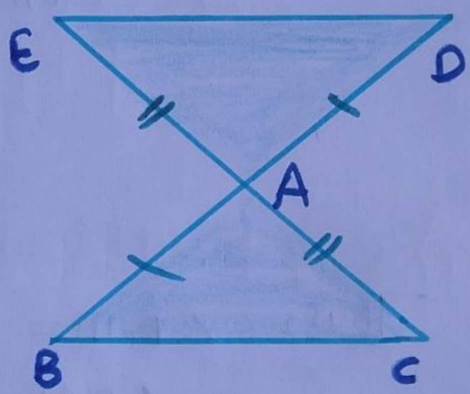
Chapter 10.
Congruent Triangles

Exercise 10.1

1.) In fig. the sides BA & CA have been produced such that BA = AD & CA = AE. Prove that segment DE ∥ BC.

→ From fig. Given that
The sides BA & CA have been produced such that BA = AD & CA = AE.



To prove: DE ∥ BC

Let us consider, ΔBAC & ΔDAE,
then from fig, BA = AD & CA = AE
But, we know that vertically opposite angles are always equal.

$$\angle BAC = \angle DAE$$

Then, by Side-Angle-Side congruence criterion,
we can write, $\boxed{\Delta BAC \cong \Delta DAE}$

But, the corresponding parts of congruent triangles are equal.

So, BC = DE & $\angle DEA = \angle BCA$, $\angle EDA = \angle CBA$

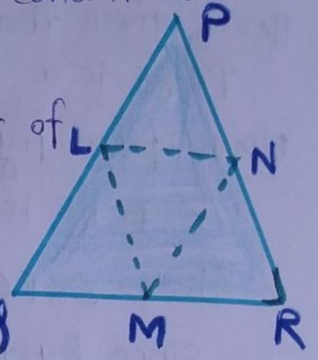
Here, the transversal DB intersects two lines DE & BC such that $\angle DEA = \angle BCA$ (alternate angles are equal)

Thus, DE ∥ BC

Hence proved.

2.) In a PQR, if PQ = QR and L, M & N are the mid-points of the sides PQ, QR & RP respectively. Prove that LN = MN

→ we will draw the diagram from given conditions.



From fig, In ΔPQR,

$PQ = QR$ and L, M, N are the midpoints of the sides PQ, QP and RP respectively.

Now, we will prove: $LN = MN$

we know that,

when two sides of the triangle are equal, then ΔPQR is also an isosceles triangle.

$$PQ = QR \ \& \ \angle QPR = \angle QRP \quad \text{--- (1)}$$

Here, points L & M are midpoints of PQ and QR respectively

Then, we can write, $PL = LQ = QM = MR = QR/2$

Now, In ΔLPN and ΔMRN,

$$LP = MR$$

and also, $\angle LPN = \angle MRN$ ∵ by (1)

$$\angle QPR = \angle LPN \text{ and } \angle QRP = \angle MRN$$

Since point 'N' is midpoint of PR ⇒ $PN = NR$

Now, By side-angle-side congruence criterion,

$$\Delta LPN \cong \Delta MRN$$

But, the corresponding parts of congruent triangles are also equal.

⇒ $\boxed{LN = MN}$

Hence proved.

3.) In fig, PQRS is a square & SRT is an equilateral triangle. Prove that i) $PT = QT$ ii) $\angle TQR = 15^\circ$

→ Here, given that

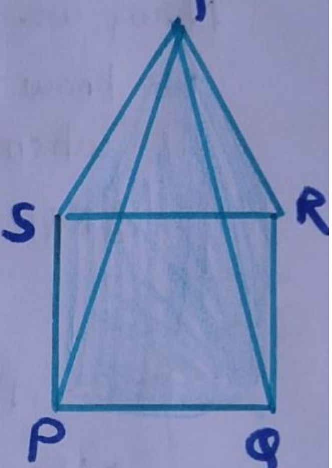
PQRS is a square & SRT is an equilateral triangle.

