Chapter 16. Circles

Exercise 16.1

1) Fill in the blanks:

i) All points lying inside/outside a circle are called interior points/ exterior points.

ii) Circles having the same centre and different radii are called concentric circles.

iii) A point whose distance from the centre of a circle is greater than its radius lies in the exterior of the circle.

iv) A continuous piece of a circle is arc of the circle.

v) An arc is a semi-circle when its ends are the ends of the diameter.

vi) The longest chord of a circle is a diameter of the circle.

vii) Segment of a circle is a region between an arc and center of the circle.

viii) A circle divides the plane, on which it lies in three parts.

2) Write the truth value (T/F) of the following with suitable reasons:

i) A circle is a plane figure.

→ True, because circle is a figure drawn in a plane.

ii) Line segment joining the center to any point on the circle is a radius of the circle.

→ True, the line joining center of circle & any point on the circle is always radius of the circle.

iii) If a circle is divided into three equal arcs each is a major arc.

→ True, Since three arcs are equal which are major arcs.

iv) A circle has only finite no. of equal chords.

→ False, because a circle has infinite no. of equal chords.

v) A chord of a circle, which is twice as long as its radius is the diameter of the circle.

→ True, diameter of the circle is the largest chord which is twice radius of a circle.

vi) Sector is the region between the chord and its corresponding arc.

→ True.

vii) The degree measure of an arc is the complement of the central angle containing the arc.

→ False.

viii) The degree measure of a semicircle is 180°.

→ True.

Exercise 16.2

1) The radius of a circle is 8 cm and the length of one of its chords is 12 cm. Find the distance of the chord from the center.

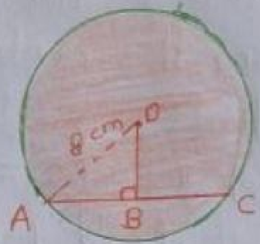
→ Here, given that

Radius of a circle = OA = 8 cm

Chord = AC = 12 cm

Now, we will draw a perpendicular from centre to the chord AC which bisects the chord.

$\Rightarrow \quad AB = BC = \frac{12}{2} = 6\text{cm}$



Now, In ΔOBA,

By Pythagoras theorem,

$OA^2 = OB^2 + AB^2$

$64 = 36 + OB^2$

$OB^2 = 64 - 36 = 28$

$\boxed{OB = \sqrt{28} = 5.291 \text{ cm}}$

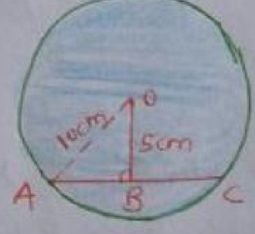
Thus, the distance of the chord from the center was found to be 5.921 cm.

2) Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 10 cm.

→ Given that,

Radius of a circle = OA = 10 cm

Distance of chord AC from 'O' = OB = 5 cm



Now, In ΔOBA, By Paythagoras Theorem

$OA^2 = OB^2 + AB^2$

$100 = AB^2 + 25$

$100 - 25 = 75$

$AB^2 = 75$

$\boxed{AB = 8.66 \text{ cm}}$

As given that, the perpendicular from the centre of a circle bisects the chord.

$\Rightarrow \quad AB = BC = 8.66$ cm

Then, $\quad AC = AB + BC = 8.66 + 8.66 = 17.32$ cm

Thus, the length of the chord AC is found to be 17.32 cm.

3.) Find the length of the chord which is at a distance of 4 cm from the centre of a circle of radius 6 cm.

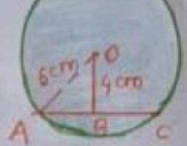
→ Given that,

Radius of a circle $= OA = 6$ cm

Distance of chord from the centre of a circle $= OB = 4$ cm

Now, In ΔOAB

By Pythagoras Theorem,



$$OA^2 = OB^2 + AB^2$$

$$36 = AB^2 + 16$$

$$AB^2 = 36 - 16 = 20$$

$$\boxed{AB = \sqrt{20} = 4.47 \text{ cm}}$$

But, we know that the perpendicular drawn from centre of a circle on a chord bisects the chord.

Then, $\quad AB = BC = 4.47$ cm

Thus, $\quad AC = AB + BC = 4.47 + 4.47 = 8.94$ cm

Thus, the length of chord AC is found to be 8.94 cm.

