

— Mechanical Properties of Solids —

We are familiar about the structure of solids. The interatomic force between is very strong so that they possess definite shape and size. But when external force is large enough, the solids too show some changes in it.

Rigid body: The body whose shape, size remains constant (same) on application of forces

Eg. stone, wood, etc.

★ **Elastic body:** The body which shape, size changes after application of forces and regains the original dimensions after removal of forces is called as elastic body.

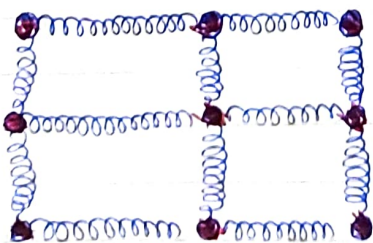
Eg. rubber, metal etc

★ **Plastic body:** The body which shape, size changes after application of external forces but do not regains original dimensions after removal of forces is called plastic body

★ **Deformation:** The changes in shape, size that produced in object are called deformation. The forces that produces deformation are called as deforming forces.

★ **Elastic behaviour of solids:**

In solids atoms are closely packed, so that every atom exerts force on neighbouring atom. The net force is always zero & substance remains in stable or equilibrium state.



When deforming forces are exerted, there is displacement in layers of atoms. But when the forces are removed, the molecules (atoms) again pulls each other like spring and

regains its original dimensions (Fig. shows spring ball model of solid.) Deformation produces when internal force is overcome by deforming forces.
stress :

When deforming forces are applied on object such that they overcome internal force so that deformation is produced in body. These forces act over the given area of body gives rise the concept of stress & it is defined as 'internal restoring force acting per unit area' of object.

$$\therefore \text{stress} = \frac{\text{internal restoring force}}{\text{Area}}$$

$$\text{stress} = \frac{\text{force}}{\text{area}}$$

$$= \frac{F}{A} = \frac{Mg}{A}$$

SI unit is N/m^2 & Dimensions are $[M^1 L^{-1} T^{-2}]$

Depending upon change in dimensions, stress is of following types

* 1) Tensile stress \Rightarrow

stress corresponding to change in length of object is known as tensile stress

It is defined as applied force per unit area

$$\therefore \text{Tensile stress} = \frac{\text{applied force}}{\text{Area}}$$

$$= F/A$$

* 2) Compressive stress or volume stress :

stress corresponding to change in volume of object is known as compressive stress. It defined as ~~that~~ applied force per unit area.

$$\therefore \text{Compressive stress} = \frac{\text{applied force}}{\text{area}} = \text{change in pressure} = dp.$$

* 3) Shearing or tangential stress :

The stress related to change in shape of body is known as shearing stress. It is defined as tangential applied force per unit area.

$$\therefore \text{Shearing stress} = \frac{\text{Tangential applied force}}{\text{Area}}$$

$$= F/A$$

* Strain :

When force acts on body, its dimensions gets changed. The change in dimension per unit original dimensions is called as strain.

$$\therefore \text{strain} = \frac{\text{Change in dimension}}{\text{original dimension}}$$

Strain is unitless and dimensionless quantity.

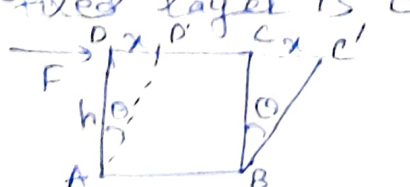
It is of three types

- * 1) Longitudinal strain: The strain corresponding to change in length is called as longitudinal strain. The change in length per unit length of object is known as longitudinal or tensile strain.
- $$\therefore \text{Longitudinal strain} = \frac{\text{Change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

- * 2) Volume strain: strain corresponding to change in volume is known as volume strain. It is defined as change in volume per unit original volume.

$$\therefore \text{Volume strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

- * 3) Shearing strain: - stress related to change in shape is called shearing strain. The lateral displacement of layer per unit distance of layer from fixed layer is called as shearing strain.



