

Chapter 20: Surface Area & Volume of A Right Circular Cone

Exercise 20.1

1.) Find the curved surface area of a cone, if its slant height is 60 cm and the radius of its base is 21 cm.

→ Here, Given that

Slant height of cone (l) = 60 cm

Radius of base of cone (r) = 21 cm

Then, Curved surface area of right circular cone } = $\pi r l$

$$= \frac{22}{7} \times 21 \times 60$$

$$= 3960 \text{ cm}^2$$

→ Thus, the curved surface area of cone is found to be 3960 cm^2 .

2.) The radius of a cone is 5 cm and vertical height is 12 cm. Find the area of the curved surface.

→ Given that,

Radius of a cone (r) = 5 cm

Vertical height of a cone (h) = 12 cm

We know that, $l^2 = r^2 + h^2$

where l is the slant height of the cone.

$$\Rightarrow l^2 = (5)^2 + (12)^2$$
$$= 25 + 144$$

$$l^2 = 169$$

$$l = 13 \text{ cm}$$

$$\begin{aligned}\text{Now, Curved surface of cone} &= \pi r l \\ &= 3.14 \times 5 \times 13 \\ &= 204.28\end{aligned}$$

Thus, the curved surface area of cone is found to be 204.28 cm^2 .

3.) The radius of a cone is 7 cm and area of curved surface is 176 cm^2 . Find the slant height.

→ Here, given that

$$\text{Radius of a cone } (r) = 7 \text{ cm}$$

$$\text{Curved surface area of a cone} = 176 \text{ cm}^2.$$

We know that,

$$\text{Curved surface area of a cone} = \pi r l$$

$$176 = \pi r l$$

$$176 = \frac{22}{7} \times 7 \times l$$

$$\boxed{l = 8 \text{ cm}}$$

Thus, the slant height of the cone is found to be 8 cm .

4.) The height of a cone is 21 cm . Find the base if the slant height is 28 cm .

→ Here, given that

$$\text{Height of a cone } (h) = 21 \text{ cm}$$

$$\text{Slant height of a cone } (l) = 28 \text{ cm}$$

$$\begin{aligned}\text{We have, } l^2 &= h^2 + r^2 \\ r^2 &= l^2 - h^2 = (28)^2 - (21)^2\end{aligned}$$

$$\boxed{r = 7\sqrt{7} \text{ cm}}$$

$$\begin{aligned}\text{Now, Area of circular base of a cone} &= \pi r^2 \\ &= \frac{22}{7} (7\sqrt{7})^2 \\ &= 1078 \text{ cm}^2.\end{aligned}$$

Thus, the area of circular base of a cone is found to be 1078 cm^2 .

5) Find the total surface area of a right circular cone with radius 6cm & height 8cm.

→ Here, Given that

Radius of a cone (r) = 6cm

Height of a cone (h) = 8cm

We have,

$$l^2 = r^2 + h^2$$

$$= 6^2 + 8^2$$

$$= 36 + 64$$

$$l^2 = 100$$

$$\boxed{l = 10 \text{ cm}}$$

Now, Total surface area of a cone = (Curved surface Area of a cone) + (Area of a circular base of a cone)

$$= \pi r l + \pi r^2$$

$$= \left(\frac{22}{7} \times 6 \times 10\right) + \left(\frac{22}{7} \times 6 \times 6\right)$$

$$= \frac{1320}{7} + \frac{792}{7}$$

$$= 301.71 \text{ cm}^2$$

Thus, the total surface area of the cone is found to be 301.71 cm^2 .

6) Find the curved surface area of a cone with base radius 5.25cm and slant height 10cm.

→ Here, given that

Base radius of a cone (r) = 5.25 cm

Slant height of a cone (l) = 10 cm

We have,

$$l^2 = h^2 + r^2$$

$$h^2 = l^2 - r^2 = 10^2 - (5.25)^2$$

$$= 100$$

We have, The curved surface area = $\pi r l$

$$= \frac{22}{7} \times 5.25 \times 10$$

$$= 165 \text{ cm}^2$$

Thus, the curved surface area of cone is found to be 165 cm^2 .

7) Find the total surface area of a cone, if its slant height is 21m & diameter of its base is 24m.