4) Two chords AB, CD of lengths 5 cm, 11 cm respectively of a circle are parallel. If the distance between AB and CD is 3 cm, find the radius of the circle.

→

Here, given that

from fig. $AB = 5cm$, $CD = 11cm$, $PQ = 3cm$

Now, we will draw perpendiculars OP and ~~OP~~ and OQ on CD and AB respectively as shown in fig.

Let us consider,

$OP = x$ cm and $OC = OA = r$ cm

But, we have

The perpendicular drawn from centre to the chord bisects it.

Hence, $CP = PD = 11/2$ cm (since $OP \perp CD$)

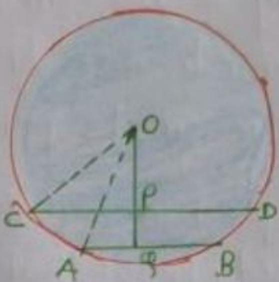
Also, $AQ = BQ = 5/2$ cm ($OQ \perp AB$)

In ΔOCP,

By Pythagoras theorem,

$$OC^2 = OP^2 + CP^2$$

$$r^2 = x^2 + (11/2)^2 \quad \text{——— (1)}$$



Now, In ΔOQA,

By Pythagoras Theorem,

$$OA^2 = OQ^2 + AQ^2$$

$$r^2 = (x+3)^2 + (5/2)^2 \quad \text{——— (2)}$$

$$(x+3)^2 + (5/2)^2 = x^2 + (11/2)^2 \quad \text{from (1) \& (2)}$$

$$x^2 + 6x + 9 + 25/4 = x^2 + 121/4$$

$$4x^2 + 24x + 36 + 25 = 4x^2 + 121$$

$$24x + 61 = 121$$

$$24x = 121 - 61$$

$$24x = 60$$

$$x = \cancel{60}^{10}/\cancel{24}_{4} = \frac{10}{4} = \frac{5}{2}$$

$$\boxed{x = 5/2 \text{ cm}}$$

Now, $r^2 = (5/2)^2 + (11/2)^2 = 25/4 + 121/4 = 146/4$

$$\boxed{r = \sqrt{146/4} \text{ cm}}$$

5) Give a method to find a centre of a given circle.

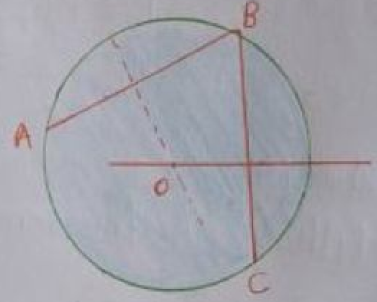
→ following steps are given to find the method of centre of a given circle.

1) Consider any three points A, B, C on a circle initially.

2) Join the points A & B and B & C so that segments AB & BC are formed.

3) Here, we can see AB and BC are the chords of a given circle.

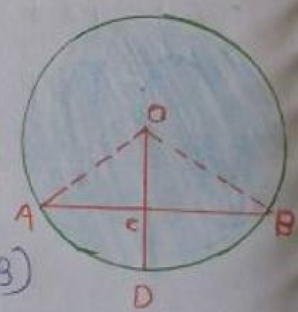
4) Now, draw perpendicular bisectors of chords AB & BC respectively & we can observe these perpendicular bisectors meet at a single point 'O'.



5) And the point 'O' is nothing but the centre of the circle, because we already know that perpendicular bisectors of a chord always pass through the centre of a circle.

6) Prove that the line joining the mid-point of a chord to the centre of the circle passes through the mid-point of the corresponding minor arc.

→ Let us consider, 'c' is the mid-point of chord AB as shown in fig.



Now, In ΔOAC & ΔOBC

$OA = OB$ = Radius of a circle

$OC = OC$ = common side

$AC = BC$ (since 'c' is the mid-point of AB)

Then, By SSS condition we can say that

$\Delta OAC \cong \Delta OBC$.

Thus, $m\angle AOC = m\angle BOC$

$\Rightarrow$ $\boxed{AD = BD}$ By CPCT

Thus, the point 'D' is the mid-point of arc AB.

Hence proved.

7) Prove that a diameter of a circle which bisects a chord of the circle also bisects the angle subtended by the chord at the centre of the circle.

$\rightarrow$ From fig, PQ is the diameter of a circle.

