Chapter 14.

Co-ordinate Geometry

Exercise 14.1

1.) On which axis do the following points lie?

i) P (5,0)

The point P(5,0) lies on X-axis.

ii) Q (0,-2)

The point Q (0,-2) lies on Y-axis (negative Y-axis)

iii) R (-4,0)

The point R (-4,0) lies on X-axis (negative X-axis)

iv) S (0,5)

The point S(0,5) lies on Y-axis.

Exercise 14.2

1) Find the distance between the following pair of points:

i) $(-6, 7)$ and $(-1, -5)$

→ Given points are $(-6, 7)$ and $(-1, -5)$.

Then, $x_1 = -6,\ y_1 = 7$

$x_2 = -1,\ y_2 = -5$

By distance formula,

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{[-1 - (-6)]^2 + (-5 - 7)^2}$$

$$PQ = \sqrt{(-1 + 6)^2 + (-5 - 7)^2} = \sqrt{(5)^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169}$$

$$\boxed{PQ = 13}$$

ii) $(a+b, b+c)$ and $(a-b, c-b)$

→ Given points are $P(a+b, b+c)$ & $Q(a-b, c-b)$.

Here, $x_1 = a+b,\ y_1 = b+c$

$x_2 = a-b,\ y_2 = c-b$

By distance formula,

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{[a-b-(a+b)]^2 + [c-b-(b+c)]^2}$$

$$PQ = \sqrt{(a-b-a-b)^2 + (c-b-b-c)^2} = \sqrt{(-2b)^2 + (-2b)^2} = \sqrt{4b^2 + 4b^2}$$

$$PQ = \sqrt{8b^2} = \sqrt{4 \times 2b^2} = 2\sqrt{2}b$$

$$\boxed{PQ = 2\sqrt{2}b}$$

iii) $(a\sin\alpha, -b\cos\alpha)$ & $(-a\cos\alpha, b\sin\alpha)$

→ Let $P(a\sin\alpha, -b\cos\alpha)$ & $Q(-a\cos\alpha, b\sin\alpha)$

Then, $x_1 = a\sin\alpha$, $y_1 = -b\cos\alpha$

$x_2 = -a\cos\alpha$, $y_2 = b\sin\alpha$

By distance formula,

$$PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \quad \text{——— ①}$$

$$(x_2-x_1)^2 = [-a\cos\alpha - a\sin\alpha]^2$$

$$= [-(a\cos\alpha + a\sin\alpha)]^2 = a^2(\cos\alpha + \sin\alpha)^2$$

$$(x_2-x_1)^2 = a^2(\cos^2\alpha + \sin^2\alpha + 2\cos\alpha\sin\alpha) = a^2(1 + 2\cos\alpha\cdot\sin\alpha) \quad \text{——— ②}$$

And

$$(y_2-y_1)^2 = [b\sin\alpha + b\cos\alpha]^2 = b^2(\sin\alpha + \cos\alpha)^2$$

$$= b^2(\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cdot\cos\alpha]$$

$$(y_2-y_1)^2 = b^2(1 + 2\sin\alpha\cdot\cos\alpha) \quad \text{——— ③}$$

Now,

$$PQ = \sqrt{a^2(1+2\cos\alpha\cdot\sin\alpha) + b^2(1+2\cos\alpha\cdot\sin\alpha)} \quad \because \text{by ①, ② \& ③}$$

$$PQ = \sqrt{(a^2+b^2)(1+2\cos\alpha\cdot\sin\alpha)}$$

iv) $(a, 0)$ & $(0, b)$

→ Let $P(a, 0)$ & $Q(0, b)$.

Then $x_1 = a$, $y_1 = 0$ and $x_2 = 0$, $y_2 = b$

By distance formula,

$$PQ = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(0-a)^2 + (b-0)^2} = \sqrt{(-a)^2 + b^2}$$

$$PQ = \sqrt{a^2+b^2}$$

2.) Find the value of a when the distance betn the points (3, a) and (4, 1) is $\sqrt{10}$.

→ Let the given points are P(3, a) and Q(4, 1).

Then, By distance formula,

$$PQ = \sqrt{10}$$

$$\sqrt{10} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$\sqrt{10} = \sqrt{(4-3)^2 + (1-a)^2}$$

$$\sqrt{10} = \sqrt{16+9-24+1+a^2-2a}$$

$$\sqrt{10} = \sqrt{2+a^2-2a}$$

$$\Rightarrow 10 = 2+a^2-2a$$

$$a^2-2a+2-10=0$$

$$a^2-2a-8=0$$

$$a^2-4a+2a-8=0$$

$$a(a-4)+2(a-4)=0$$

$$(a-4)(a+2)=0 \quad \Rightarrow a=4 \text{ and } a=-2$$

Thus, the two values of a are 4 and −2.

3.) If the points (2, 1) and (1, −2) are equidistant from the point (x, y), show that $x+3y=0$

→ Let us consider, the points P(2, 1) and Q(1, −2) are equidistant from point R(x, y).

Then, $PR = QR$

$$PR = \sqrt{(x-2)^2 + (y-1)^2} \quad \because \text{By distance formula}$$

$$PR = \sqrt{x^2+4-4x+y^2+1-2y}$$

$$PR = \sqrt{x^2+y^2-4x-2y+5}$$

And $QR = \sqrt{(x-1)^2 + (y+2)^2}$ ∵ By distance formula

$$= \sqrt{x^2+1-2x+y^2+4+4y}$$

$$QR = \sqrt{x^2+y^2-2x+4y+5}$$

But, PR = QR

$$\Rightarrow \sqrt{x^2+y^2-2y-4x+5} = \sqrt{x^2+y^2+4y-2x+5}$$

$$\Rightarrow x^2+y^2-2y-4x+5 = x^2+y^2+4y-2x+5$$

$$-2y-4x = 4y-2x$$

$$2x-4y-2y-4x = 0$$

$$-2x-6y = 0$$

$$-2x = 6y$$

$$x = -3y$$

$$\Rightarrow \boxed{x+3y=0}$$

Hence proved.

4) Find the value of x, y if the distances of point (x,y) from $(-3,0)$ as well as from $(3,0)$ are 4.

→ Let the given points are $P(x,y)$, $Q(-3,0)$ & $R(3,0)$.

Then, $PQ = \sqrt{(x+3)^2+(y-0)^2}$ ∵ By distance formula

$$4 = \sqrt{x^2+9+6x+y^2}$$

$$16 = x^2+9+6x+y^2$$

$$x^2+y^2 = 7-6x \quad \text{——} \quad (1)$$

Now,

$$PR = \sqrt{(x-3)^2+(y-0)^2}$$

$$4 = \sqrt{x^2+9-6x+y^2}$$

$$16 = x^2+9-6x+y^2$$

$$x^2+y^2 = 7+6x \quad \text{——} \quad (2)$$

from (1) & (2) $\Rightarrow$

$$7-6x = 7+6x$$

$$-6x-6x = 0$$

$$-12x = 0$$

$\boxed{x=0}$ put in (1)

$$\Rightarrow 0+y^2 = 7-6x = 7-0$$

$$y^2 = 7 \quad \boxed{y = \pm\sqrt{7}}$$

Hence, the required two points are $(12, \sqrt{7})$ & $(12, -\sqrt{7})$.

5) The length of a line segment is of 10 units and the coordinates of one end-point are $(2, -3)$. If the abscissa of the other end is 10, find the ordinate of the other end.

→ Given that, the length of the segment is 10 units.
And co-ordinate of end point is $(2, -3)$ and the abscissa of other end point is 10.
Let 'k' be the ordinate of other end point.
Then, By distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$10 = \sqrt{(10-2)^2 + (k+3)^2}$$

$$10 = \sqrt{8^2 + (k+3)^2}$$

$$\Rightarrow 100 = 8^2 + (k+3)^2 = 64 + k^2 + 9 + 6k$$

$$100 - 64 = k^2 + 6k + 9$$

$$k^2 + 6k - 27 = 0$$

$$k^2 + 9k - 3k - 27 = 0$$

$$k(k+9) - 3(k+9) = 0$$

$$(k+9)(k-3) = 0$$

$$\boxed{k = -9} \text{ or } \boxed{k = 3}$$

Hence, the ordinates of other end can be 3 or -9.

6.) Show that the points $A(-4, -1)$, $B(-2, -4)$, $C(4, 0)$ & $D(2, 3)$ are the vertices of points of a rectangle.

→ Let given points are: $A(-4, -1)$, $B(-2, -4)$, $C(4, 0)$ & $D(2, 3)$

To prove: Given points are vertices of a rectangle.

By distance formula,

Length of a side $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$l(AB) = \sqrt{(-2+4)^2+(-4+1)^2} = \sqrt{4+9} = \sqrt{13}$ units

$l(BC) = \sqrt{(4+2)^2+(0+4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

$l(CD) = \sqrt{(2-4)^2+(3-0)^2} = \sqrt{4+9} = \sqrt{13}$ units

$l(AD) = \sqrt{(2+4)^2+(3+1)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

Now, for diagonals,

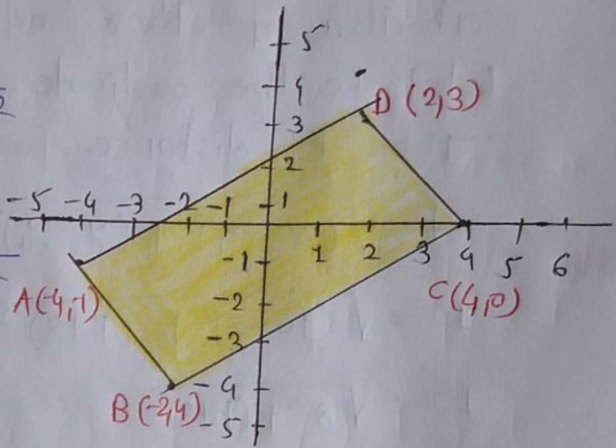
$l(\text{diagonal } BD) = \sqrt{(2+2)^2+(3+4)^2}$

$= \sqrt{16+49} = \sqrt{65}$

$l(\text{diagonal } AC) = \sqrt{(4+4)^2+(0+1)^2}$

$= \sqrt{64+1}$

$= \sqrt{65}$ units



Thus, as the opposite sides are equal & also the diagonals are equal.

Hence, the given points are the vertices of the rectangle.

Hence proved.