→ Here, given that

$$\text{Slant height of a cone } (l) = 21\text{m}$$

$$\text{Diameter of base of a cone} = 24\text{m}$$

$$\text{Then, Radius of base of a cone} = 12\text{m.}$$

$$\text{Total surface area of cone} = \text{CSA} + \text{BSA}$$

$$= \pi r l + \pi r^2$$

$$= \left(\frac{22}{7} \times 12 \times 21\right) + \left(\frac{22}{7} \times 12 \times 12\right)$$

$$= 1244.57 \text{ m}^2$$

Thus, the total surface area of a cone is found to be 1244.57 m^2 .

8) The area of the curved surface of a cone is $60\pi \text{ cm}^2$. If the slant height of the cone be 8cm, find the radius of the base.

→ Here, given that

$$\text{Curved surface area of a cone} = 60\pi \text{ cm}^2$$

$$\text{Slant height of a cone } (l) = 8\text{cm}$$

$$\text{We have, Curved surface area of a cone} = \pi r l$$

$$\pi r l = 60\pi$$

$$r l = 60$$

$$r = 60/8$$

$$\boxed{r = 7.5 \text{ cm}}$$

Thus, the radius of the base is found to be 7.5cm.

9.) The curved surface area of a cone is 4070 cm^2 and diameter is 70 cm . What is slant height? ($\pi = \frac{22}{7}$)

→ Here, given that

$$\text{Curved surface area of a cone} = 4070 \text{ cm}^2$$

$$\text{Diameter of the cone} = 70 \text{ cm}$$

$$\text{Then, Radius of base of cone} = 35 \text{ cm}$$

$$\text{Now, Curved surface area of cone} = \pi r l$$

$$\pi r l = 4070$$

$$\frac{22}{7} (35) \times l = 4070$$

$$\boxed{l = 37 \text{ cm}}$$

→ Thus, the slant height of the cone is found to be 37 cm .

10.) The radius & slant height of a cone are in the ratio $4:7$. If its curved surface area is 792 cm^2 , find its radius. ($\pi = \frac{22}{7}$)

→ Here, given that

Radius & slant height of a cone are in the ratio = $4:7$

$$\text{Curved surface area of cone} = 792 \text{ cm}^2$$

If $4x$ is the radius then slant height is $7x$.

$$\text{Now, Curved surface area of cone} = \pi r l$$

$$\frac{22}{7} \times (4x) \times (7x) = 792$$

$$x^2 = 9$$

$$\boxed{x = 3 \text{ cm}}$$

$$\text{Then, Radius of a cone} = 4x = 4 \times 3 = 12 \text{ cm}$$

$$\text{Slant Height of a cone} = 7x = 7 \times 3 = 21 \text{ cm}$$

Exercise 20-2

7. Find the volume of the right circular cone with:

i) Radius 6cm, height 7cm

ii) Radius 3.5cm, height 12cm

iii) Height is 21cm, slant height 28cm

→

i) Given that

Radius of a cone (r) = 6cm

Height of a cone (h) = 7cm

Volume of a right circular cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} (3.14) (6)^2 (7)$$

$$= 264 \text{ cm}^3$$

Thus, the volume of a cone is found to be 264 cm^3 .

ii) Given that,

Radius of a cone (r) = 3.5cm

Height of a cone (h) = 12cm

Then, Volume of a right circular cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} (3.14) (3.5)^2 (12)$$

$$= 154 \text{ cm}^3$$

Thus, the volume of a cone is found to be 154 cm^3 .

iii) Given that,

Height of a cone (h) = 21cm

Slant height of a cone (l) = 28cm

We have, $l^2 = h^2 + r^2$

$$r^2 = l^2 - h^2 = (28)^2 - (21)^2$$

$$\boxed{r = 7\sqrt{7}}$$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} (3.14) (7\sqrt{7})^2 (21)$$

$$= 7546 \text{ cm}^3.$$

(2)

- 2.) Find the capacity in litres of a conical vessel with
- Radius 7cm, slant height 25cm.
 - height 12cm, slant height 13cm.