Now, from fig.

As PQRS is a square ⇒ $PQ = QR = RS = SP$

Also, $\angle SPQ = \angle QRS = \angle PQR = \angle RSP = 90^\circ$ — ①

since By the properties of square.



Also, from fig..

ΔSRT is an equilateral triangle.

⇒ $SR = RT = TS$

and $\angle TSR = \angle SRT = \angle RTS = 60^\circ$ — ②

since, By the properties of equilateral triangle.

from ① & ② ⇒ $PQ = QR = SP = SR = RT = TS$ — ③

But, from fig.

$\angle TSP = \angle TSR + \angle RSP = 60^\circ + 90^\circ = 150^\circ$

Also, $\angle TRQ = \angle TRS + \angle SRQ = 60^\circ + 90^\circ = 150^\circ$

⇒ $\angle TSP = \angle TRQ = 150^\circ$ — ④

Then, By Side-Angle-Side criterion

$\boxed{\Delta TSP \cong \Delta TRQ}$

But, we know that, the corresponding parts of congruent triangles are equal.

Thus, $\boxed{PT = QT}$

Hence proved.

Now, In ΔTQR,

$QR = TR$ ∵ by ③

$\Rightarrow \Delta TQR$ is an isosceles triangle.

$\angle QTR = \angle TQR$ (since angles opposite to equal sides are also equals.)

we know that, the sum of all three interior angles of a triangle is $180°$.

$\Rightarrow \angle QTR + \angle TQR + \angle TRQ = 180°$

$2\angle TQR + 150° = 180°$ by ④

$2\angle TQR = 30°$

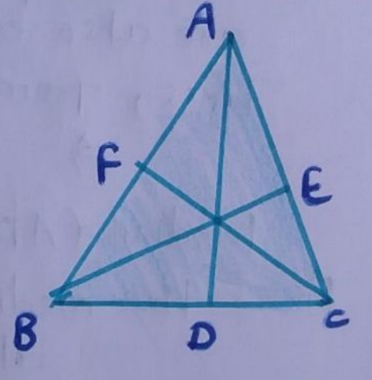
$\boxed{\angle TQR = 15°}$

Hence proved.

4) Prove that the medians of an equilateral triangle are equal.

→ Let us consider an equilateral triangle ΔABC as shown in fig.

Let D, E, F are the midpoints of sides BC, CA & AB respectively.



Then from fig, AD, BE and CF are medians of ΔABC.

Then, for midpoint D: $BD = DC$

for midpoint E: $CE = EA$

for midpoint F: $AF = FB$

As ΔABC is an equilateral triangle

$\Rightarrow AB = BC = CA$ —— ①

Also, $BD = DC = CE = EA = AF = FB$ —— ②

again $\angle ABC = \angle BCA = \angle CAB = 60°$ —— ③

since, By the properties of an equilateral triangle.

Let us consider ΔABD and ΔBCE,

$AB = BC$ by ①

$BD = CE$ by ②

$\angle ABD = \angle BCE$ by ③

Then, By Side-Angle- Side congruence criterion,

$\Delta ABD \cong \Delta BCE$

$\Rightarrow \boxed{AD = BE}$ ——— ④

Since, corresponding parts of an equilateral triangles are equal.

Now, In ΔBCE and ΔCAF,

$$\left.\begin{array}{l} BC = CA \\ \angle BCE = \angle CAF \\ CE = AF \end{array}\right\} \because \text{by } ①, ③ \,\&\, ②$$

Then, By Side-Angle - Side congruence relation,

$\Delta BCE \cong \Delta CAF$

$\Rightarrow \boxed{BE = CF}$ ——— ⑤

Since, corresponding parts of an equilateral triangles are also equal.

Thus, from ④ & ⑤

$\Rightarrow AD = BE = CF.$

Median $\cancel{(}$(AD) = Median (BE) = Median CF

Thus, the medians of an equilateral triangles are also equal.

Hence proved.