8.) Prove that, the points $A(1,7)$, $B(4,2)$, $C(-1,-1)$ and $D(-4,4)$ are the vertices of a square.

→ Given points are $A(1,7)$, $B(4,2)$, $C(-1,-1)$ & $D(-4,4)$.

To prove: the given points are vertices of a square.

By distance formula,

Length of a side $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$l(AB) = \sqrt{(4-1)^2+(2-7)^2} = \sqrt{9+25} = \sqrt{34}$ units

$l(BC) = \sqrt{(-1-4)^2+(-2-1)^2} = \sqrt{25+9} = \sqrt{34}$ units

$l(CD) = \sqrt{(-4+1)^2+(4+1)^2} = \sqrt{9+25} = \sqrt{34}$ units

$l(DA) = \sqrt{(-4-1)^2+(4-7)^2} = \sqrt{25+9} = \sqrt{34}$ units

Now, length of diagonals

$l(\text{diagonal } BD) = \sqrt{(-4-4)^2+(4-2)^2} = \sqrt{64+4} = \sqrt{68}$ units

$l(\text{diagonal } AC) = \sqrt{(-1-1)^2+(-1-7)^2} = \sqrt{4+64} = \sqrt{68}$ units

Now, as the opposite sides are equal and also the diagonals are equal.

And all sides are having equal length.

Hence, given vertices are the vertices of square.

Hence proved.

9) Prove that the points (3,0), (6,4) and (-1,3) are vertices of a right-angled isosceles triangle.

→ Let us consider, A(3,0), B(6,4) & C(-1,3) are the vertices of a ~~right-angled~~ is triangle.

By distance formula,

Length of a side $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$l(AB) = \sqrt{(6-3)^2+(4-0)^2} = \sqrt{9+16} = \sqrt{25}$ units

$l(BC) = \sqrt{(-1-6)^2+(3-4)^2} = \sqrt{49+1} = \sqrt{50}$ units

$l(AC) = \sqrt{(-1-3)^2+(3-0)^2} = \sqrt{16+9} = \sqrt{25}$ units

Here, $l(AB) = l(AC)$ & hence $\triangle ABC$ is an isosceles triangle.

By Paythagoras Theorem,

$BC^2 = AB^2 + AC^2$

$(\sqrt{50})^2 = (\sqrt{25})^2 + (\sqrt{25})^2$

$50 = 25+25$

$50 = 50$

Thus, $BC^2 = AB^2 + AC^2$

Hence, the given points are vertices of right-angled isosceles triangle.

Hence proved.

11.) Prove that, the points $(2a, 4a)$, $(2a, 6a)$ and $(2a+\sqrt{3}a, 5a)$ are the vertices of an equilateral triangle.

→ Let us consider ΔABC with vertices $A(2a, 4a)$, $B(2a, 6a)$ & $C(2a+\sqrt{3}a, 5a)$.

Now, By distance formula,

Length of a side $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$l(AB) = \sqrt{(2a-2a)^2 + (6a-4a)^2} = \sqrt{(2a)^2} = 2a$ units

$l(BC) = \sqrt{(2a+\sqrt{3}a-2a)^2 + (5a-6a)^2} = \sqrt{(4a^2)} = 2a$ units

$l(AC) = \sqrt{(2a+\sqrt{3}a-2a)^2 + (5a-4a)^2} = \sqrt{(4a^2)} = 2a$ units

Now, here all the sides of ΔABC are equal & hence ΔABC is a equilateral triangle.

Thus, the given vertices are vertices of equilateral triangle.

Hence proved.

12.) Prove that the points $(2,3)$, $(-4,-6)$ and $(1, 3/2)$ do not form a triangle.

→ Let us consider, $A(2,3)$, $B(-4,-6)$ and $C(1, 3/2)$ are the vertices of a triangle.

By distance formula,

Length of a side $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$l(AB) = \sqrt{(-4-2)^2 + (-6-3)^2} = \sqrt{36+81} = \sqrt{117}$ units

$l(BC) = \sqrt{(1+4)^2 + (\frac{3}{2}+6)^2} = \sqrt{25+56.25} = \sqrt{81.25}$ units

$l(AC) = \sqrt{(1-2)^2 + (\frac{3}{2}-3)^2} = \sqrt{1+2.25} = \sqrt{2.25}$ units

Here, as the sum of two sides of a triangle is not greater than third side.

Hence, the given vertices do not form a triangle.

Hence proved.

14) Show that the quadrilateral whose vertices are $(2,-1)$, $(3,4)$, $(-2,3)$ and $(-3,-2)$ is a rhombus.

→ Let us consider the vertices $A(2,-1)$, $B(3,4)$, $C(-2,3)$ and $D(-3,-2)$ respectively.

Then, By distance formula,

$$l(AB) = \sqrt{(3-2)^2+(4-(-1))^2} = \sqrt{(1)^2+(5)^2} = \sqrt{1+25} = \sqrt{26} \text{ units}$$

$$l(BC) = \sqrt{(3-(-2))^2+(4-3)^2} = \sqrt{(5)^2+(1)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$

$$l(CD) = \sqrt{(-2-(-3))^2+(3-2)^2} = \sqrt{(-5)^2+(1)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$

$$l(AD) = \sqrt{(-3-2)^2+(-2-(-1))^2} = \sqrt{(-5)^2+(-1)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$

Here, as $AB = BC = CD = AD$

Hence, we can say that

Quadrilateral ABCD is a rhombus.

15) Two vertices of an isosceles triangle are $(2,0)$ & $(2,5)$. Find the third vertex if the length of the equal sides is 3.

→ Let, the given vertices are $A(2,0)$ & $B(2,5)$.

Let us consider the ~~ve~~ third ~~vertices~~ vertex is $C(x,y)$.

Then, By distance formula,

$$l(AB) = \sqrt{(2-2)^2+(5-0)^2} = \sqrt{0^2+5^2} = \sqrt{25} = 5 \text{ units}$$

$$l(BC) = \sqrt{(x-2)^2+(y-5)^2} = \sqrt{x^2-4x+4+y^2-10y+25}$$

$$= \sqrt{x^2-4x+y^2-10y+29} \text{ units}$$

$$l(AC) = \sqrt{(x-2)^2+(y-0)^2} = \sqrt{(x^2-4x+4+y^2)} \text{ units}$$

Given that,

$$AC = BC = 3$$

$$\text{So, } AC^2 = BC^2 = 9$$

$$x^2-4x+4+y^2 = x^2-4x+y^2-10y+29$$

$$10y = 25$$

$$y = 25/10 = 2.5$$

And, $AC^2 = 9$

$x^2 - 4x + 4 + y^2 = 9$

$x^2 - 4x + 4 + (2.5)^2 = 9$

$x^2 - 4x + 4 + 6.25 = 9$

$x^2 - 4x + 1.25 = 0$

Now, $D = (-4)^2 - 4(1)(1.25)$

$= 16 - 5$

$\boxed{D = 11}$

Now, roots are, $x = \frac{-(-4) + \sqrt{11}}{2} = \frac{(4 + 3.31)}{2} = 3.65$

And $x = \frac{-(-4) - \sqrt{11}}{2} = \frac{(4 - 3.31)}{2} = 0.35$

Hence, the third vertex may be $(3.65, 2.5)$ or $(0.35, 2.5)$.

16.) Which point on X-axis is equidistant from $(5,9)$ and $(-4,6)$?

→ Let $A(5,9)$ and $B(-4,6)$ are given points. which are
Let $C(x,0)$ is the point on X-axis which is equidistant from given points.

Now, $AC = \sqrt{(x-5)^2 + (0-9)^2} = \sqrt{x^2 - 10x + 25 + 81}$

$AC = \sqrt{x^2 - 10x + 106}$

And $BC = \sqrt{(x-(-4))^2 + (0-6)^2} = \sqrt{x^2 + 8x + 16 + 36}$

$= \sqrt{x^2 + 8x + 52}$

As $AC = BC$ ∵ from given condition

$\Rightarrow AC^2 = BC^2$

$x^2 - 10x + 106 = x^2 + 8x + 52$

$18x = 54$

$\boxed{x = 3}$

Thus, the point on the X-axis is $C(3,0)$ which is equidistant from points $(5,9)$ & $(-4,6)$.

17) P.T. the points $(-2,5)$, $(0,1)$ & $(2,-3)$ are collinear.

→ Let us consider $A(-2,5)$, $B(0,1)$ & $C(2,-3)$ are the given points.

Then, By distance formula,

$$AB = \sqrt{(0-(-2))^2 + (1-5)^2} = \sqrt{2^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

$$BC = \sqrt{(2-0)^2 + (-3-1)^2} = \sqrt{(2)^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

$$AC = \sqrt{(2-(-2))^2 + (-3-5)^2} = \sqrt{4^2 + (-8)^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5} \text{ units}$$

Now, we can write

$$AB + BC = AC$$

$$2\sqrt{5} + 2\sqrt{5} = 4\sqrt{5}$$

$$4\sqrt{5} = 4\sqrt{5}$$

Hence, we can conclude that the given points $(-2,5)$, $(0,1)$ & $(2,-3)$ are collinear.

18) The coordinates of the point P are $(-3,2)$. Find the coordinates of the point Q which lies on the line joining P and origin such that $OP = OQ$.

→ Let us consider, the coordinates of point $Q(x,y)$.

As point Q lies on the line joining points P & O. with $PO = OQ$.

Then, according to mid-point formula,

$$\frac{(x-3)}{2} = 0 \quad \& \quad \frac{(y+2)}{2} = 0$$

$$\therefore \boxed{x = 3} \quad \& \quad \boxed{y = -2}$$

Hence, the ~~points~~ co-ordinates of point Q are $(3,-2)$.

19.) Which point on the Y-axis is equidistant from (2,3) and (-4,1).

→ Let us consider the given points as A(2,3) & B(-4,1).

Let C(0,y) be the point on Y-axis which is equidistant from given two points.

Now, By distance formula,

$$AC = \sqrt{(0-2)^2 + (y-3)^2} = \sqrt{y^2 - 6y + 9 + 4} = \sqrt{y^2 - 6y + 13}$$

And, $$BC = \sqrt{(0-(-4))^2 + (y-1)^2} = \sqrt{y^2 - 2y + 1 + 16} = \sqrt{y^2 - 2y + 17}$$

From given condition,

$$AC = BC$$

$$\Rightarrow AC^2 = BC^2$$

$$y^2 - 6y + 13 = y^2 - 2y + 17$$

$$\Rightarrow -4y = 4$$

$$\boxed{y = -1}$$

Thus, the point on the Y-axis is (0,-1).

