{Area of} \\ \text{smaller square}\end{array}\right)$

$= (25.12 + 16)$

$= 41.12 \text{ cm}^2$

Thus,

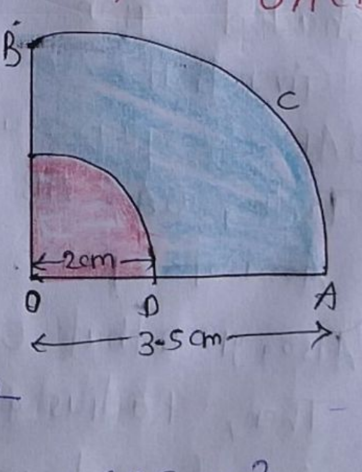
The area of shaded region $= \left(\begin{array}{c}\text{Area of square} \\ ABCD\end{array}\right) - \left(\begin{array}{c}\text{Area of} \\ \text{unshaded region}\end{array}\right)$

$= (196 - 41.12)$

$= 154.88 \text{ cm}^2$

Thus, the area of shaded region is found to be 154.88 cm^2.

19) OACB is a quadrant of a circle with center O and radius 3.5 cm. If OD = 2 cm, find the area of the i) quadrant OACB ii) shaded region.



→ Here, given that

Radius of small quadrant $= r = 2 \text{ cm}$

Radius of big quadrant $= R = 3.5 \text{ cm}$

i) Area of quadrant OACB $= \frac{1}{4}\pi R^2$

$= \frac{1}{4}\left(\frac{22}{7}\right)(3.5)^2$

$= 269.5/28 = 9.625 \text{ cm}^2$

ii) Area of shaded region = Area of big quadrant − Area of small quadrant

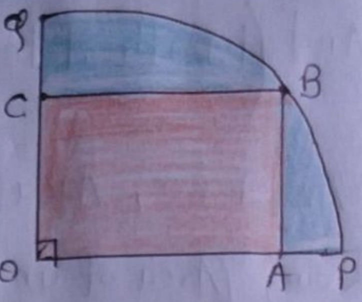
$= \frac{1}{4}\pi(R^2 - r^2) = \frac{1}{4}\left(\frac{22}{7}\right)(3.5^2 - 2^2)$

$= \frac{1}{4}\left(\frac{22}{7}\right)(12.25 - 4)$

$= \frac{1}{4}\left(\frac{22}{7}\right)(8.25) = 6.482 \text{ cm}^2$

20) A square OABC is inscribed in quadrant OPBQ of a circle. If OA = 21cm, find the area of the shaded region.

→ Given that,



Side of the square = 21cm = OA

Area of the square = $OA^2 = 21^2 = 441\,cm^2$

Diagonal of the square = $OB = \sqrt{2}\,OA = 21\sqrt{2}$ cm

From fig,

The diagonal of the square is nothing but the radius of the circle, $r = 21\sqrt{2}$ cm.

Hence, Area of the quadrant $= \frac{1}{4}\pi r^2 = \frac{1}{4}\left(\frac{22}{7}\right)(21\sqrt{2})^2$

$= 693\,cm^2$

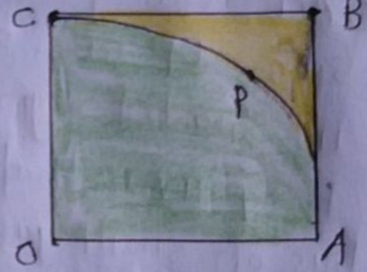
Thus, $\left(\text{The area of shaded region}\right) = \left(\text{Area of quadrant}\right) - \left(\text{Area of the square}\right)$

$= 693 - 441$

$= 252\,cm^2$

21) OABC is a square of side 7cm. If OAPC is a quadrant of a circle with centre 'O', then find the area of the shaded region.

→ Given that,



OABC is square with side = 7cm

Then, OA = AB = BC = OC = 7 cm

Area of square OABC = $(7)^2 = 49\,cm^2$

Also, given that OAPC is a quadrant of circle with centre 'O',

Then, Radius of quadrant = OA = OC = 7 cm

Area of quadrant OAPC $= \frac{90}{360} \times \pi r^2$

$= \frac{1}{4}\left(\frac{22}{7}\right) \times (7)^2$

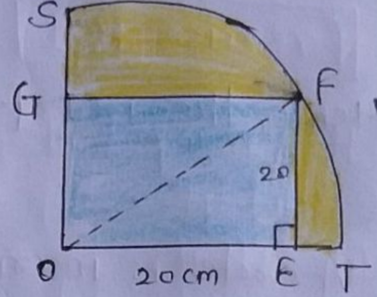
$= \frac{77}{2} = 38.5\,cm^2$

Thus, Area of shaded region = (Area of square OABC) − (Area of quadrant OAPC)

$= (49 - 38.5)$

$= 10.5 \text{ cm}^2$

22) OE = 20cm. In sector OSFT, square OEFG is inscribed. Find the area of the shaded region.



→ from fig, we have

OEFG is a square of side 20cm.