5.) In a ΔABC, if $\angle A = 120°$ & $AB = AC$. find $\angle B$ & $\angle C$.

→ Here, In ΔABC

$\angle A = 120°$ & $AB = AC$

Then ΔABC is an isosceles triangle.

$\Rightarrow \boxed{\angle B = \angle C}$ ——— ①

Since, angles opposite to equal sides are also equal.
But, the sum of all three interior angles of a triangle is 180°.

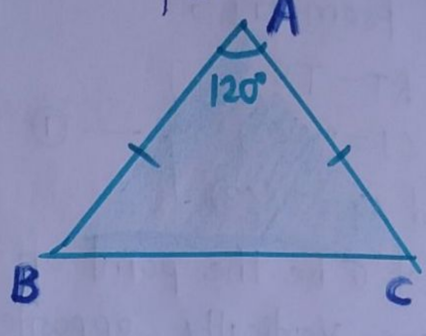
$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$

$\angle A + \angle B + \angle B = 180^\circ$ $\because$ by ①

$120^\circ + 2\angle B = 180^\circ$

$2\angle B = 60^\circ$

$\boxed{\angle B = 30^\circ}$



Thus, $\angle B = \angle C = 30^\circ$.

6.) In a ΔABC, if $AB = AC$ & $\angle B = 70^\circ$, find $\angle A$.

$\longrightarrow$ Given that, In ΔABC,

$AB = AC$ and $\angle B = 70^\circ$

But, angles opposite to equal sides are also equals.

$\Rightarrow \angle B = \angle C$

Thus, $\boxed{\angle B = \angle C = 70^\circ}$

Also, the sum of three angles of a triangle is 180°.

$\angle A + \angle B + \angle C = 180^\circ$

$\angle A + 70^\circ + 70^\circ = 180^\circ$

$\angle A = 180^\circ - 140^\circ$

$\boxed{\angle A = 40^\circ}$

Exercise 10.2

1.) In fig., it is given that $RT = TS$, $\angle 1 = 2\angle 2$ and $\angle 4 = 2(\angle 3)$.

Prove that $\Delta RBT \cong \Delta SAT$.

→ From fig.,

$$\left.\begin{aligned} RT &= TS \\ \angle 1 &= 2(\angle 2) \\ \text{and } \angle 4 &= 2(\angle 3) \end{aligned}\right\} \text{—— (1)}$$

Let 'O' be the point of intersection of RB and SA.

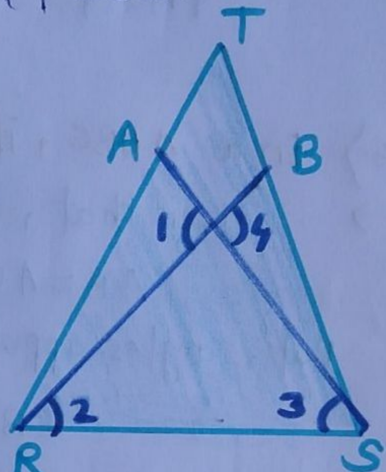
Here, Vertically opposite angles are $\angle AOR$ & $\angle BOS$.

$\Rightarrow \angle AOR = \angle BOS$

or $\angle 1 = \angle 4$

$\Rightarrow 2(\angle 2) = 2(\angle 3)$ ∵ from (1)

$\Rightarrow \boxed{\angle 2 = \angle 3}$ —— (2)



In ΔTRS, we have $RT = TS$

$\Rightarrow \Delta TRS$ is an isosceles triangle

$\Rightarrow \angle TRS = \angle TSR$ —— (3)

Since, angles opposite to equal sides are also equal.