20.) The three vertices of a parallelogram are (3,4), (3,8) and (9,8). Find the fourth vertex.

→ Let the given points be A(3,4), B(3,8) & C(9,8).

Let D(x,y) be the fourth vertex.

We have,

The diagonals of a parallelogram bisects each other.

Hence, the mid-point of AC & mid-point of BC is the same point.

By mid-point formula,

midpoint of $AC = \left(\frac{3+9}{2}, \frac{4+8}{2}\right) = (6,6)$

And midpoint of $BD = \left(\frac{3+x}{2}, \frac{8+y}{2}\right)$

& the midpoint of BD is (6,6).

Hence, we can write

$$\frac{(3+x)}{2} = 6 \quad \& \quad \left(\frac{8+y}{2}\right) = 6$$

$$3+x=12 \qquad 8+y=12$$

$$\boxed{x=9} \qquad \boxed{y=4}$$

Hence, the fourth vertex is D (9,4).

21.) Find a point which is equidistant from the points A(-5,4) and B(-1,6). How many such points are there.

→ Let us consider the point P(x,y) is the point which is equidistant from points A(-5,4) & B(-1,6).

Then, required point is the mid point

$$(x,y) = \left(\left(\frac{-5-1}{2}\right), \left(\frac{4+6}{2}\right)\right) = \left(\frac{-6}{2}, \frac{10}{2}\right) = (-3,5)$$

Hence, the required point is (-3,5).

Now, we have AP = BP

$$\Rightarrow AP^2 = BP^2$$

$$(x+5)^2 + (y-4)^2 = (x+1)^2 + (y-6)^2$$

$$x^2 + 25 + 10 + y^2 - 8y + 16 = x^2 + 2x + 1 + y^2 - 12y + 16$$

$$10x + 41 - 8y = 2x + 37 - 12y$$

$$8x + 4y + 4 = 0$$

$$2x + y + 1 = 0$$

Hence, all the points which lies on the line $(2x+y+1=0)$ are equidistant from points A and B.

22.) The center of a circle is (2a, a-7). Find the values of 'a' if the circle passes through the point (11, -9) & has diameter $10\sqrt{2}$ units.

→ Here, given that

diameter of the circle = d = $10\sqrt{2}$ units

Radius of the circle $= r = 5\sqrt{2}$ units

Let 'O' be the center of a circle with coordinates $O(2a, a-7)$ & the circle which passes through the point $P(11, -9)$.

Then, OP is the radius of the circle.

$OP = 5\sqrt{2}$

$OP^2 = (5\sqrt{2})^2 = 50$

$\Rightarrow (11-2a)^2 + (-9-a+7)^2 = 50$

$121 - 44a + 4a^2 + 4 + a^2 + 4a = 50$

$5a^2 - 40a + 75 = 0$

$a^2 - 8a + 15 = 0$

$(a-5)(a-3) = 0$

$\Rightarrow \boxed{a=5}$ or $\boxed{a=3}$

24.) Find the value of k, if the point P(0,2) is equidistant from (3,k) & (k,5).

→ Let P(0,2) is the point which is equidistant from the points A(3,k) & B(k,5).

$\Rightarrow PA = PB$ ∵ By distance formula

$(PA)^2 = (PB)^2$

$(3-0)^2 + (k-2)^2 = (k-0)^2 + (5-2)^2$

$9 + k^2 + 4 - 4k - k^2 - 9 = 0$

$4 - 4k = 0$

$-4k = -4$

$\boxed{k=1}$

Thus, the value of $\boxed{k=1}$

25.) If $(-4,3)$ & $(4,3)$ are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the i) interior ii) exterior of the triangle

→ Let us consider, $B(-4,3)$ & $C(4,3)$ be the given two vertices of the equilateral triangle.

Let $A(x,y)$ be the third vertex.

Let us consider that, $AB=BC$

$\Rightarrow AB^2 = BC^2$ ∵ By distance formula

$(-4-x)^2+(3-y)^2 = (4+4)^2+(3-3)^2$

$16+x^2+8x+9+y^2-6y=64$

$x^2+y^2+8x-6y=39$

Let us consider that, $AB = AC$

$\Rightarrow AB^2=AC^2$ ∵ By distance formula

$(-4-x)^2+(3-y)^2=(4-x)^2+(3-y)^2$

$16+x^2+8x+9+y^2-18y = 16+x^2-8x+9+y^2-6y$

$16x=0$

$\boxed{x=0}$

Now, $BC=AC$

$\Rightarrow BC^2=AC^2$ ∵ By distance formula

$(4+4)^2+(3-3)^2 = (4-0)^2+(3-y)^2$

$64+0 = 16+9+y^2-6y$

$64 = 16+(3-y)^2$

$(3-y)^2 = 48$

$3-y = \pm 4\sqrt{3}$

$\boxed{y = 3\pm4\sqrt{3}}$

Thus, the coordinates of third vertex are

i) When origin lies in the interior of the triangle is $(0, 3-4\sqrt{3})$.

ii) When origin lies in the exterior of the triangle is $(0, 3+4\sqrt{3})$.

28.) Find a point on the X-axis which is equidistant from the points (7,6) & (-3,4).

→ Let us consider, the given points be A (7,6) & B(-3,4).

Let P(x, 0) is the point on the X-axis such that

$$PA = PB$$

$$\Rightarrow (PA)^2 = (PB)^2 \quad \because \text{By distance formula}$$

$$(x-7)^2 + (0-6)^2 = (x+3)^2 + (0-4)^2$$

$$x^2 + 49 - 14x + 36 = x^2 + 9 + 6x + 16$$

$$-20x = -60$$

$$\boxed{x = 3}$$

Thus, the point which lies on X-axis is (3,0).

Exercise 14.3

1.) Find the coordinates of the point which divides the line segment joining (-1,3) and (4,-7) internally in the ratio 3:4.

→ Let the point P(x,y) be the required point.

By the section formula,

$$x = \frac{mx_2 + nx_1}{m+n} \quad \& \quad y = \frac{my_2 + ny_1}{m+n}$$

$x_1 = -1$, $y_1 = 3$ and $x_2 = 4$, $y_2 = -7$ and $m:n = 3:4$

Then, $x = \frac{3(4) + 4(-1)}{3+4}$ ~~x 3~~ and $y = \frac{3(-7) + 4(3)}{3+4}$

$$x = \frac{12-4}{7} \qquad y = \frac{-21+12}{7}$$

$$\boxed{x = 8/7} \qquad \boxed{y = -9/7}$$

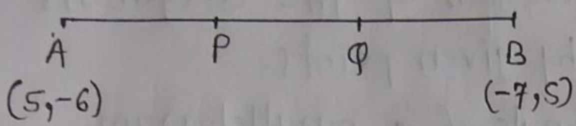
Hence, the required point with co-ordinates is

$$P(x,y) \equiv P(8/7, -9/7)$$

2.) Find the points of trisection of the line segment joining the points.

→ i) (5, −6) and (−7, 5)

→ • Let us consider the points P and Q are the points of trisection of AB such that $AP = PQ = QB$.



• That means, point 'P' divides the line segment AB internally in the ratio 1 : 2.

• Then by section formula,

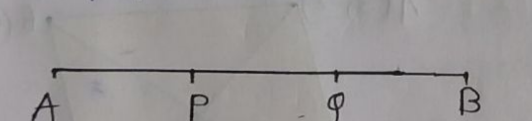
$$P(x, y) = \left(\frac{2(-7)+5}{2+1}, \frac{2(5)+1(-6)}{2+1} \right) = \left(1, -\frac{7}{3}\right)$$

• Also, point Q divides segment AB internally in the ratio 2:1.
Then by section formula,

$$Q(x, y) = \left(\frac{2(-7)+1(5)}{2+1}, \frac{2(5)+1(-6)}{2+1} \right) = \left(-3, \frac{4}{3}\right)$$

ii) (3, −2) and (−3, −4)

→ • Let us consider the points P & Q are the points of trisection of AB such that $AP = PQ = QB$.



• Here, point P divides AB internally in the ratio of 1:2.
• Then by section formula,

$$P(x, y) = \left(\frac{1(-3)+2(3)}{1+2}, \frac{1(-4)+(-2)}{1+2} \right) = \left(1, -\frac{8}{3}\right)$$

• Now, Q divides the segment AB internally in the ratio 2:1.
Then By section formula,

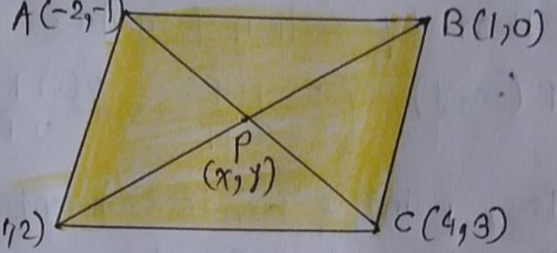
$$Q(x, y) = \left(\frac{2(-3)+1(3)}{2+1}, \frac{2(-4)+1(-2)}{2+1} \right) = \left(-1, \frac{-10}{3}\right)$$

3.) Find the coordinates of the point where the diagonals of the parallelogram formed by joining the points $(-2,-1)$, $(1,0)$, $(4,3)$ and $(1,2)$ meet.

→ Let us consider the given points are $A(-2,-1)$, $B(1,0)$, $C(4,3)$ and $D(1,2)$.

Let $P(x,y)$ is the point of intersection of the diagonals of the parallelogram formed by given points.

But, we know that, the diagonals of a parallelogram bisects each other.



$\therefore \quad x = \frac{-2+4}{2} = \frac{2}{2} = 1$

$\therefore \quad y = \frac{-1+3}{2} = \frac{2}{2} = 1$

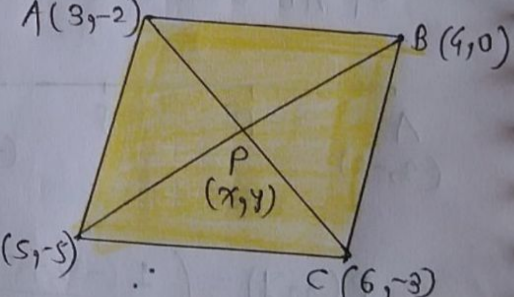
Hence, the co-ordinates of point P are $(1,1)$.