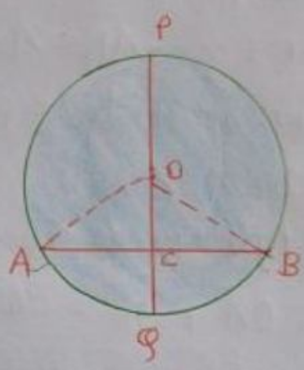
The diameter PQ bisects the chord AB at point 'C'.

Now, In ΔBOC and ΔAOC,

$OA = OB$ (since Radius of a circle)

$OC = OC$ (since common side)

$AC = BC$ (Given)



Then, By SSS condition,

$\boxed{\Delta AOC \cong \Delta BOC}$

Thus, $\angle AOC = \angle BOC$

Thus, PQ bisects $\angle AOB$. Hence proved.

Exercise 16.3

1.) Three girls Ishita, Isha and Nisha are playing a game by standing on a circle of radius 20m drawn in a park. Ishita throws a ball to Isha, Isha to Nisha and Nisha to Ishita. If the distance between Ishita and Isha and and between Isha and Nisha is 24m each, what is the distance between Ishita and Nisha.

→ Here, given that

Let us consider R, S and M are the positions of Ishita, Isha and Nisha respectively.

R → Ishitha
S → Isha
M → Nisha

As, OA is a perpendicular bisector on RS,

Then, $AR = AS = 24/2 = 12cm$

Radius of circle $= OR = OS = OM = 20cm$

Now, In ΔOAR,

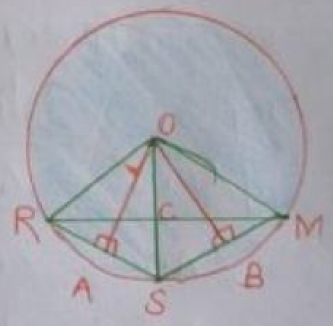
By Pythagoras Theorem,

$$OA^2 + AR^2 = OR^2$$

$$OA^2 + 12^2 = 20^2$$

$$OA^2 = 400 - 144 = 256$$

$$\boxed{OA = 16m}$$



From fig., we can observe that OABC is a kite and $OA = OC$ & $AB = BC$.

We have, the diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangle is bisected by another diagonal.

Thus, In ΔRSM, $\angle RCS = 90°$ & $RC = CM$

But, $A(\Delta ORS) = A(\Delta ORS)$

$$\Rightarrow \frac{1}{2} \times OA \times RS = \frac{1}{2} \times RC \times OS$$

$$\Rightarrow OA \times RS = RC \times OS$$

$$16 \times 24 = RC \times 20$$

$$\boxed{RC = 19.2cm}$$

Since, RC = CM ⇒ RM = 2(19·2) = 38·4 cm

Thus, the distance between Ishita and Nisha is 38·4m

2) A circular park of radius 40m is situated in a colony. Three boys Ankur, Amit and Anand are siting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. find the length of the string of each phone.

⟶ Here, from given & from figure,

$AB = BC = CA$

Thus, ABC is an equilateral triangle.

Radius of circle $= OA = 40$ m

But, we have the medians of a equilateral triangle pass through the circumcentre and intersect each other at the ratio 2:1.

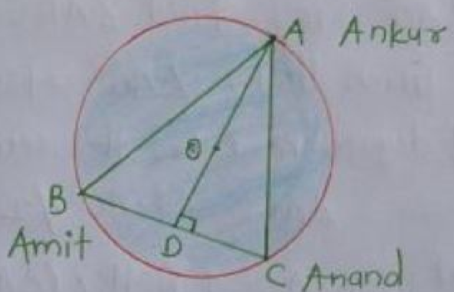
Here, AD is the median of equilateral triangle ABC.

Thus, $\frac{OA}{OD} = \frac{2}{1}$

$\frac{40}{OD} = \frac{2}{1}$

$\Rightarrow \boxed{OD = 20m}$

Thus, $AD = OA + OD = (40 + 20)m = 60m$



Now, In ΔADC,

By Pythagoras Theorem,

$AC^2 = AD^2 + DC^2$

$AC^2 = 60^2 + (AC/2)^2$

$AC^2 = 3600 + AC^2/4$

$3/4\ AC^2 = 3600$

$AC^2 = 4800$

$\boxed{AC = 40\sqrt{3}m}$

Thus, the length of string of each phone will be $40\sqrt{3}$m.

Exercise 16.4

1) In fig., O is the centre of the circle, if $\angle APB = 50°$, find $\angle AOB$ and $\angle OAB$.