$\Rightarrow \angle TRS = \angle TRB + \angle 2$ —— (4)

and $\angle TSR = \angle TSA + \angle 3$ —— (5)

from (3), (4) & (5) $\Rightarrow \angle TRB + \angle 2 = \angle TSA + \angle 3$

$\Rightarrow \angle TRB = \angle TSA$ from (2)

Now, consider, ΔRBT & ΔSAT

$\Rightarrow RT = ST$ by (1)

& $\angle TRB = \angle TSA$ by (2)

Then, by Angle-Side-Angle criterion of congruence

$\Rightarrow \boxed{\Delta RBT \cong \Delta SAT}$

Hence proved.

2.) Two lines AB and CD intersect at 'O' such that BC is equal & parallel to AD. Prove that the lines AB and CD bisect at 'O'.

→ Given that, two lines AB & CD intersect at 'O' so that BC is equal and parallel to AD.

From fig. lines AB and CD intersects at point 'O'.

$$\left.\begin{aligned} \Rightarrow\ & BC \parallel AD \\ \text{and}\ \ & BC = AD \end{aligned}\right\} \text{— ①}$$

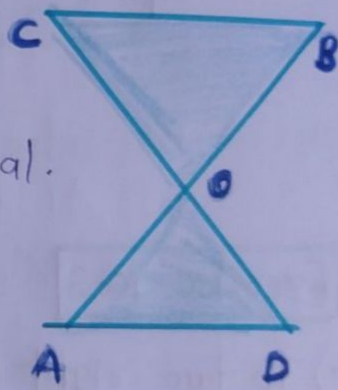
From fig, $AD \parallel BC$ & CD is a transversal.

$\Rightarrow \angle OCB = \angle ODA$

$AD = BC$ by ①

Again, $AD \parallel BC$ & AB is a transversal.

$\Rightarrow \angle OBC = \angle OAD$



Now, By angle-side-angle criterion,

$\Delta AOD \cong \Delta BOC$

Then, $OA = OB$ and $OD = OC$

Thus, AB and CD bisects each other at point 'O'.

Hence proved.

3.) BD and CE are bisectors of $\angle B$ and $\angle C$ of an isosceles ΔABC with $AB = AC$. Prove that $BD = CE$.

→ Here, given that

BD and CE are bisectors of $\angle B$ and $\angle C$ of an isosceles ΔABC with $AB = AC$.

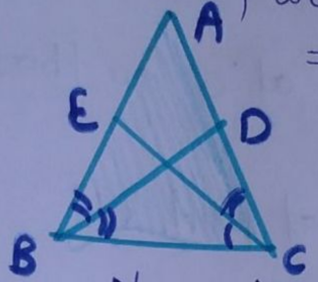
$\Rightarrow \angle ABC = \angle ACB$ —— ①

since, angles opposite to equal sides are also equal.

As BD & CE are bisectors of $\angle B$ & $\angle C$, we can write

$\angle ABD = \angle DBC = \angle BCE = \angle ECA = \angle B/2 = \angle C/2$ —— (2)

Now, we consider, $\Delta EBC = \Delta DCB$



$\Rightarrow \angle EBC = \angle DCB$ by (1)

$BC = BC$ since common side

$\angle BCE = \angle CBD$ ∵ by (2)

Now, by Angle-Side-Angle criterion of congruence,

$\boxed{\Delta EBC \cong \Delta DCB}$

Again, corresponding parts of congruent triangles are equal.

$\Rightarrow CE = BD$

or $\boxed{BD = CE}$

Hence proved.

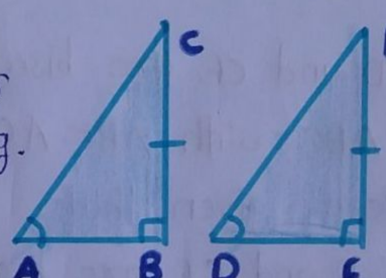
Exercise 10.3

1.) In two right triangles one side an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

→ Given that,

In two right triangles one side an acute angle of one are equal to the corresponding side and angle of the other.

Consider two right angle triangles ΔABC & ΔDEF as shown in fig.



$$\Rightarrow \left.\begin{array}{l}\angle B = \angle E = 90^\circ \\ AB = DE \\ \angle C = \angle F\end{array}\right\} \text{— (1)}$$

Here, two right angle triangles ΔABC & ΔDEF.