4.) Prove that the points $(3,-2)$, $(4,0)$, $(6,-3)$ and $(5,-5)$ are the vertices of a parallelogram.

→ • Let the given points are $A(3,-2)$, $B(4,0)$, $C(6,-3)$ & $D(5,-5)$

• Let the point $P(x,y)$ be the point of intersection of the diagonals AC & BD of ABCD.

Then, by mid-point formula,



$x = \frac{3+6}{2} = \frac{9}{2}$

$y = \frac{-2-3}{2} = \frac{-5}{2}$

Thus, mid-point of AC is $\left(\frac{9}{2}, \frac{-5}{2}\right)$.

And mid-point of BD is given by,

$x = \frac{5+4}{2} = \frac{9}{2}$

$y = \frac{-5+0}{2} = \frac{-5}{2}$

Thus, mid-point of BD is $\left(\frac{9}{2}, \frac{-5}{2}\right)$.

Thus, we can say that, the diagonals AC & BD bisects each other.
And we also know that, the diagonals of a parallelogram bisects each other.
Thus, $\square$ABCD is a parallelogram.

5) If $P(9a-2, -b)$ divides the line segment joining $A(3a+1, -3)$ and $B(8a, 5)$ in the ratio $3:1$, find the value of a and b.

→ Here, given that the point $P(9a-2, -b)$ divides the line segment joining $A(3a+1, -3)$ and $B(8a, 5)$ in the ratio $3:1$.
By Section formula,
The co-ordinates of point 'P' are given by

$$9a-2 = \frac{3(8a)+1(3a+1)}{3+1}$$

And $$-b = \frac{3(5)+1(-3)}{3+1}$$

$\Rightarrow (9a-2)\times 4 = 24a+3a+1$

$36a-8 = 27a+1$

$9a = 9$

$\boxed{a=1}$

And

$4(-b) = 15-3$

$-4b = 12$

$\boxed{b=-3}$

Hence, the values of a and b are found to be 1 and -3 respectively.

6.) If (a, b) is the mid-point of the line segment joining the points $A(10, -6)$, $B(k, 4)$ and $a-2b = 18$, find the value of k & the distance AB.

→ Here, given that the point (a, b) is the mid-point of the line segment joining the points $A(10, -6)$, $B(k, 4)$ & $a-2b = 18$.

Then, $(a, b) = \left(\frac{10+k}{2}, \frac{-6+4}{2}\right)$

$\Rightarrow a = \frac{10+k}{2}$ & $b = -1$

$2a = 10 + k$

$\boxed{k = 2a - 10}$

But, given that, $a - 2b = 18$

put $\boxed{b = -1}$ $\Rightarrow$ $a - 2(-1) = 18$

$a + 2 = 18$

$\boxed{a = 16}$

Thus, $k = 2a - 10$

$k = 2(16) - 10 = 32 - 10$

$\boxed{k = 22}$

Hence, $AB = \sqrt{(22-10)^2 + (4+6)^2}$

$= \sqrt{(12)^2 + (10)^2}$

$= \sqrt{144 + 100}$

$\boxed{AB = 2\sqrt{61} \text{ units}}$

7.) Find the ratio in which the point $(2, y)$ divides the line segment joining the points $A(-2, 2)$ and $B(3, 7)$. Also find the value of y.

→ Let us consider, the point $P(2, y)$ divides the line segment joining the points $A(-2, 2)$ & $B(3, 7)$ in the ratio $k:1$.

Then, By section formula,

$$P\left(\frac{3k + (-2)(1)}{k+1}, \frac{7k + 2(1)}{k+1}\right) = P\left(\frac{3k-2}{k+1}, \frac{7k+2}{k+1}\right)$$

But, given that, P has coordinates $(2, y)$.

Then, $2 = \frac{(3k-2)}{(k+1)}$ & $y = \frac{7k+2}{k+1}$

$2k + 2 = 3k - 2$

$2k + 2 = 3k - 2$

$\boxed{k = 4}$

$y = \frac{7(4) + 2}{4+1}$

$y = \frac{28+2}{5}$

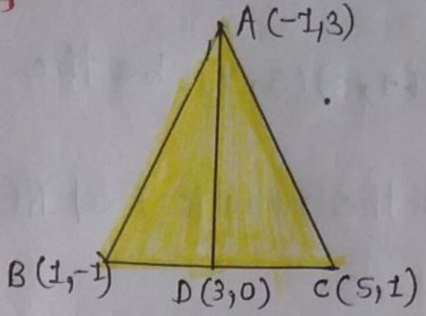
$y = \frac{30}{5}$

$\boxed{y = 6}$

⇒ Hence, the ratio is $4:1$ and $\boxed{y = 6}$

8.) If $A(-1,3)$, $B(1,-1)$ and $C(5,1)$ are the vertices of a triangle ABC, find the length of median through A.

→



• Let us consider, AD be the median through vertex A.

• Let us consider, AD is the median & point 'D' is the mid-point of BC.

Then, By mid-point formula

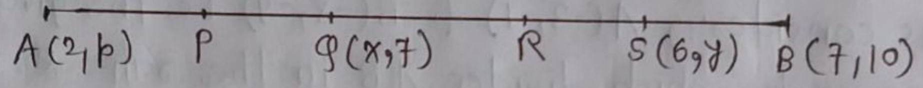
$$D\left(\frac{1+5}{2}, \frac{-1+1}{2}\right) = D(3,0)$$

Thus, length of median AD

$$= \sqrt{(3+1)^2+(0-3)^2}$$
$$= \sqrt{4^2+(-3)^2}$$
$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ units.}$$

9.) If the points P, Q(x,7), R, S(6,y) in this order divide the line segment joining A(2,p) & B(7,10) in 5 equal parts, find x, y & p.

→



From given condition,

$$AP = PQ = QR = RS = SB$$

Then, point 'Q' is the mid-point of A & S.

$$\Rightarrow x = \left(\frac{2+6}{2}\right) = \frac{8}{2} = 4$$

$$y\ 7 = \left(\frac{y+p}{2}\right)$$

$$y + p = 14 \quad \text{———} ①$$

Now, point 'S' divides QB in the ratio 2:1.

$$y = \frac{2(10)+1(7)}{2+1} = \frac{20+7}{3} = \frac{27}{3} = 9$$

from ① ⇒ $y + p = 14$

$$9 + p = 14$$

$$\boxed{p = 5}$$

Thus, $x = 4$, $y = 9$ and $p = 5$

11.) i) In what ratio is the segment joining the points $(-2,-3)$ & $(3,7)$ divides by the y-axis? Also, find the coordinates of the point of division.

→ Let us consider, $P(-2,-3)$ & $Q(3,7)$ be the given points.

Let us suppose, y-axis divides PQ in the ratio $k:1$ at $R(0,y)$.

Then, By section formula

$$\left[\frac{3k+(-2)(1)}{k+1}, \frac{7k+(-3)1}{k+1}\right]$$

$$\Rightarrow \frac{3k+(-2)}{k+1} = 0$$

$$3k-2=0$$

$$\boxed{k=2/3}$$

Thus, the ratio is 2:3

put $k=2/3$ in co-ordinates of R.

$\Rightarrow R(0,1)$

ii) In what ratio is the line segment joining $(-3,-1)$ & $(-8,-9)$ divided at the point $(-5,-21/5)$?

→ Let us consider, $A(-3,-1)$ & $B(-8,-9)$ be the given points.

Let P be any point which divides AB in the ratio $k:1$.

Then, By section formula,

$$\left[\frac{-8k-3}{k+1}, \frac{-9k-1}{k+1}\right]$$

But, from given condition,

$$\frac{-8k-3}{k+1} = -5$$

$$-8k-3=-5k-5$$

$$3k=2$$

$$\boxed{k=2/3}$$

Thus, the point P divides AB in the ratio 2:3.

12.) If the mid-point of the line joining $(3,4)$ & $(K,7)$ is (x,y) and $2x+2y+1=0$, find the value of K.

→ As given that, (x,y) is the mid-point.

$x = \frac{(3+K)}{2}$ & $y = \frac{(4+7)}{2} = \frac{11}{2}$ ∵ By mid-point formula

But, given that the mid-point lies on the line $2x+2y+1=0$.

$\Rightarrow 2\frac{(3+K)}{2} + 2\left(\frac{11}{2}\right) + 1 = 0$

$3+K+11+1=0$

$\Rightarrow \boxed{K=-15}$

13.) Find the ratio in which the point $P(3/4, 5/12)$ divides the line segments joining the point. $A(1/2, 3/2)$ & $B(2,-5)$.

→ Given points are $A(1/2, 3/2)$ & $B(2,-5)$.

Let us consider, point $P(3/4, 5/12)$ divides the line segment AB in the ratio $k:1$.

Then, $P\left(\frac{3}{4}, \frac{5}{12}\right) = \left(\frac{2k+1/2}{k+1}\right), \left(\frac{2k+3/2}{k+1}\right)$

Now, equating abscissa,

$\frac{3}{4} = \frac{(2k+1/2)}{(k+1)}$

$3(k+1) = 4(2k+1/2)$

$3k+3 = 8k+2$

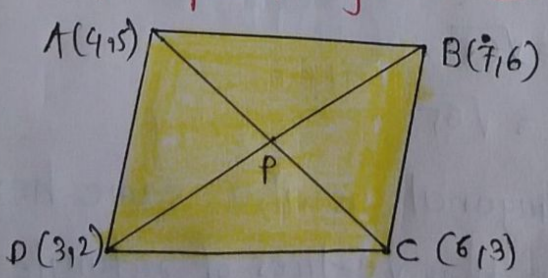
$5k = 1$

$\boxed{k=1/5}$

Thus, the ratio in which the point $P(3/4, 5/12)$ divides is $1:5$.

14.) Prove that the points $(4,5)$, $(7,6)$, $(6,3)$, $(3,2)$ are the vertices of a parallelogram. Is it a rectangle?

→



Let us consider, the given points are $A(4,5)$, $B(7,6)$, $C(6,3)$ & $D(3,2)$.

And point 'P' is the intersection of AC and BD.

Then, co-ordinates of mid-point of AC are: $\left(\frac{4+6}{2}, \frac{5+3}{2}\right) = (5,4)$

Co-ordinates of mid point of BD are: $\left(\frac{7+3}{2}, \frac{6+2}{2}\right) = (5,4)$

Hence, it concludes that, the mid-point of AC and BD are same.