→ Given that, $\angle APB = 50°$

By the theorem of degree measure,

$$\angle AOB = 2\angle APB$$

$$\angle AOB = 2 \times 50° = 100°$$

Again, $OA = OB$ (since radius of circle)

Then, $\angle OAB = \angle OBA$ (since angles opposite to equal sides)

Let us consider, $\angle OAB = m$

Then, By angle sum propeoty, we can write

$$\angle OAB + \angle OBA + \angle AOB = 180°$$

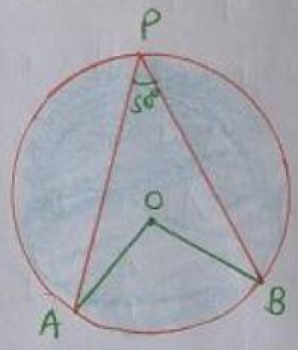
$$m + m + 100° = 180°$$

$$2m = 180° - 100°$$

$$2m = 80°$$

$$\boxed{m = 40°}$$

$$\boxed{\angle OAB = \angle OBA = 40°}$$



2) In fig., it is given that, 'O' is the centre of the circle and $\angle AOC = 150°$. Find $\angle ABC$.

→ Given that, $\angle AOC = 150°$

By degree measure theorem,

$$\angle ABC = R(\angle AOC)/2$$

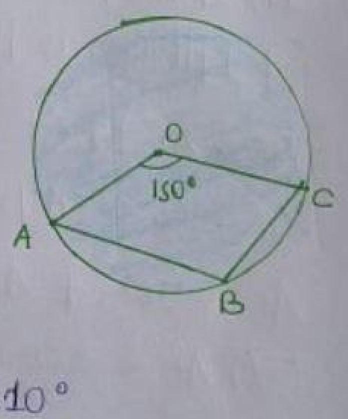
But, $\angle AOC + \text{Reflex}(\angle AOC) = 360°$

$$150° + \text{reflex}(\angle AOC) = 360°$$

$$\text{reflex}(\angle AOC) = 360° - 150° = 210°$$

Thus, $\angle ABC = 210°/2 = 105°$

$$\boxed{\angle ABC = 105°}$$



3) In fig., O is the centre of the circle. find $\angle BAC$.

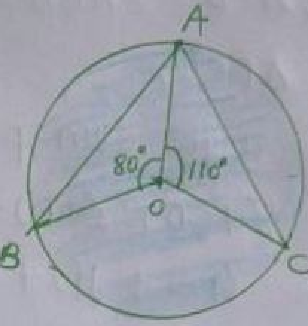
→ Given that, $\angle AOB = 80°$ and $\angle AOC = 110°$

Thus, $\angle AOB + \angle AOC + \angle BOC = 360°$

$80° + 100° + \angle BOC = 360°$

$\angle BOC = 360° - 80° - 110° = 170°$

$\boxed{\angle BOC = 170°}$



By theorem of degree measure,

$\angle BOC = 2 \angle BAC$

$170° = 2(\angle BAC)$

$\angle BAC = 170°/2 = 85°$

$\boxed{\angle BAC = 85°}$

4) If 'O' is the centre of the circle, find the value of 'x' in each of the following fig.

→ i) From fig., $\angle AOC = 135°$

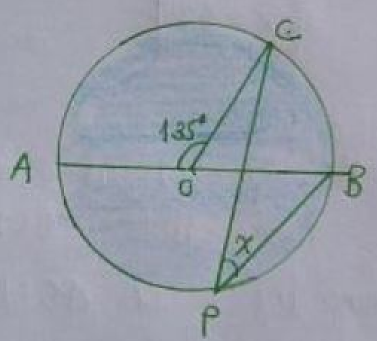
The linear pair angles are $\angle AOC$ & $\angle BOC$,

$\angle AOC + \angle BOC = 180°$

$135° + \angle BOC = 180°$

$\angle BOC = 180° - 135°$

$\boxed{\angle BOC = 45°}$



By degree measure theorem,

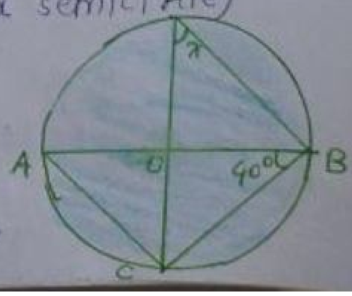
$\angle BOC = 2 \angle CPB$

$45° = 2x$

$\boxed{x = 45°/2}$

ii) From given, $\angle ABC = 40°$

$\angle ACB = 90°$ (since angle subtended in a semicircle)



Now, In ΔABC,

$\angle CAB + \angle ACB + \angle ABC = 180°$

$\angle CAB + 90° + 40° = 180°$

$\angle CAB = 180° - 90° - 40° = 50°$

Now, $\angle CDB = \angle CAB$ since angles on same segment.