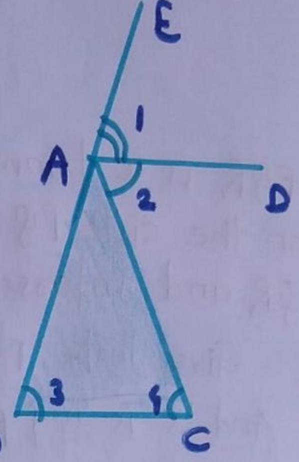
from (1), By Angle-Angle-Side congruence criterion,

$\boxed{\Delta ABC \cong \Delta DEF}$

Hence proved.

2.) If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

→ Let us consider a ΔABC such that AD is the angle bisector of exterior angle $\angle EAC$ and $AD \parallel BC$.



From fig,

$\angle 1 = \angle 2$ since AD is bisector of $\angle EAC$

$\angle 1 = \angle 3$ since corresponding angles

$\angle 2 = \angle 4$ since alternate angles

$\Rightarrow \angle 3 = \angle 4$

$\Rightarrow \boxed{AB = AC}$

As the two sides AB & AC of ΔABC are equal we can say that ΔABC is an isosceles triangle.

Hence proved.

3.) In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

→ Let us consider ΔABC which is isosceles triangle.

$\Rightarrow AB = AC$ and $\angle B = \angle C$

Given that, the vertex angle A is twice the sum of the base angles B and C.

$\Rightarrow \angle A = 2(\angle B + \angle C)$

$\Rightarrow \angle A = 2(2\angle B)$

$\angle A = 4(\angle B)$

But, the sum of all three angles of triangle is 180°.

$\angle A + \angle B + \angle C = 180°$

$4\angle B + \angle B + \angle B = 180°$

$6\angle B = 180°$

$\boxed{\angle B = 30°}$

But, $\angle B = \angle C$

$\Rightarrow \angle B = \angle C = 30^\circ$

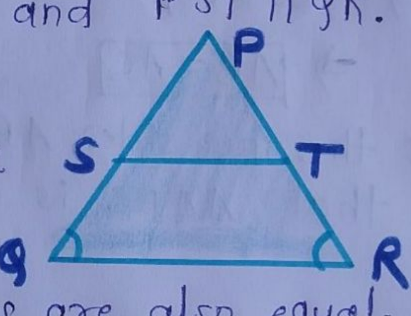
Also, $\angle A = 4\angle B = 120^\circ$

Thus, the angles of required triangle are 30°, 30° and 120° respectively.

4.) PQR is a triangle in which PQ = PR and is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT.

→ Given that, PQR is a triangle in which PQ = PR.

And 'S' is any point on the side PQ and $ST \parallel QR$.



From fig, $PQ = PR$

$\Rightarrow \Delta PQR$ is an isosceles triangle

$\Rightarrow \angle PQR = \angle PRQ$

since, angle opposite to equal sides are also equal.

Now, $\angle PST = \angle PQR$ and $\angle PTS = \angle PRQ$

$ST \parallel QR$ & corresponding angles formed are equal.

since, $\angle PQR = \angle PRQ$

$\angle PST = \angle PTS$

In ΔPST, $\angle PST = \angle PTS$

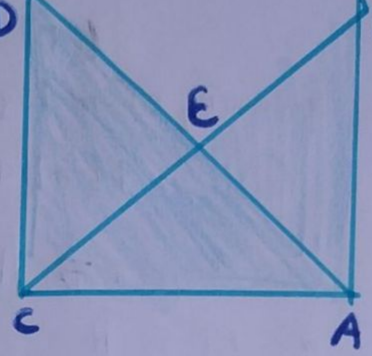
$\Rightarrow \Delta PST$ is also an isosceles triangle.

Thus, $\boxed{PS = PT}$

Hence Proved.