→ i) Given that,

Radius of conical vessel (r) = 7cm

Slant height of conical vessel (l) = 25cm

We have, $l^2 = h^2 + r^2$

$$h^2 = l^2 - r^2 = (25)^2 - (7)^2$$

$$\boxed{h = 24 \text{ cm}}$$

Then, Volume of a conical vessel = $\frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} (3.14) (7)^2 (24)$$
$$= 1232 \text{ cm}^3$$

Thus, the volume of a conical vessel is found to be 1232 cm^3 or 1.232 litres.

ii) Given that,

Height of a conical vessel (h) = 12cm

Slant height of a conical vessel (l) = 13cm

We have, $l^2 = h^2 + r^2$

$$r^2 = l^2 - h^2 = (13)^2 - (12)^2$$

$$\boxed{r = 5 \text{ cm}}$$

Volume of a conical vessel = $\frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} (3.14) (5)^2 (12)$$
$$= 314.28 \text{ cm}^3$$

Thus, the volume of a conical vessel is found to be 314.28 cm^3 or 0.314 litres.

3.) Two cones have their heights in the ratio 1:3 & the radii of their bases in the ratio 3:1. Find the ratio of their volumes.

→ Given that,

Two cones have their heights in the ratio 1:3 & the radii of their bases in the ratio 3:1.

If h is the height of first cone then $3h$ is the height of second cone.

Similarly, if $3r$ is the radius of first cone then r is the height of second cone.

Then, Volume of first cone (V_1) = $\frac{1}{3}\pi (3r)^2 h$

Volume of second cone (V_2) = $\frac{1}{3}\pi r^2 (3h)$

Now,
$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi (3r)^2 h}{\frac{1}{3}\pi r^2 (3h)}$$

$$\frac{V_1}{V_2} = \frac{9r^2 h}{3r^2 h} = \frac{9}{3} = \frac{3}{1}$$

→ Thus, the required ratio is found to be 3:1.

4.) The radius and the height of a right circular cone are in the ratio 5:12, if its volume is 314 m^3 , find the slant height and the radius. (Use $\pi = 3.14$)

→ Here, Given that

The radius & height of a right circular cone are in the ratio 5:12.

$$\text{Volume of a cone} = 314 \text{ m}^3$$

If radius of a circular cone is $5x$ then its height will be $12x$.

We have,
$$l^2 = h^2 + r^2$$

$$l^2 = (12x)^2 + (5x)^2$$

$$l^2 = 25x^2 + 144x^2$$

$$\boxed{l = 13x}$$

And hence slant height of a right circular cone is

Given that, Volume of cone = 314 m^3

$$\text{Then } \frac{1}{3} \pi r^2 h = 314$$

$$\frac{1}{3} (3.14) (5x)^2 (12x) = 314$$

$$x^3 = 1$$

$$\boxed{x = 1}$$

Then, Radius of a cone (r) = $5x = 5 \text{ m}$

Slant height of a cone (l) = 13 m

5.) The radius and height of a right circular cone are in the ratio $5:12$ & its volume is 2512 cm^3 . Find the slant height & radius of the cone. ($\pi = 3.14$).

→ Given that,

The radius & height of a right circular cone are in the ratio $5:12$.

Volume of right circular cone = 2512 cm^3 .

If $5y$ is the radius of cone then $12y$ is the height of cone respectively.

Radius of cone (r) = $5y$

Height of cone (h) = $12y$

We have, $l^2 = r^2 + h^2$

$$l^2 = (12y)^2 + (5y)^2$$

$$l^2 = 25y^2 + 144y^2$$

$$\boxed{l = 13y}$$

Volume of the cone = 2512

$$\frac{1}{3} \pi r^2 h = 2512$$

$$\frac{1}{3} (3.14) (5y)^2 (12y) = 2512$$

$$y^3 = 8 \Rightarrow \boxed{y = 2 \text{ cm}}$$

(12)

Thus, Radius of cone = $5y = 5(2) = 10\text{cm}$

and Slant height of cone (l) = $13y = 13(2) = 26\text{cm}$

6.) The ratio of volumes of two cones is $4:5$ and the ratio of the radii of their bases is $2:3$, find the ratio of their vertical heights.

→ Given that,

The ratio of volumes of two cones is $4:5$ and

the ratio of the radii of their bases is $2:3$.

If $2x$ is the radius of first cone then $3x$ is the radius of second cone.

Similarly, If $4y$ is the volume of first cone then $5y$ is the volume of second cone.

Thus, Radius of first cone (r_1) = $2x$

Radius of second cone (r_2) = $3x$

Volume of first cone (V_1) = $4y$

Volume of second cone (V_2) = $5y$.