$\Rightarrow \boxed{x = 50^\circ}$

iii) From fig., $\angle ABD = 40^\circ$ and $\angle CPD = 110^\circ$

$\angle ACD = \angle ABD = 40^\circ$ (since angle in same segment)

In ΔPCD,

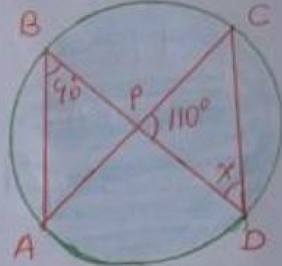
By angle sum property,

$\angle PCD + \angle CPD + \angle PDC = 180^\circ$

$400 + 110 + x = 180$

$x = 180^\circ - 150^\circ = 30^\circ$

$\boxed{x = 30^\circ}$



iv) From fig., $\angle BAC = 52^\circ$

$\angle BDC = \angle BAC = 52^\circ$ (since angles in same segment)

$OD = OC$ (radii of same circle)

$\angle ODC = \angle OCD$ (since opposite angle to equal radii)

$\boxed{x = 52^\circ}$

5) 'o' is the circumcentre of the triangle ABC and OD is perpendicular on BC. Prove that $\angle BOD = \angle A$.

→ From fig., In ΔOBD & ΔOCD,

$OB = OC$ (since radii of circle)

$\angle ODB = \angle ODC$ (since each angle of 90°)

$OD = OD$ (since common side)

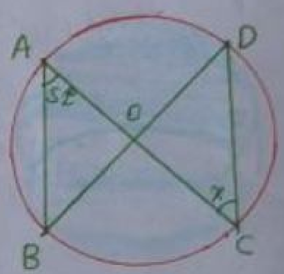
Thus, By RHS condition,

$\Delta OBD \cong \Delta OCD$

Thus, $\boxed{\angle BOD = \angle COD}$

By theorem of degree measure,

$\angle BOC = 2\angle BAC$



$2\angle BOD = \angle COD$

$\Rightarrow 2\angle BOD = 2\angle BAC$

$\boxed{\angle BOD = \angle BAC}$

Hence proved.

6.) In fig., O is the centre of the circle, BO is the bisector of $\angle ABC$. Show that $AB = AC$.

$\rightarrow$ As, BO is the angle bisector of $\angle ABC$,

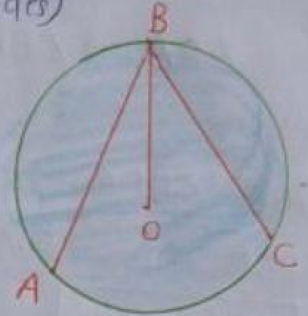
Then, $\angle ABO = \angle CBO$ — ①

From fig, $OB = OA = OC$ (Radii of circle)

$\angle OAB = \angle OCB$ (angles opposite to equal sides) — ②

$\angle ABO = \angle OAB$ (angles opposite to equal sides)

Thus, $\angle OAB = \angle OCB$ from ① & ②



Now, In $\triangle OAB$ & $\triangle OCB$,

$\angle OAB = \angle OCB$

$OB = OB$

$\angle OBA = \angle OBC$ (given)

Then, By AAS condition, $\boxed{\triangle OAB \cong \triangle OCB}$

Thus, $\boxed{AB = BC}$

Hence proved.

7.) In fig., 'O' is the centre of the circle, then prove that $\angle x = \angle y + \angle z$.