Exercise 10.4

1.) In fig, it is given that $AB = CD$ and $AD = BC$.
Prove that $\Delta ADC \cong \Delta CBA$.



→ From fig, we can write
$AB = CD$ and $AD = BC$
Let us consider ΔADC and ΔCBA.

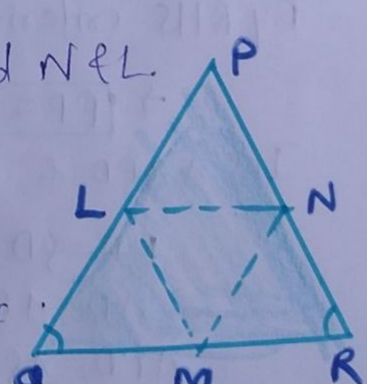
$$\Rightarrow \left.\begin{array}{l} AB = CD \\ BC = AD \end{array}\right\} \text{ since given}$$

And $AC = AC$ ∵ common side
Then, By Side-Side-Side criterion of congruence,

$\Rightarrow \boxed{\Delta ADC \cong \Delta ABC}$ Hence proved.

2.) In a ΔPQR, if $PQ = QR$ and L, M and N are the mid-points of the sides PQ, QR and RP respectively.
Prove that $LN = MN$.

→ Given that, In a ΔPQR, $PQ = QR$.
Also, L, M and N are mid-points of side PQ, QR & RP respectively.
In fig., we join L & M, M & N and N & L.



Then, $PL = LQ$
$QM = MR$
and $RN = NP$
Since, L, M & N are midpoints of sides PQ, QR and RP respectively.

$$\left.\begin{array}{l} \text{Also, } PQ = QR \\ \Rightarrow PL = LQ = QM = MR = PN = LR \end{array}\right\} \text{— (1)}$$

By mid-point theorem
$MN \parallel PQ$ and $MN = PQ/2$

$$MN = PL = LQ \text{ ——— } ②$$

Simillarly, we can write

$$LN \parallel QR \text{ and } LN = \frac{1}{2}(QR)$$

$$LN = QM = MR \text{ ——— } ③$$

from ①, ② and ③, we have

$$PL = LQ = QM = MR = MN = LN$$

$$\Rightarrow \boxed{LN = MN}$$

Hence proved.

Exercise 10.5

1.) ABC is a triangle and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.

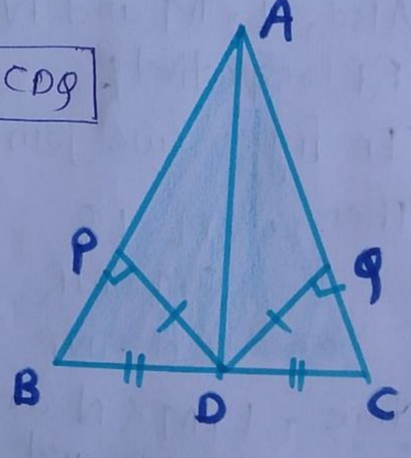
→ Given that, D is the midpoint of BC & PD = DQ in ΔABC.

from fig, In ΔBDP & ΔCDQ,

$$\left.\begin{array}{l} PD = QD \\ BD = DC \\ \angle BPD = \angle CQD = 90^\circ \end{array}\right\} \text{(D is mid-point)}$$

By RHS criterion $\Rightarrow \boxed{\Delta BDP \cong \Delta CDQ}$

$$\Rightarrow \boxed{BP = CQ} \text{ ——— } ①$$



In ΔAPD & ΔAQD

$$\left.\begin{array}{l} PD = QD \\ AD = AD \\ APD = AQD = 90^\circ \end{array}\right\}$$

Then by RHS criterion $\Rightarrow \Delta APD \cong \Delta AQD$

$$\Rightarrow \boxed{PA = QA} \text{ ——— } ②$$

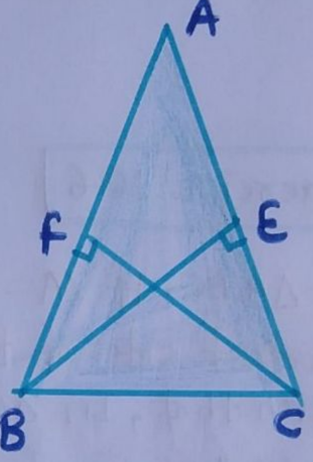
$$① + ② \Rightarrow \quad BP + PA = CQ + QA$$

$$\Rightarrow \boxed{AB = AC}$$

As the two sides of a triangle are equal &
hence ΔABC is an isosceles triangle.