Hence, $\square ABCD$ is a parallelogram.

Now, $AC = \sqrt{(6-4)^2 + (3-5)^2} = \sqrt{(2)^2 + (-2)^2}$

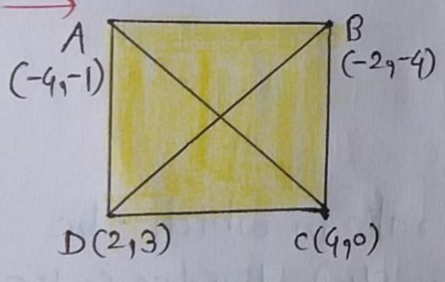
$AC = \sqrt{4+4} = \sqrt{8}$ units

And, $BD = \sqrt{(7-3)^2 + (6-2)^2} = \sqrt{(4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$ units

Since, $AC \neq BD$

Hence, ABCD is not a rectangle.

17.) Prove that the points $(-4,-1)$, $(-2,-4)$, $(4,0)$ & $(2,3)$ are the vertices of a rectangle.



• Let us consider the given points are $A(-4,-1)$, $B(-2,-4)$, $C(4,0)$ & $D(2,3)$.

Then, By mid-point formula,

Coordinates of the mid point of AC $\Big\} = \left(\frac{-4+4}{2}, \frac{-1+0}{2}\right) = \left(0, \frac{-1}{2}\right)$

Co-ordinates of mid-point of BD $\Big\} = \left(\frac{-2+2}{2}, \frac{-4+3}{2}\right) = (0, -1/2)$

Thus, we can say that AC & BD having same mid point.

Thus, By distance formula,

$AC = \sqrt{(4+4)^2 + (0+1)^2} = \sqrt{65}$

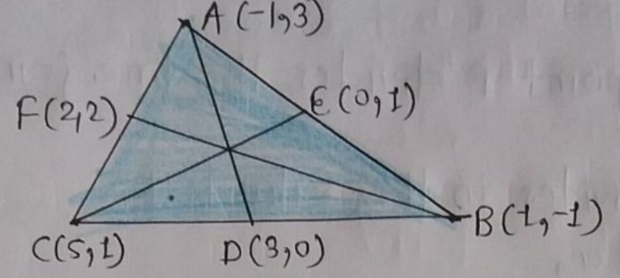
$BD = \sqrt{(-2-2)^2 + (-4-3)^2} = \sqrt{65}$

The lengths of diagonals is also same here.

Hence, the given points are the vertices of rectangle only.

18.) Find the length of the medians of a triangle whose vertices are $A(-1,3)$, $B(1,-1)$ & $C(5,1)$.

→



• Let AD, BF & CE are the medians of ΔABC as shown in fig

Co-ordinates of D are $\left(\frac{5+1}{2}, \frac{1-1}{2}\right) = (3,0)$

Co-ordinates of E are $\left(\frac{-1+1}{2}, \frac{3-1}{2}\right) = (0,1)$

Co-ordinates of F are $\left(\frac{5-1}{2}, \frac{1+3}{2}\right) = (2,2)$

Now, By distance formula,

$l(AD) = \sqrt{(-1-3)^2 + (3-0)^2} = 5$ units

$l(BF) = \sqrt{(2-1)^2 + (2+1)^2} = \sqrt{10}$ units

$l(CE) = \sqrt{(5-0)^2 + (1-1)^2} = 5$ units

19.) Find the ratio in which the line segment joining the points $A(3,-3)$ & $B(-2,7)$ is divided by X-axis. Also, find the coordinates of the point of division.

→ Let us consider the point on X-axis is $(x,0)$.

And this point divides the line segment AB in the ratio of $k:1$.

By section formula,

$$0 = \frac{(7k-3)}{(k+1)} \Rightarrow \boxed{k = 3/7}$$

Thus, the line segment AB is divided by X-axis in the ratio 3:7.

20.) find the ratio in which the point $P(x, 2)$ divides the line segment joining the points $A(12, 5)$ & $B(4, -3)$. Also, find the value of K.

→ Let us consider the point P' divides the line joining points A & B.

Let us consider, it divides in the ratio $K:1$.

By section formula,

$$2 = \frac{(-3K+5)}{(K+1)}$$

$$2(K+1) = -3K+5$$

$$2K+2 = -3K+5$$

$$5K = 3$$

$$\boxed{K = 3/5}$$

Thus, point 'P' divides the line segment AB in the ratio 3:5.

Then,

$$x = \frac{12+60}{8} = \frac{72}{8} = 9$$

Thus, the co-ordinates of point P are (9,2).

21.) find the ratio in which the point $P(-1, y)$ lying on the line segment joining $A(-3, 10)$ & $B(6, -8)$ divides it. Also, find the value of y.

→ Let us consider, the point 'P' divides $A(-3, 10)$ & $B(6, -8)$ in the ratio of $K:1$.

Given that, point P' is having coordinates $(-1, y)$.

By Section formula,

$$-1 = 6K-3/(K+1)$$

$$-(K+1) = 6K-3$$

$$7K = 2$$

$$\boxed{K = 2/7}$$

Thus, the point 'P' divides AB in the ratio 2:7.

Then,

$$y = \frac{(-8K+10)}{(K+1)} = \frac{-8(2/7)+10}{2/7+1}$$

$$y = \frac{-16+70}{2+7} = \frac{54}{9} = 6 \quad \boxed{y = 6}$$

Thus, the y-coordinate of point P is 6.

22.) Find the coordinates of a point A, where AB is the diameter of circle whose center is $(2,-3)$ & B is $(1,4)$.

→ Let us consider, the coordinates of point A are (x,y).

If AB is the diameter, then the center is at the mid-point of the diameter.

Then, $(2,-3) = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$

$2 = x+1/2$ & $-3 = y+4/2$

$4 = x+1$ $\quad$ $-6 = y+4$

$\boxed{x=3}$ $\quad$ $\boxed{y=-10}$

Thus, the coordinates of point A are $(3,-10)$.

23.) If the points $(-2,1)$, $(1,0)$, $(x,3)$ & $(1,y)$ form a parallelogram find the values of x & y.

→ Let us consider the given points are $A(-2,1)$, $B(1,0)$, $C(x,3)$ & $D(1,y)$ & which are the points of parallelogram.

But, we know that

The diagonals of a parallelogram bisects each other.

Thus,

$$\left.\begin{matrix}\text{Co-ordinates of mid-point}\\ \text{of AC}\end{matrix}\right\} = \left\{\begin{matrix}\text{Co-ordinates of mid-point}\\ \text{of BD}\end{matrix}\right.$$

$$\left(\frac{x-2}{2}, \frac{3-1}{2}\right) = \left(\frac{1+1}{2}, \frac{y+0}{2}\right)$$

$$\left(\frac{x-2}{2}, 1\right) = \left(1, \frac{y}{2}\right)$$

$\frac{x-2}{2} = 1$ & $\frac{y}{2} = 1$

$\boxed{y=2}$

$x = 2+2$

$\boxed{x=4}$

Hence, the coordinates are here

$$\boxed{\begin{matrix}x=4\\ y=2.\end{matrix}}$$

25.) In what ratio does the point (-4, 6) divide the line segment joining the points A(-6, 10) & B(3, -8).

→ Let us consider the point (-4, 6) divide the line segment AB in the ratio k:1.

By section formula,

$$(-4, 6) = \left(\frac{3k-6}{k+1}, \frac{-8k+10}{k+1}\right)$$

$$-4 = \frac{3k-6}{k+1} \Rightarrow -4k-4 = 3k-6$$

$$7k = 2$$

$$k:1 = 2:7$$

Thus, the ratio in which the point (-4, 6) divides the line segment AB is 2:7.

26.) Find the ratio in which the y-axis divides the line segment joining the points (5, -6) & (-1, -4). Also, find the coordinates of the point of division.

→ Let us consider, P(5, -6) & Q(-1, -4) be the given points. Let y-axis divide the line segment PQ in the ratio k:1.

By section formula,

for x-coordinate $\frac{-k+5}{k+1} = 0$

$$-k+5 = 0$$

$$\boxed{k=5}$$

Thus, the y-axis divides the given points in the ratio 5:1.

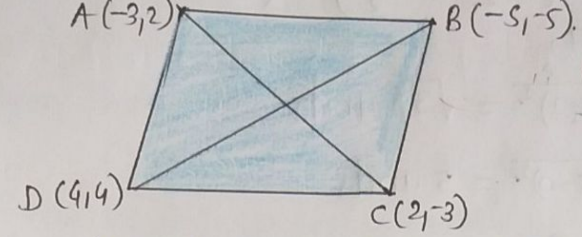
Now, for coordinates of point of division:

$$\left(\frac{-5+5}{5+1}, \frac{-4\times5-6}{5+1}\right) = \left(0, \frac{-13}{3}\right)$$

Thus, the coordinates of the point of division are (0, -13/3).

27) Show that A(-3, 2), B(-5, 5), C(2, -3) & D(4, 4) are the vertices of a rhombus.

→



Here, the given points are. A(-3,2), B(-5,5), C(2,-3) & D(4,4).

Then, co-ordinates of mid-point of AC are:

$$\left(\frac{-3+2}{2}, \frac{2-3}{2}\right) = \left(\frac{-1}{2}, \frac{-1}{2}\right)$$

And,

Co-ordinates of mid-point of BD are:

$$\left(\frac{-5+4}{2}, \frac{-5+4}{2}\right) = \left(\frac{-1}{2}, \frac{-1}{2}\right)$$

Hence, the mid-point of both the diagonals is same.

⇒ ABCD is a parallelogram.

By distance formula,

$$AB = \sqrt{(-5+3)^2 + (-5-2)^2} = \sqrt{4+49} = \sqrt{53}$$

$$BC = \sqrt{(-5-2)^2 + (-5+3)^2} = \sqrt{49+4} = \sqrt{53}$$

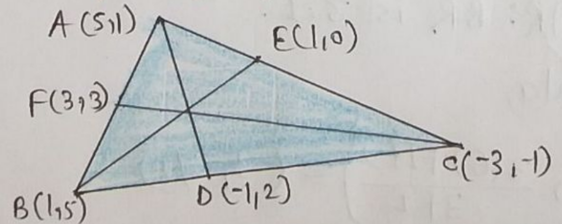
⇒ AB = BC

Thus, ABCD is a parallelogram with adjacent sides equal

Thus, ABCD is a rhombus.