$\rightarrow$ From fig,

$\angle 3 = \angle 4$ (since angles in same segment)

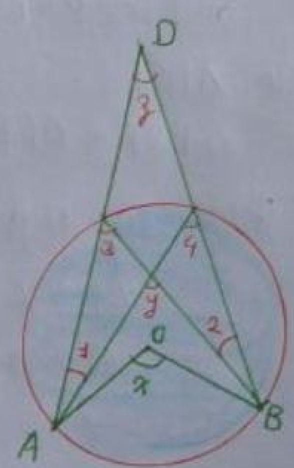
$\angle x = 2(\angle 3)$ ($\because$ By degree measure theorem)

$\angle x = \angle 3 + \angle 3$

$\angle x = \angle 3 + \angle 4$

Again, $\angle y = \angle 3 + \angle 1$ ($\because$ exterior angles)

$\angle 3 = \angle y - \angle 1$



$\angle 4 = \angle 3 + \angle 1$ ($\because$ exterior angles)

Thus, $\angle x = \angle y - \angle 1 + \angle 3 + \angle 1$

$\angle x = \angle y + \angle 3 + \angle 1 - \angle 1$

$\boxed{\angle x = \angle y + \angle 3}$

Hence proved.

Exercise 16.5

1.) In fig., ΔABC is an equilateral triangle. find $m\angle BEC$.

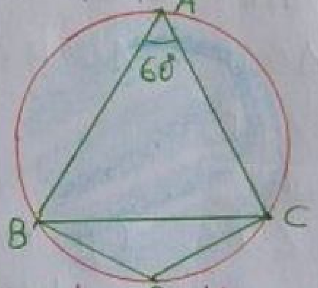
→ Given that,

ΔABC is an equilateral triangle.

we know that, each angle of an equilateral triangle is of $60°$.

Thus, $\angle BAC + \angle BEC = 180°$

($\because$ opposite angles of quadrilateral)



$60° + \angle BEC = 180°$

$\angle BEC = 180° - 60°$

$\boxed{\angle BEC = 120°}$

2.) In fig., ΔPQR is an isosceles triangle with $PQ = PR$ and $m\angle PQR = 35°$. find $m\angle QSR$ and $m\angle QTR$.

→ Here, given that

ΔPQR is an isosceles triangle with $PQ = PR$

and $m\angle PQR = 35°$.

In ΔPQR,

$\angle PQR = \angle PRQ = 35°$ (since angles opposite to equal sides)

Then, By angle sum property,

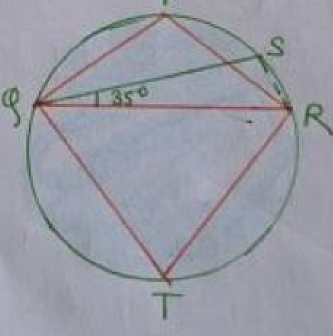
$\angle P + \angle Q + \angle R = 180°$

$\angle P + 35° + 35° = 180°$

$\angle P + 70° = 180°$

$\angle P = 180° - 70°$

$\boxed{\angle P = 110°}$



Now, in □SQTR,

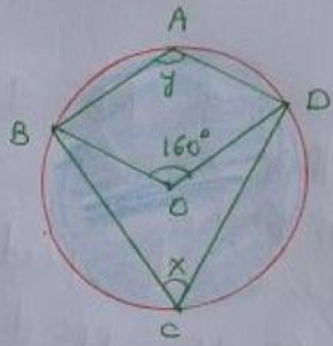
$\angle QSR$ & $\angle QTR$ are opposite angles of quadrilateral.

$\angle QSR + \angle QTR = 180^\circ$

$110^\circ + \angle QTR = 180^\circ$

$\boxed{\angle QTR = 70^\circ}$

3) In fig, O is the centre of the circle. If $\angle BOD = 160^\circ$, find the values of x and y.



→ Now, from fig.

$\angle BOD = 160^\circ$

By theorem of degree measure,

$\angle BOD = 2\angle BCD$

$160^\circ = 2x$

$\boxed{x = 80^\circ}$

In □ABCD,

$\angle BAD$ and $\angle BCD$ are opposite angles of cyclic quadrilateral.

$\angle BAD + \angle BCD = 180^\circ$

$y + x = 180^\circ$

$y + 80^\circ = 180^\circ$

$\boxed{y = 100^\circ}$

Thus, $\boxed{x = 80^\circ,\ y = 100^\circ}$

4.) In fig., ABCD is a cyclic quadrilateral. If $\angle BCD = 100^\circ$ & $\angle ABD = 70^\circ$, find $\angle ADB$.

→ from fig.,

In quadrilateral ABCD,

$\angle DCB$ & $\angle BAD$ are opposite angles of cyclic quadrilateral.