2) ABC is a triangle in which BE and CF are respectively, the perpendiculars to the sides AC and AB. If $BE = CF$, prove that ΔABC is isosceles.

→ Given that,
ABC is a triangle in which BE and CF are perpendiculars to the sides AC and AB respectively.



From fig. In ΔBCF & ΔCBE,

$\angle BFC = \angle CEB = 90°$

$BC = CB$ ∵ common side

also, $CF = BE$

Then, by RHS congruence criterion,

~~AAP~~ $\boxed{\Delta BFC \cong \Delta CEB}$

$\Rightarrow \angle FBC = \angle EBC$

$\Rightarrow \boxed{\angle ABC = \angle ACB}$

The sides opposite to equal angles are also equal.

$\Rightarrow$ Two sides of ΔABC are equal.

Thus, ΔABC is an equilateral triangle.

Hence proved.

3) If perpendiculars from any point within an angle on its arms are congruent. Prove that it lies on the bisector of that angle.

→ Let us consider $\angle ABC$ & BP is one arm ~~of $\angle A$~~ within the angle as shown in fig.

we drawn perpendiculars PN and PM on the arms BC and BA respectively.

Now, In ΔBPM & ΔBPN

$\angle BMP = \angle BNP = 90^\circ$

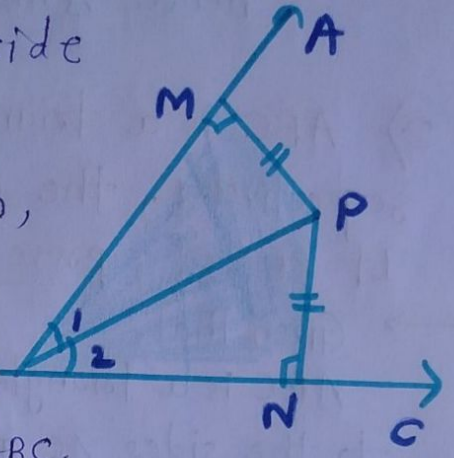
$BP = BP$ ∵ common side

$NP = MP$

Then, By RHS congruence criterion,

$\Delta BPM \cong \Delta BPN$

$\Rightarrow \boxed{\angle MBP = \angle NBP}$



Thus ~~As~~, BP is the angle bisector of $\angle ABC$.

Hence proved.

Exercise 10.6

1.) In ΔABC, if $\angle A = 40^\circ$ and $\angle B = 60^\circ$. Determine the longest & shortest sides of the triangle.

→ Given that, In ΔABC, $\angle A = 40^\circ$ & $\angle B = 60^\circ$

But, the sum of all three angles of a triangle is 180°.

$\angle A + \angle B + \angle C = 180^\circ$

$40^\circ + 60^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 100^\circ = 80^\circ$

$\boxed{\angle C = 80^\circ}$

Here $80^\circ > 60^\circ > 40^\circ \Rightarrow \angle C > \angle B > \angle A$

Thus, $\angle C$ is a greater angle & $\angle A$ is smaller angle.

Now, $\angle A < (\angle B) < (\angle C)$

Thus, the side opposite to greater angle is larger & side opposite to smaller angle is smaller.

Thus, $BC < AC < AB$

Hence, AB is the longest and BC is the shortest side.

2.) In a ΔABC, if $\angle B = \angle C = 45^\circ$, which is the longest side?