29.) Find the lengths of the median of a ΔABC having vertices at A(5,1), B(1,5) & C(-3,-1).

→



Given vertices of ΔABC are A(5,1), B(1,5) & C(-3,-1).

Let AD, BE & CF are the medians of ΔABC.

Coordinates of D are: $\left(\frac{1-3}{2}, \frac{5-1}{2}\right) = (-1, 2)$

Co-ordinates of E are: $\left(\frac{5-3}{2}, \frac{1-1}{2}\right) = (1,0)$

Co-ordinates of F are: $\left(\frac{5+1}{2}, \frac{1+5}{2}\right) = (3,3)$

By distance formula,

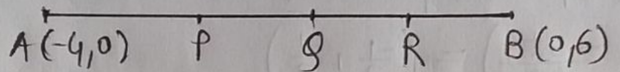
$$l(AD) = \sqrt{(5+1)^2 + (1-2)^2} = \sqrt{37} \text{ units}$$

$$l(BE) = \sqrt{(1-1)^2 + (5-0)^2} = 5 \text{ units}$$

$$l(CF) = \sqrt{(3+3)^2 + (3+1)^2} = 2\sqrt{13} = \sqrt{52} \text{ units}$$

30.) Find the coordinates of the point which divide the line segment joining the points $(-4,0)$ & $(0,6)$ in four equal parts.

→



Let us consider, the given points are $A(-4,0)$ & $B(0,6)$.

Let P, Q & R are the points which divides line-segemt AB in four equal parts.

we have, $AP : PB = 1:3$

By section formula,

$$P\left(\frac{1(0)+3(-4)}{1+3}, \frac{1(6)+3(0)}{1+3}\right) = \left(-3, \frac{3}{2}\right)$$

But, Q is the midpoint of AB.

By section formula,

$$Q\left(\frac{-4+0}{2}, \frac{0+6}{2}\right) = (-2,3)$$

Now, the ratio $AR : BR$ is $3:1$.

By section formula,

$$R\left(\frac{3(0)+1(-4)}{3+1}, \frac{3(6)+1(0)}{3+1}\right) = (-1, 9/2)$$

Exercise 14.4

1. Find the centroid of the triangle whose vertices are:

i) $(1,4)$, $(-1,-1)$ & $(3,-2)$

→ Given that, the three vertices of a given triangle are $(1,4)$, $(-1,-1)$ & $(3,-2)$.

Then, by the centroid formula,

For three vertices (x_1,y_1), (x_2,y_2) & (x_3,y_3):

$$G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Thus, $G \equiv \left(\frac{1-1+3}{3}, \frac{4-1-2}{3}\right) \equiv \left(\frac{3}{3}, \frac{1}{3}\right) \equiv (1, 1/3)$

Hence, co-ordinates of a centroid are $(1, 1/3)$.

ii) $(-2,3)$, $(2,-1)$ & $(4,0)$

→ Given that, the three vertices of a given triangle are $(-2,3)$, $(2,-1)$ & $(4,0)$.

Then, by the centroid formula,

For three vertices (x_1,y_1), (x_2,y_2) & (x_3,y_3):

$$G \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Thus, $G \equiv \left(\frac{-2+2+4}{3}, \frac{3-1+0}{3}\right) \equiv \left(\frac{4}{3}, \frac{2}{3}\right)$

Hence, coordinates of a centroid are $(4/3, 2/3)$.

2.) Two vertices of a triangle are (1,2), (3,5) & its centroid is at the origin. Find the coordinates of the third vertex.

→ Let us consider the third vertex be (x,y).

Then by Centroid formula,

for three vertices of a triangle:

$(x,y),(1,2),(3,5)$

$$\Rightarrow \quad G \equiv \left(\frac{x+1+3}{3}, \frac{y+2+5}{3}\right) \equiv \left(\frac{x+4}{3}, \frac{y+7}{3}\right)$$

But, given that, the coordinates of centroid are $G \equiv (0,0)$

$$\Rightarrow \quad \frac{x+4}{3} = 0 \quad \text{and} \quad \frac{y+7}{3} = 0$$

$$x+4=0 \quad \text{and} \quad y+7=0$$

$$\boxed{x=-4} \quad \text{and} \quad \boxed{y=-7}$$

Thus, the co-ordinates of third vertex is $(-4,-7)$.

3.) Find the third vertex of a triangle, if two of it's vertices are at (-3,1) & (0,-2) and the centroid at the origin.

→ Let us consider the third vertex of a triangle is (x,y).

Then by Centroid formula,

for three vertices of a triangle:

$(x,y), (-3,1)$ & $(0,-2)$

$$\Rightarrow \quad G \equiv \left(\frac{x-3+0}{3}, \frac{y+1-2}{3}\right) \equiv \left(\frac{x-3}{3}, \frac{y-1}{3}\right)$$

But, given that the centroid of a triangle is at origin.

$G \equiv (0,0)$

$$\Rightarrow \quad \frac{x-3}{3} = 0 \quad \& \quad \frac{y-1}{3} = 0$$

$$x-3=0 \quad \& \quad y-1=0$$

$$\boxed{x=3} \quad \& \quad \boxed{y=1}$$

Thus, the coordinates of third vertex is $(3,1)$.

4.) A(3, 2) & B(−2, 1) are two vertices of a triangle ABC whose centroid G has the co-ordinates (5/3, −1/3). Find the co-ordinates of the third vertex C of the triangle.

→ Let us consider the third point or vertex of a triangle is (x, y).

Then, By Centroid formula:

For three vertices of a triangle: $(3, 2)$, $(-2, 1)$, (x, y)

$$G \equiv \left(\frac{x+3-2}{3}, \frac{y+2+1}{3}\right) \equiv \left(\frac{x+1}{3}, \frac{y+3}{3}\right)$$

But, Given that the coordinates of centroid G are $(5/3, -1/3)$.

$$\Rightarrow \frac{x+1}{3} = \frac{5}{3} \quad \text{and} \quad \frac{y+3}{3} = \frac{-1}{3}$$

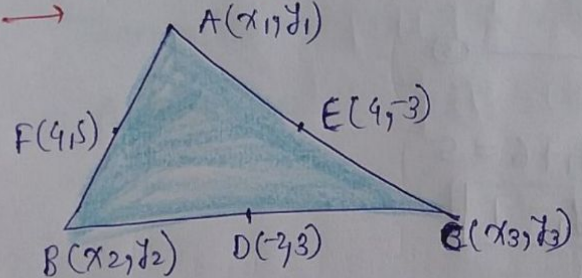
$$\Rightarrow x+1 = 5 \quad \text{and} \quad y+3 = -1$$

$$\boxed{x = 4} \qquad \boxed{y = -4}$$

Thus, the coordinates of third vertex are $(4, -4)$.

5.) If (−2, 3), (4, −3) and (4, 5) are mid-point of the sides of a triangle, find the coordinates of it's centroid.

→



• Let us consider the three vertices of a triangle are $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$.

• Let the points $F(4, 5)$, $D(-2, 3)$ & $E(4, -3)$ are mid-points of sides AB, BC & AC respectively.

For point D: By mid-point formula

$$D \equiv \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right) \equiv (-2, 3)$$

$$\Rightarrow \frac{x_2+x_3}{2} = -2 \quad \& \quad \frac{y_2+y_3}{2} = 3$$

$$\Rightarrow x_2+x_3 = -4 \quad \& \quad y_2+y_3 = 6 \quad \text{——} (1)$$

for mid-point F:

$$F \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \equiv (4,5)$$

$$\Rightarrow \frac{x_1+x_2}{2} = 4 \quad \& \quad \frac{y_1+y_2}{2} = 5$$

$$\Rightarrow x_1+x_2 = 8 \quad \& \quad y_1+y_2 = 10 \quad \text{---} \; (2)$$

for mid-point E:

$$E \equiv \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right) \equiv (4,-3)$$

$$\Rightarrow \frac{x_1+x_3}{2} = 4 \quad \& \quad \frac{y_1+y_3}{2} = -3$$

$$\Rightarrow x_1+x_3 = 8 \quad \& \quad y_1+y_3 = -6 \quad \text{---} \; (3)$$

from (1), (2) and (3):

$$x_2+x_3+x_1+x_3+x_1+x_2 = -4+8+8$$

$$2x_2+2x_3+2x_1 = 12$$

$$x_2+x_3+x_1 = 6 \quad \text{---} \; (4)$$

and $\cdot\; y_2+y_3+y_1+y_3+y_1+y_2 = 6-6+10$

$$2y_1+2y_2+2y_3 = 10$$

$$y_1+y_2+y_3 = 5 \quad \text{---} \; (5)$$

from (1), (4) & (5): $x_1 - 4 = 6$ $\qquad$ $y_1 + 6 = 5$

$$\boxed{x_1 = 10} \qquad \boxed{y_1 = -1}$$

And $x_2 + 8 = 6$ $\qquad$ $y_3 + 10 = 5$

$$\boxed{x_2 = -2} \qquad \boxed{y_3 = -5}$$

Hence, co-ordinates of point B $(-2,1)$ & C$(-2,-5)$.

Thus, the three vertices of a triangle ABC are found to be A $(10,-1)$, B$(-2,1)$ & C$(-2,-5)$.

Hence, centroid of a triangle is given by

$$\left(\frac{10-2-2}{3}, \frac{-1+11-5}{3}\right) = (2, 5/3)$$

Exercise 14.5

1) Find the area of a triangle whose vertices are

i) $(6,3)$, $(-3,5)$ & $(4,-2)$

$\rightarrow$ Let us consider, the three vertices of a given triangle are $A(6,3)$, $B(-3,5)$ & $C(4,-2)$

Then, By area of a triangle

$$A(\Delta ABC) = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1+y_2)]$$

$$A(\Delta ABC) = \frac{1}{2}[6(5+2) + (-3)(-2-3) + 4(3-5)]$$

$$= \frac{1}{2}[6\times 7 - 3\times(-5) + 4(-2)]$$

$$= \frac{1}{2}[42+15-8]$$

$$\boxed{A(\Delta ABC) = 49/2 \text{ sq. units}}$$

ii) $(a, c+a)$, (a,c) & $(-a, c-a)$.

$\rightarrow$ Let us consider, $A = (x_1, y_1) = (a, c+a)$

$B = (x_2, y_2) = (a, c)$

and $C = (x_3, y_3) = (-a, c+a)$ be the vertices of a triangle.