Let us consider, h_1 & h_2 are the heights of first cone & second cone respectively.

Then $V_1 = \frac{1}{3}\pi r_1^2 h_1$ & $V_2 = \frac{1}{3}\pi r_2^2 h_2$

$$\text{Thus, } \frac{V_1}{V_2} = \frac{4}{5} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{r_1^2 h_1}{r_2^2 h_2}$$

$$\frac{V_1}{V_2} = \frac{4}{5} = \frac{4x^2 h_1}{9x^2 h_2} = \frac{4h_1}{9h_2}$$

$$\Rightarrow \frac{4}{5} = \frac{4h_1}{9h_2}$$

$$\boxed{\frac{h_1}{h_2} = \frac{9}{5}}$$

Thus, the ratio of heights given two cones is found to be $9:5$.

7) A cylinder and a cone have equal radii of their bases and equal heights. Show that their volumes are in the ratio 3:1

→ Here, Given that

A cylinder & a cone have equal radii of their bases & equal heights.

Let 'r' be the radius of cone & cylinder.

Let 'h' be the height of cone & cylinder.

$$\text{Then, } \frac{\text{Volume of Cylinder}}{\text{Volume of Cone}} = \frac{\pi r^2 h}{\frac{1}{3} \pi r^2 h} = \frac{3}{1}$$

Thus, the ratio of the volumes is found to be 3:1.

Exercise VSAGs

1.) The height of a cone is 15cm. If its volume is $500\pi \text{ cm}^3$, then find the radius of its base.

→ Given that, Height of a cone (h) = 15cm

$$\text{Volume of a cone (V)} = 500\pi \text{ cm}^3$$

Let 'r' be the radius of base of a cone.

$$\text{Then, } \text{Volume of a cone (V)} = \frac{1}{3} \pi r^2 h$$

$$500\pi = \frac{1}{3} \pi r^2 (15)$$

$$500 = \frac{1}{3} r^2 (15)$$

$$500 = 5r^2$$

$$100 = r^2$$

$$\boxed{r = 10 \text{ cm}}$$

Thus, the radius of the base is found to be 10cm.

2) If the volume of a right circular cone of height 9 cm is $48\pi \text{ cm}^3$, find the diameter of its base.

→ Here given that,

$$\text{Height of a cone (h)} = 9 \text{ cm}$$

$$\text{Volume of a cone (V)} = 48\pi \text{ cm}^3$$

Let r be the radius of base of a cone.

$$\text{Then, Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$48\pi = \frac{1}{3}\pi r^2 (9)$$

$$48 = 3r^2$$

$$16 = r^2$$

$$\boxed{r = 4 \text{ cm}}$$

Thus, Radius of base of a cone is found to be 4 cm.

Then diameter of base of a cone = $2r = 8 \text{ cm}$.

3) If the height & slant height of a cone are 21 cm & 28 cm respectively, find its volume.

→ Here, given that

$$\text{Height of a cone (h)} = 21 \text{ cm}$$

$$\text{Slant height of a cone (l)} = 28 \text{ cm}$$

$$\text{we have, } l^2 = h^2 + r^2$$

$$r^2 = l^2 - h^2 = (28)^2 - (21)^2$$

$$\boxed{r = 7\sqrt{7} \text{ cm}}$$

$$\text{Then, Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi (7\sqrt{7})^2 (21)$$

$$= 2401\pi \text{ cm}^3$$

Thus, the volume of a cone is found to be $2401\pi \text{ cm}^3$.

4.) The height of a conical vessel is 3.5 cm. If its capacity is 3.3 litres of milk. Find the diameter of its base.

→ Here, given that

Height of a conical vessel (h) = 3.5 cm

Volume / Capacity of a conical vessel (V) = 3.3 litres.
= 3300

Now, Volume of a cone = $\frac{1}{3}\pi r^2 h$

$$3.3 = \frac{1}{3} \cdot \left(\frac{22}{7}\right) r^2 (3.5)$$

$$3300 = \frac{1}{3} \left(\frac{22}{7}\right) r^2 (3.5)$$

$$r^2 = 900$$

$$\boxed{r = 30 \text{ cm}}$$

Thus, the radius of base of cone is found to be 30 cm.

→ Then, diameter of base of cone = $2r = 2 \times 30 = 60 \text{ cm}$.