$\angle DCB + \angle BAD = 180^\circ$

$100^\circ + \angle BAD = 180^\circ$

$\boxed{\angle BAD = 180^\circ}$

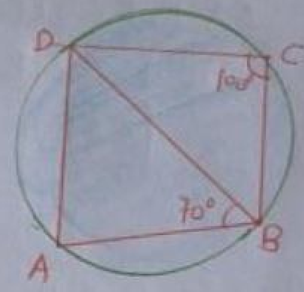
Now, In ΔBAD,

By angle sum property,

$\angle ADB + \angle DAB + \angle ABD = 180^\circ$

$\angle ADB + 80^\circ + 70^\circ = 180^\circ$

$\boxed{\angle ADB = 30^\circ}$



5.) If ABCD is a cyclic quadrilateral in which AD ∥ BC. Prove that $\angle B = \angle C$.

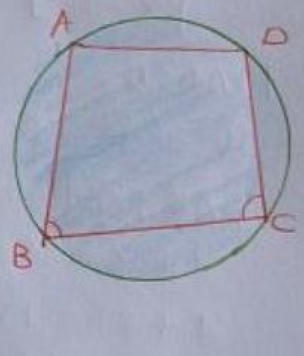
→ Given that,

ABCD is a cyclic quadrilateral with AD ∥ BC.

ABCD is a cyclic quadrilateral.

$\angle A$ and $\angle C$ are opposite angles of cyclic quadrilateral.

$\boxed{\angle A + \angle C = 180^\circ}$ —— ①



$\angle A$ and $\angle B$ are cointerior angles.

$\therefore \boxed{\angle A + \angle B = 180^\circ}$ —— ②

Thus, $\Rightarrow \boxed{\angle B = \angle C}$ from ① & ②

Hence proved.

6) In fig., O is the centre of the circle. find $\angle CBD$.

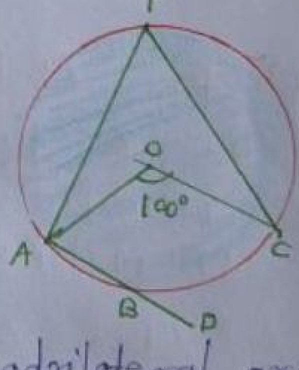
→ Given that, $\angle BOC = 100^\circ$

Then, By theorem of degree measure,

$\boxed{\angle AOC = 2\angle APC}$

$100^\circ = 2(\angle APC)$

or $\boxed{\angle APC = 50^\circ}$



Thus, as opposite angles of cyclic quadrilateral are sums to 180°) ~~equal~~)

$\angle APC + \angle ABC = 180^\circ$

$50^\circ + \angle ABC = 180^\circ$

$\boxed{\angle ABC = 130^\circ}$

Now, $\angle ABC$ & $\angle CBD$ are linear pair angles.

Thus, $\angle ABC + \angle CBD = 180^\circ$

$130^\circ + \angle CBD = 180^\circ$

$\boxed{\angle CBD = 50^\circ}$

7) In a cyclic quadrilateral ABCD, if $m\angle A = 3(m\angle C)$. Find $m\angle A$

→ We know that,

In a cyclic quadrilateral the sum of opposite angles is 180°.

Thus, $\angle A + \angle C = 180^\circ$

Again, Given that $m\angle A = 3(m\angle C)$

$\angle A = 3(\angle C)$

$3\angle C + \angle C = 180^\circ$

$4\angle C = 180^\circ$

$\boxed{\angle C = 45^\circ}$

Thus, $\angle A = 3(\angle C) = 3(45^\circ) = 135^\circ$

$\boxed{m\angle A = 135^\circ}$

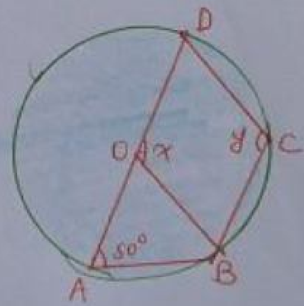
11) In fig., O is the centre of the circle $\angle DAB = 50^\circ$. Calculate the value of x and y.

→ Here, given that $\angle DAB = 50^\circ$

Then, By degree measure theorem,

$\angle BOD = 2\angle BAD$

$x = 2(50^\circ) = 100^\circ$



As, ABCD is a cyclic quadrilateral.

$\angle A + \angle C = 180^\circ$

$50^\circ + y = 180^\circ$

$\boxed{y = 130^\circ}$