Then Area of triangle ABC is given by,

$$A(\Delta ABC) = \frac{1}{2}[a[c-(c-a)) + a(c-a-(c+a)) + (-a)(c+a-a)]$$

$$= \frac{1}{2}[a(c-c+a) + a(c-a-c-a) + (-a)(c)]$$

$$= \frac{1}{2}[a\times a + a(-2a) - a\times a]$$

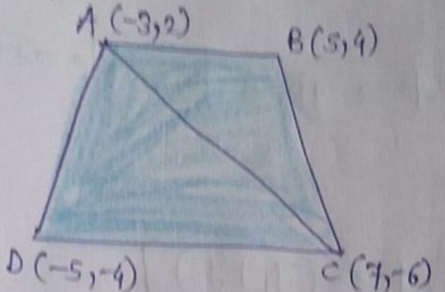
$$= \frac{1}{2}(a^2 - 2a^2 - a^2)$$

$$= \frac{1}{2}(-2a)^2$$

$$\boxed{A(\Delta ABC) = 2a^2}$$

2) Find the area of the quadrilaterals, the co-ordinates of whose vertices are i) $(-3,2), (5,4), (7,-6)$ & $(-5,-4)$.

→



- Let us consider the four vertices of a given quadrilateral are $A(-3,2)$, $B(5,4)$, $C(7,-6)$ & $D(-5,-4)$.

$$A(\triangle ABC) = \frac{1}{2}[-3(4+6)+5(-6-2)+7(2-4)]$$

$$= \frac{1}{2}[-3\times 1 + 5(-8) + 7(-2)]$$

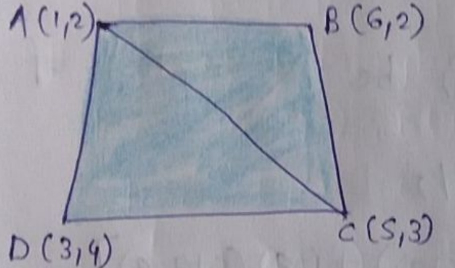
$$= \frac{1}{2}[-30 - 40 - 14]$$

$$= \frac{1}{2}(= -42$$

$$\boxed{A(\triangle ABC) = -42}$$

ii) $(1,2), (6,2), (5,3)$ & $(3,4)$

→ Let us consider the ~~th~~ four vertices of a given quadrilateral are $A(1,2)$, $B(6,2)$, $C(5,3)$ & $D(3,4)$.



$$A(\triangle ABC) = \frac{1}{2}[1(2-3)+6(3-2)+5(2-2)]$$

$$= \frac{1}{2}[-1+6(1)+0]$$

$$= \frac{1}{2}[-1+6]$$

$$\boxed{A(\triangle ABC) = 5/2}$$

Now, $A(\triangle ADC) = \frac{1}{2}[1(3-4)+5(4-2)+3(2-3)]$

$$= \frac{1}{2}[-1(5)(2)+3(-1)]$$

$$= \frac{1}{2}[-1+10-3]$$

$$= 1/2\,(6)$$

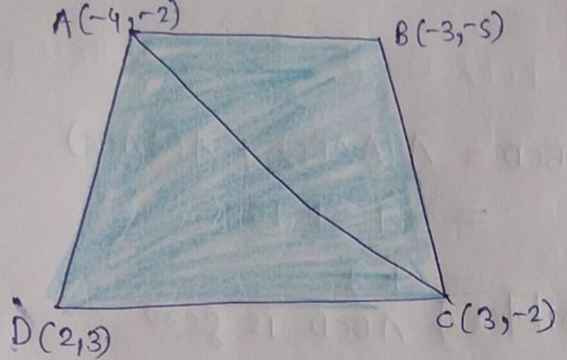
$$\boxed{A(\triangle ADC) = 3}$$

Total $A(\square ABCD) = A(\triangle ABC) + A(\triangle ADC)$

$$= \left(\frac{5}{2}+3\right) = \frac{11}{2} \text{ sq. units}$$

iii) $(-4,-2)$, $(-3,-5)$, $(3,-2)$, $(2,3)$

→ Let us consider the four vertices of a quadrilateral are $A(-4,-2)$, $B(-3,-5)$, $C(3,-2)$ & $D(2,3)$.



Then $A(\Delta ABC) = \frac{1}{2}[(-4)(-5+2)+2(-2+2) + 3(-2-3)]$

$= \frac{1}{2}[-4(5)+2(0)+3(-5)]$

$= \frac{1}{2}(\cancel{-35})(21)$

But, area is always positive.

Hence, $A(\Delta ABC) = \cancel{35/2} = 21/2$

Now, $A(\Delta ADC) = \frac{1}{2}[-4(3+2)+2(-2+2)+3(-2-3)]$

$= \frac{1}{2}[-4(5)+2(0)+3(-5)]$

$= \frac{1}{2}(-35)$

But, Area is always positive. $\Rightarrow \boxed{A(\Delta ADC) = 35/2}$

Thus, total area of quadrilateral ABCD,

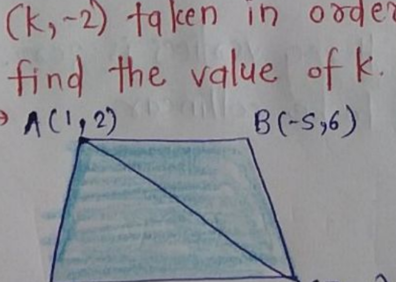
$A(\square ABCD) = A(\Delta ABC) + A(\Delta ADC)$

$= 21/2 + 35/2$

$= 56/2$

$\boxed{A(\square ABCD) = 28 \text{ sq. units}}$

3.) The four vertices of a quadrilateral are $(1,2)$, $(-5,6)$, $(7,-4)$ & $(k,-2)$ taken in order. If the are of the quadrilateral is zero, find the value of k.

→ 

Let us consider the given four vertices of quadrilateral are $A(1,2)$, $B(-5,6)$, $C(7,-4)$ & $D(k,-2)$

Then, $A(\Delta ABC) = \frac{1}{2}[(1)(6+4) - 5(4+2) + 7(2-6)]$

$= \frac{1}{2}[10+30-28]$

$A(\Delta ABC) = \frac{1}{2}(12) = 6$ sq. units

Now, $A(\Delta ACD) = \frac{1}{2}[(1)(-4+2) + 7(-2-2) + k(2+4)]$

$= \frac{1}{2}[-2 + 7(-4) + k(6)]$

$= \frac{1}{2}(-30 + 6k)$

$\boxed{A(\Delta ACD) = 3k - 15}$

Now, total area of quadrilateral $ABCD = A(\Delta ABC) + A(\Delta ADC)$

$= 6 + 3k - 15$

$= 3k - 9$

But, given that area of quadrilateral ABCD is zero.

$\therefore \quad 3k - 9 = 0$

$3k = 9$

$\boxed{k = 3}$

5.) Show that, the following sets of points are collinear.

a) (2,5), (4,6) & (8,8)

→ The given three points are said to be collinear only when the area of triangle formed by them is zero.

Here, A(2,5), B(4,6) & C(8,8)

Then, By area of triangle formed,

$A(\Delta ABC) = \frac{1}{2}[2(6-8) + 4(8-5) + 8(5-6)]$

$= \frac{1}{2}[2(-2) + 4(3) + 8(-1)]$

$= \frac{1}{2}(-4 + 12 - 8) = \frac{1}{2}(0)$

$\boxed{A(\Delta ABC) = 0}$

As the area of triangle formed by given three vertices is zero. Hence, the given three points are said to be collinear.

b) (1,-1), (2,1) & (4,5)

→ Let A(1,-1), B(2,1) & C(4,5)

Then, By area of triangle formed by vertices A, B & C.

$$A(\Delta ABC) = \frac{1}{2}[1(1-5) + 2(5+1) + 4(-1-1)]$$

$$= \frac{1}{2}[-4+12-8]$$

$$= 1/2 (0)$$

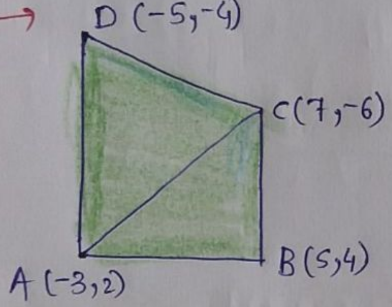
$$\boxed{A(\Delta ABC) = 0}$$

As the area of the triangle ABC formed by given three vertices is zero.

Hence, given three points are said to be collinear.

6.) Find the area of quadrilateral ABCD, the coordinates of whose vertices are A(-3,2), B(5,4), C(7,6) & D(-5,4).

→



• Let us consider a quadrilateral ABCD as shown in fig. Now, we joined two vertices A & C So that two triangles are formed.

$$\text{Now, } A(\Delta ABC) = \frac{1}{2}[-3(4+6) + 5(-6-2) + 7(2-4)]$$

$$= \frac{1}{2}[-30-40-14]$$

$$= \frac{1}{2}(84) = 42 \text{ sq. units}$$

$$\text{And, } A(\Delta ACD) = \frac{1}{2}[-3(-6+4) + 7(-4-2) - 5(2+6)]$$

$$= \frac{1}{2}[6-42-40]$$

$$= \frac{1}{2}(76) = 38 \text{ sq. units}$$

Hence, $A(\square ABCD) = 42 + 38 = 80$ sq. units.

7) In ΔABC, the co-ordinates of vertex A are $(0,-1)$, $D(1,0)$ & $E(0,1)$ respectively the mid-points of the sides AB & AC respectively. If F is the mid-point of side BC, find the $A(\Delta DEF)$.

$\rightarrow$ Let us consider $B(a,b)$ & $C(p,q)$ are the two remaining vertices of a given triangle.

Given that, $A(0,-1)$ & $D(1,0)$.

And 'D' is the mid-point of AB:

$$D = \left(\frac{0+a}{2}, \frac{-1+b}{2}\right) = \left(\frac{a}{2}, \frac{b-1}{2}\right)$$

$$(1,0) = \left(a/2, \frac{b-1}{2}\right)$$

$$\Rightarrow a/2 = 1 \quad \text{and} \quad \frac{b-1}{2} = 0$$

$$\boxed{a=2} \qquad b-1=0 \Rightarrow \boxed{b=1}$$

Thus, co-ordinates of point $B(2,1)$.