$\therefore \text{shearing strain} = \frac{\text{lateral displacement}}{\text{distance from fix layer}}$

$$= \frac{x}{h} = \tan \alpha \approx \alpha \quad (\text{for small angle})$$

* Hooke's law:

Within elastic limit, stress is directly proportional to strain.

$$\therefore \text{stress} \propto \text{strain}$$

$$\therefore \text{stress} = M \cdot \text{strain}$$

Where, $M =$ constant of proportionality
 $=$ Modulus of elasticity

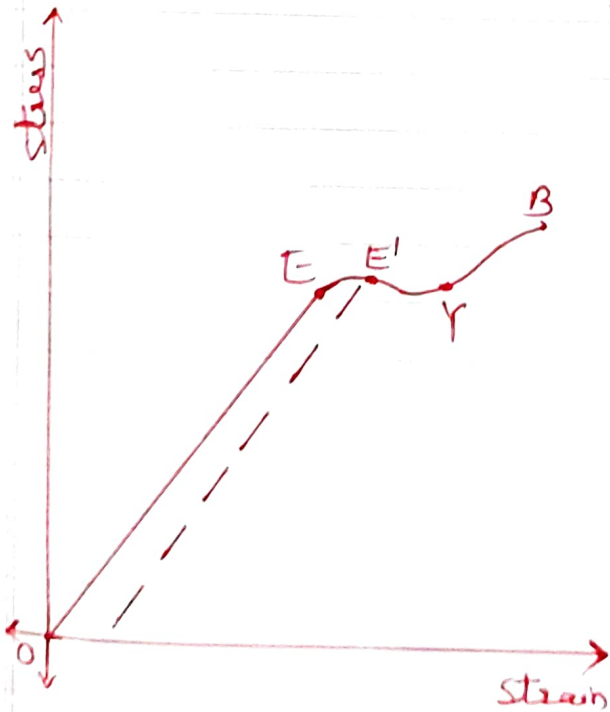
Modulus of elasticity is defined as stress per unit strain -

$$\therefore \text{Modulus of elasticity} = \frac{\text{stress}}{\text{strain}}$$

\therefore SI unit is N/m^2 & dimensions $[M^1 L^{-1} T^{-2}]$

* stress-strain curve:-

A thin wire is suspended from rigid support & load is attached to its one end, the extension is recorded in each half kilogram addition of load. The graph of applied load (stress) and strain is plotted, till the wire breaks.



- The region OE of graph represents that stress is directly proportional to strain i.e. Hooke's law is obeyed.
- This region is called as elastic region & stress at point E is elastic stress.
- Region E'E' indicates the permanent deformation state called plastic region.
- Region E'Y is known as yield region & corresponding stress is called as yield strength.

Further increase in load will form neck in wire & will break finally for certain load called as breaking stress and corresponding point is called as breaking point.

Elastomers :- Substances that can stretched to considerably high value (i.e. large strain) are called as elastomers. Eg. tissues of aorta, rubber.

* Moduli of elasticity :

From Hooke's law, we have defined modulus of elasticity as the ratio of stress to strain.

Due to three types of stress and strain, modulus of elasticity is also studied in three types.

* 1. Young's modulus :-

Modulus of elasticity related to change in length is known as Young's modulus. It is denoted by 'Y'.

The ratio of longitudinal stress to longitudinal strain is called as Young's modulus.

$$\begin{aligned} \therefore \text{Young's modulus} &= \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} \\ &= \frac{F/A}{\Delta l/l} \end{aligned}$$

$$Y = \frac{Fl}{A \Delta l} = \frac{mg l}{\pi r^2 \Delta l}$$

It is applicable for solids only.