→ Given that, In ΔABC $\angle B = \angle C = 45^\circ$.

But, we know that

The sum of all three angles of a triangle is 180°.

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 45^\circ + 45^\circ = 180^\circ$$

$$\angle A = 180^\circ - 90^\circ$$

$$\boxed{\angle A = 90^\circ}$$

$$\Rightarrow \angle B = \angle C < \angle A$$

Thus, BC is the longest side here in ΔABC.

3.) In a ΔABC, side AB is produced to D so that $BD = BC$. If $\angle B = 60^\circ$ and $\angle A = 70^\circ$. Prove that : i) $AD > CD$ ii) $AD > AC$

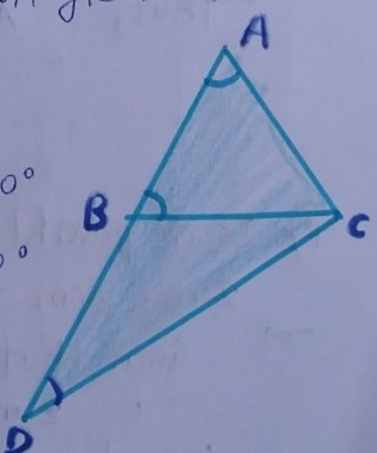
→ Given that, In ΔABC

Side AB is produced to D so that $BD = BC$.

Also, $\angle B = 60^\circ$ and $\angle A = 70^\circ$

Here, we are joining point C and D.

But, the sum of all three triangles of a triangle is 180°.



$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$70^\circ + \angle B + \angle C = 180^\circ$$

$$70^\circ + 60^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 130^\circ = 50^\circ$$

$$\boxed{\angle C = 50^\circ}$$

$$\angle ACB = 50^\circ \text{ ——— } (1)$$

And also, In ΔBDC

$$\angle DBC = 180^\circ - \angle ABC$$

$$\angle DBC = 180^\circ - 60^\circ = 120^\circ$$

$\angle DBA$ is a straight line here

and $BD = BC$

The angles opposite to equal sides are also equal.

$$\Rightarrow \angle BCD = \angle BDC$$

$$\angle DBC + \angle BCD + \angle BDC = 180^\circ$$

(Since, the sum of all three angles of a triangle is 180°)

$$120^\circ + \angle BCD + \angle BCD = 180^\circ$$

$$120^\circ + 2\angle BCD = 180^\circ$$

$$2\angle BCD = 180^\circ - 120^\circ = 60^\circ$$

$$\boxed{\angle BCD = 30^\circ}$$

$$\angle BCD = \angle BDC = 30^\circ \text{ --- } ②$$

Now, In ΔADC, $\angle DAC = 70^\circ$

$\angle ADC = 30^\circ$ by ②

$\Rightarrow \angle ACD = \angle ACB + \angle BCD = 50^\circ + 30^\circ = 80^\circ$ from ① & ②

Now, $(\angle ADC) < (\angle DAC) < (\angle ACD)$

$$AC < DC < AD$$

Since, side opposite to greater angle is longer and the side opposite to smaller angle is shorter.

Thus, $AD > CD$ and $AD > AC$

Hence proved.

4.) Is it possible to draw a triangle with sides of length 2cm, 3cm and 7cm?

→ Given that, the length of sides of a triangle be 2cm, 3cm and 7cm.

we already know that, A triangle can be drawn only when the sum of any two sides is greater than the third side.

Now, we will check the sides according to rule.

$2+3 \not> 7$ or $2+3 < 7$

$2+7 > 3$ and $3+7 > 2$

Here $2+3 \not> 7$

and hence the triangle does not exist.

Chapter 11. Co-ordinate Geometry

Exercise 11.1

1) Plot the following points on the graph paper:

i) (2,5) ii) (4,-3) iii) (-5,-7) iv) (7,-4) v) (-3,2) vi) (7,0)

vii) (-4,0) viii) (0,7) ix) (0,-4) x) (0,0)

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