Then, diagonal of square = $\sqrt{2}$ side

$= 20\sqrt{2}$ cm

(The radius of quadrant) = (Diagonal of the square)

Radius of the quadrant = $20\sqrt{2}$ cm

Then, (Area of shaded region) = (Area of quadrant) − (Area of square)

$= \frac{1}{4}(\pi r^2) - (\text{side})^2$

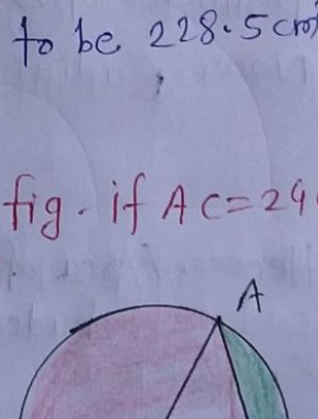
$= \frac{1}{4}\left(\frac{22}{7}\right)(20\sqrt{2})^2 - (20)^2$

$= \frac{1}{4}\left(\frac{22}{7}\right)(800) - 400$

$= 400 \times \frac{4}{7} = 1600/7 = 228.5 \text{ cm}^2$

Thus, the area of shaded region is found to be 228.5 cm^2.

23) Find the area of the shaded region in fig. if AC = 24 cm, BC = 10 cm & 'O' is the centre of the circle.



→ Given that,

AC = 24cm, BC = 10cm

AB is the diameter of the circle.

$\angle ACB = 90°$

By Pythagoras Theorem,

$AB^2 = AC^2 + BC^2 = 24^2 + 10^2 = 576 + 100 = 676$

$\boxed{AB = 26 \text{ cm}}$

Hence, the radius of the circle $= \frac{26}{2} = 13$ cm.

$$\begin{pmatrix}\text{The area of shaded} \\ \text{region}\end{pmatrix} = \begin{pmatrix}\text{Area of} \\ \text{semi-circle}\end{pmatrix} - \begin{pmatrix}\text{Area of triangle} \\ \text{ACB}\end{pmatrix}$$

$$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times b \times h$$

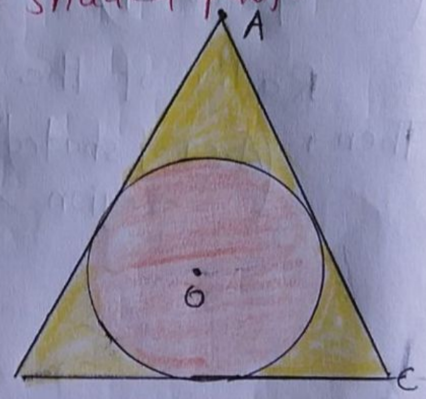
$$= \frac{1}{2}\left(\frac{22}{7}\right)(13)^2 - \frac{1}{2} \times 10 \times 24$$

$$= 265.33 - 120$$

$$= 145.33 \text{ cm}^2$$

Hence, the area of shaded region is found to be 145.33 cm^2.

24) A circle is inscribed in an equilateral triangle ABC of side 12cm, touching its sides. Find the radius of the inscribed circle and the area of the shaded part.



→ Here, given that

An equilateral triangle with side = 12cm

$$A(\text{equilateral triangle}) = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \left(\frac{\sqrt{3}}{4}\right)(12)^2$$

$$= 36\sqrt{3} \text{ cm}^2$$

Perimeter of $\Delta ABC = 3 \times 12 = 36$ cm

Now, Radius of incircle $= \dfrac{\text{Area of triangle}}{\frac{1}{2}(\text{perimeter of triangle})}$

$$= \frac{36\sqrt{3}}{1/2 \times 36} = 2\sqrt{3} \text{ cm}$$

Hence, $\begin{pmatrix}\text{Area of} \\ \text{shaded part}\end{pmatrix} = \begin{pmatrix}\text{Area of} \\ \text{equilateral triangle}\end{pmatrix} - \begin{pmatrix}\text{Area of} \\ \text{circle}\end{pmatrix}$

$$= 36\sqrt{3} - \pi r^2$$

$$= 36(1.732) - (3.14)(2\sqrt{3})^2$$

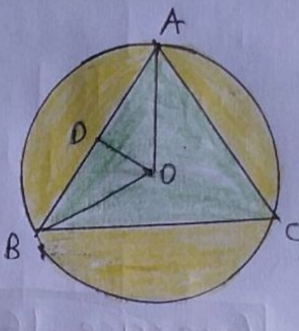
$$= 62.352 - 37.68$$

$$= 62.352 - 37.68$$

$$= 24.672 \text{ cm}^2$$

Hence, the area of shaded part is 24.672 cm^2.

25) In fig. an equilateral triangle ABC of side 6 cm has been inscribed in a circle. find the area of shaded region. (Take $\pi = 3.14$)



→ Given that,

Side of equilateral triangle = 6 cm

And, Area of equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4}(6)^2$

$= 9\sqrt{3}\ cm^2$

Let us consider 'o' be the centre of circle and OA, OB are the radii of circle.

In ΔBOD, $\sin 60^\circ = BD/OB$

$\sqrt{3}/2 = 3/OB$

$\boxed{OB = 2\sqrt{3}\,cm = r}$

Then,

$\begin{pmatrix}\text{The area of shaded} \\ \text{region}\end{pmatrix} = \begin{pmatrix}\text{Area of the} \\ \text{circle}\end{pmatrix} - \begin{pmatrix}\text{Area of equilateral} \\ \text{triangle}\end{pmatrix}$

$= \pi r^2 - 9\sqrt{3}$

$= 3.14 \times (2\sqrt{3})^2 - 9\sqrt{3}$

$= 3.14 \times 12 - 9 \times 1.732$

$= 37.68 - 15.588$

$= 22.092\ cm^2$

Hence, the area of shaded region is found to be $22.092\ cm^2$.