* 2. Bulk modulus :

Modulus of elasticity related to change in volume is known as bulk modulus. It is denoted by K. The ratio of volume stress to volume strain is called as bulk modulus.

$$\therefore \text{Bulk modulus} = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{dP}{\Delta V/V}$$

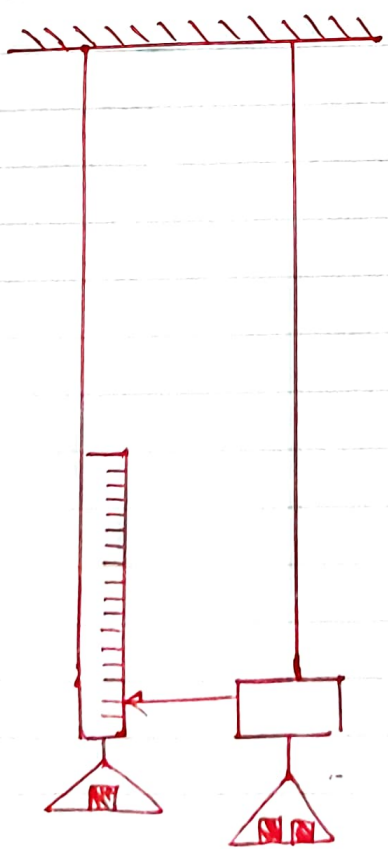
★ 3. Modulus of rigidity or shear modulus:
Modulus of elasticity related to change in shape is known as modulus of rigidity. It is denoted by η . It is ratio of shear stress to shear strain.

$$\therefore \text{Shear modulus, } \eta = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$= \frac{F/A}{\tan \theta} = \frac{F}{A \tan \theta} \approx \frac{F}{A \theta}$$

★ Determination of Young's modulus of material of wire:

The diagram below shows arrangement for measuring Young's modulus of wire. It consists of two wires P & Q, (P - reference wire, Q - experimental wire) suspended from rigid support with hanger to lower end. Vernier scale is attached to wire Q and main scale to wire P.



Procedure

1. Level the lengths of wire by spirit bottle (level)
2. Now add load of $\frac{1}{2}$ kg in hanger of wire B. Wait for a minute to allow wire to extend.
3. Now spirit level will be displaced. Adjust the vernier scale to main scale of wire P so level the wires P & Q.
4. Measure the readings in vernier scale & main scale.
5. Record the observation.
6. Repeat the procedure for 6 readings

Using formula $Y = \frac{mgL}{\pi r^2 \Delta l}$ Find the Young's modulus of material of wire.

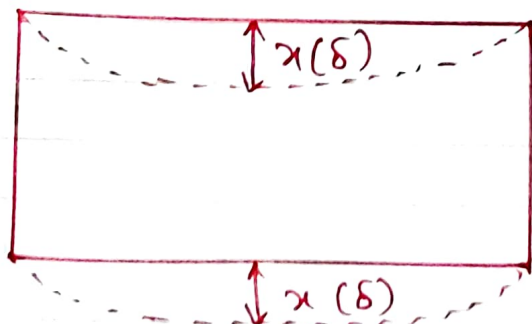
(Where L = Original length, r = radius of wire)

* Applications of elastic behaviour of material :
Elastic behaviour of materials is studied before they are used for specific purpose.

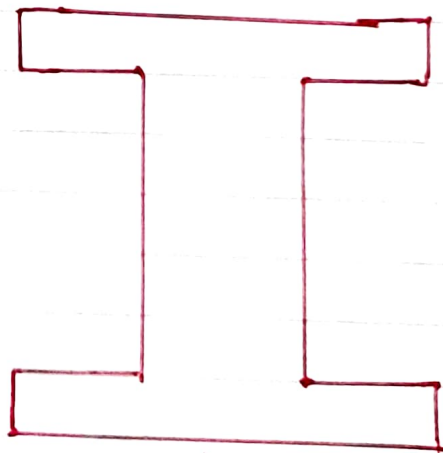
- strength of cables used in cranes is studied then the diameter is selected.
- Cable used in suspension typed bridge are selected with having large values of 'Y' & ultimate stress.
- Ropes are used in form of braided threads combined together.
- A bar used in constructions has specific diameter
- Instead of using bars, I-beams are advised to use for constructions, where heavy load is to be carried by beams.

Ordinary rods/beams bends due to its own weight and saggd from middle. The sag or bending is given as

$$\delta = \frac{wl^3}{4bd^3Y}$$



i) Bending of beam.



ii) I-beam.