Now, E is the mid-point of AC:

$$E = \left(\frac{0+p}{2}, \frac{-1+q}{2}\right) = (0,1)$$

$$\frac{0+p}{2} = 0 \quad \text{and} \quad \frac{-1+q}{2} = 1$$

$$\boxed{p=0} \quad \text{and} \quad -1+q=2$$

$$\boxed{q=3}$$

Thus, co-ordinates of point $C(0,3)$.

Again, F is the mid-point of BC:

$$F = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1,2)$$

$$\Rightarrow A(\Delta DEF) = \frac{1}{2}[1(1-2) + 0(2-0) + 1(0-1)]$$

$$= \frac{1}{2}[-1+0-1]$$

$$= 1/2(2)$$

$$\boxed{A(\Delta DEF) = 1 \text{ sq. units}}$$

8.) Find the area of the triangle PQR with $Q(3,2)$ & the mid-points of the sides through Q being $(2,-1)$ & $(1,2)$.

→ Let us consider the co-ordinates of points P and R as $P(x_1, y_1)$ & $R(x_2, y_2)$

Let 'E' be the mid-point of PQ &
'F' be the mid-point of QR respectively.

$\frac{x_1+3}{2} = 1, \quad \frac{y_1+2}{2} = 2$ & $\frac{x_2+3}{2} = 2, \quad \frac{y_2+2}{2} = -1$

$x_1+3 = 2, \quad y_1+2 = 4$ & $x_2+3 = 4, \quad y_2+2 = -2$

$\boxed{x_1 = 1}, \quad \boxed{y_1 = 2}$ & $\boxed{x_2 = 1}, \quad \boxed{y_2 = -4}$

Thus, points P & R having co-ordinates
$P(1,2)$ & $R(1,-4)$.

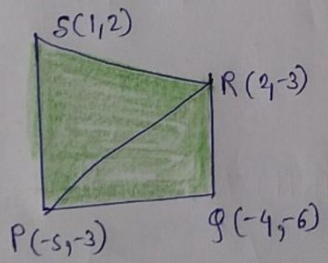
Then, $A(\Delta PQR) = \frac{1}{2}[3\times2 + (-1)(-4) + 2 - (-1\times2 + 1\times2 + 3\times(-4))]$

$= \frac{1}{2}[6+4+2-(-2+2-12)]$

$= \frac{1}{2}(24)$

$\boxed{A(\Delta PQR) = 12 \text{ sq. units}}$

9.) If $P(-5,-3)$, $Q(-4,-6)$, $R(2,-3)$ & $S(1,2)$ are the vertices of a quadrilateral PQRS, find its area.

→ 

• Let us consider the co-ordinates $P(-5,-3)$, $Q(-4,-6)$, $R(2,-3)$ & $S(1,2)$ are the vertices of a quadrilateral PQRS.

• We joined points P & R.

Now, $A(\Delta PSR) = \frac{1}{2}[-5(2+3) + 1(-3+3) + 2(-3-2)]$

$= \frac{1}{2}[-5\times5 + 1\times0 + 2\times(-5)]$

$= \frac{1}{2}[-25+0-10]$

$= \frac{1}{2}(-35) = 35/2$

$\boxed{A(\Delta PSR) = 35/2}$

Now, $A(\Delta PQR) = \frac{1}{2}[-5(-6+3) - 4(-3+3) + 2(-3+6)]$

$= \frac{1}{2}[-5(-3) - 4\times 0 + 2\times 3]$

$= \frac{1}{2}[15 + 0 + 6]$

$\boxed{A(\Delta PQR) = \frac{21}{2}}$

Thus, $A(\square PQRS) = A(\Delta PSR) + A(\Delta PQR)$

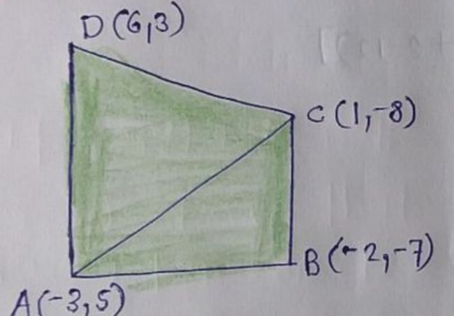
$= 35/2 + 21/2$

$= 56/2.$

$\boxed{A(\square PQRS) = 28 \text{ sq. units}}$

10.) If $A(-3,5)$, $B(-2,-7)$, $C(1,-8)$ & $D(6,3)$ are the vertices of a quadrilateral ABCD, find area.

→ Given vertices of a quadrilateral are $A(-3,5)$, $B(-2,-7)$, $C(1,-8)$ & $D(6,3)$.



Now, we joined vertices A & C

Then, $A(\Delta ABC) = \frac{1}{2}$ is given by

$A(\Delta ABC) = \frac{1}{2}[-3(-7+8) + (-2)(-8-5) + 1(5+7)]$

$= \frac{1}{2}[-3(1) + (-2)(-13) - 1(12)]$

$A(\Delta ABC) = \frac{1}{2}[-3 + 26 + 12] = \frac{1}{2}(35)$

$A(\Delta ADC) = \frac{1}{2}[-3(3+8) + 6(-8-5) + 1(5-3)]$

$= \frac{1}{2}[-3(11) + 6(-13) + 1(2)]$

$= \frac{1}{2}[-33 - 78 + 2]$

$A(\Delta ADC) = \frac{1}{2}(-109) = \frac{109}{2}$

$A(\square ABCD) = A(\Delta ABC) + A(\Delta ADC)$

$= \frac{35 + 109}{2} = \frac{144}{2} = 72$ sq. units

Thus, area of quadrilateral ABCD is 72 sq. units.

11.) For what value of a the points $(a,1)$, $(1,-1)$ & $(11,4)$ are collinear?

→ Here, the given points are $(a,1)$, $(1,-1)$ & $(11,4)$.

$$A(\Delta ABC) = \frac{1}{2}[a(-1-4)+1(4-1)+11(1+1)]$$

$$= \frac{1}{2}[-5a+3+22]$$

$$= \frac{1}{2}[-5a+25)$$

As the given point are collinear that means the area of a triangle formed by vertices A,B,C is zero.

$$\therefore A(\Delta ABC) = 0$$

$$\frac{1}{2}(-5a+25)=0$$

$$-5a+25=0$$

$$25=5a$$

$$\boxed{a=5}$$

12.) Prove that the points (a,b), (a_1,b_1) & $(a-a_1, b-b_1)$ are collinear if $ab_1 = a_1b$.

→ Let us consider the given points be $A(a,b)$, $B(a_1,b_1)$ & $C(a-a_1, b-b_1)$ be the given points.

Then $A(\Delta ABC) = \frac{1}{2}[a(b_1-(b-b_1)) + a_1(b-b_1-b) + (a-a_1)(b-b_1)]$

$$= \frac{1}{2}\{a(b_1-b+b_1) + a_1(-b_1) + ab - ab_1 - a_1b + a_1b_1]$$

$$= \frac{1}{2}[ab_1 - ab + ab_1 - a_1b_1 + ab - ab_1 - a_1b + a_1b_1$$

$$= \frac{1}{2}(ab_1 - a_1b)$$

Here, $ab_1 = a_1b$ only when $A(\Delta ABC)=0$

$$\therefore A(\Delta ABC) = \frac{1}{2}(0) = 0$$

Hence, given points are collinear only when $ab_1 = a_1b$.

13.) If the vertices of a triangle are $(1,-3)$, $(4,p)$ & $(-9,7)$ and it's area is 15 sq. units, find the value of p.

→ Let us consider, the given points are

$A(1,-3)$, $B(4,p)$ & $C(-9,7)$.

Then, $A(\Delta ABC) = \frac{1}{2}[1(p-7) + 4(7+3) - 9(-3-p)]$

$$15 = \frac{1}{2}[p-7+40+27+9p]$$

$$15 = \frac{1}{2}(10p+60)$$

$\Rightarrow$ $15 = \frac{1}{2}(10p+60)$ or $15 = \frac{1}{2}(-10p-60)$.

$30 = 10p+60$ or $30 = -10p-60$

$10p = -30$ or $10p = -90$

$\boxed{p=-3}$ or $\boxed{p=-9}$.

14.) If (x,y) be on the line joining the two points $(1,-3)$ & $(-4,2)$. Prove that $x+y+2=0$.

→ Let us consider the given points are

$A(x,y)$, $B(1,-3)$ & $C(-4,2)$.

Then, $A(\Delta ABC) = \frac{1}{2}[x(-3-2) + 1(2-y) + (-4)(y+3)]$

$$= \frac{1}{2}[-5x+2-y-4y-12]$$

$$A(\Delta ABC) = \frac{1}{2}[-5x-5y+10]$$

As the given points are collinear $\Rightarrow$ $A(\Delta ABC) = 0$

$$\frac{1}{2}(-5x-5y-10) = 0$$

$$-5x-5y-10 = 0$$

$$-5(x+y+2) = 0$$

$\boxed{x+y+2=0}$ Hence proved

15.) Find the value of k if points $(k,3)$, $(6,-2)$ & $(-3,4)$ are collinear.

→ Let us consider the points given are $A(k,3)$, $B(6,-2)$ & $C(-3,4)$

Then $A(\Delta ABC) = \frac{1}{2}\{k(-2-4) + 6(4-3) + (-3)(3+2)\}$

$= \frac{1}{2}(-6k + 6 - 15)$

$= \frac{1}{2}(-6k - 9)$

But given points are collinear ⇒ $A(\Delta ABC) = 0$

$\therefore \frac{1}{2}(-6k-9) = 0$

$-6k = 9$

$k = 9/6 = -3/2$ $\boxed{k = -3/2}$

16.) Find the value of k, if points $A(7,-2)$, $B(5,1)$ & $C(3,2k)$ are collinear.

→ Let us consider the given points are $A(7,-2)$, $B(5,1)$ & $C(3,2k)$ are collinear.

Then, $A(\Delta ABC) = \frac{1}{2}\{7(1-2k) + 5(2k+2) + 3(-2-1)\}$

$= \frac{1}{2}(7 - 14k + 10k + 10 - 9\}$

$= \frac{1}{2}(-4k + 8)$

But, $A(\Delta ABC) = 0$

$\frac{1}{2}(-4k+8) = 0$

$-4k + 8 = 0$

$-4k = -8$

$\boxed{k = 2}$