Chapter 9. Triangle & it's Angles

Exercise 9.1

1.) In ΔABC, if $\angle A = 55^\circ$, $\angle B = 40^\circ$ find $\angle C$.

→ Given that, $\angle A = 55^\circ$, $\angle B = 40^\circ$

But, the sum of all angles of a triangle is 180°.

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$55^\circ + 40^\circ + \angle C = 180^\circ$$

$$95^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 95^\circ$$

$$\boxed{\angle C = 85^\circ}$$

2.) If the angles of a triangle are in the ratio $1:2:3$, determine three angles.

→ Here, given that the angles of a triangle are in the ratio $1:2:3$.

Let us consider these angles be x, $2x$ & $3x$ respectively.

Then, $\angle x + 2x + 3x = 180^\circ$

$$6x = 180^\circ$$

$$\boxed{x = 30^\circ}$$

$2x = 30(2) = 60^\circ$

& $3x = 3(30) = 90^\circ$

The required angles are 30°, 60°, 90° respectively.

3) The angles of a triangle are $(x-40)^\circ$, $(x-20)^\circ$ & $(\frac{1}{2}x-10)^\circ$. find the value of x.

→ Given that, the three angles of a triangle are $(x-40)^\circ$, $(x-20)^\circ$ & $(\frac{1}{2}x-10)^\circ$

But, the sum of all angles of triangle is 180°.

$$(x-40)^\circ + (x-20)^\circ + (\tfrac{1}{2}x-10)^\circ = 180^\circ$$

$$\frac{5}{2}x - 70^\circ = 180^\circ$$

$$\frac{5}{2}x = 180^\circ + 70^\circ$$

$$\frac{5}{2}x = 250^\circ$$

$$5x = 500^\circ$$

$$\boxed{x = 100^\circ}$$

4) The angles of a triangle are arranged in ascending order of magnitude. If the difference between the two consecutive angles is 10°, find the three angles.

→ Given that, the difference betn the two consecutive angles is 10°.

Let x, $x+10^\circ$ & $x+20^\circ$ be the required consecutive angles.

Then,
$$x + x + 10^\circ + x + 20^\circ = 180^\circ$$

$$3x + 30^\circ = 180^\circ$$

$$3x = 180^\circ - 30^\circ$$

$$3x = 150^\circ$$

$$\boxed{x = 50^\circ}$$

Thus,
$$x = 50^\circ$$

$$x + 10^\circ = 50^\circ + 10^\circ = 60^\circ$$

$$x + 20^\circ = 50^\circ + 20^\circ = 70^\circ.$$

5) Two angles of a triangle are equal & the third angle is greater than each of those angles by $30°$. Determine the angles of the triangle.

→ Here, given that

Two angles of a triangle are equal & the third angle is greater than each of those angle by $30°$.

Then, if one angle is x then the remaining angles are x, $x+30°$ respectively.

But, In a triangle the sum of all three angles is $180°$.

$$\Rightarrow x + x + x + 30° = 180°$$

$$3x + 30° = 180°$$

$$3x = 150°$$

$$\boxed{x = 50°}$$

$$x + 30° = 50° + 30° = 80°$$

Then remaining angles are $x = 50°$, $x = 50°$, $80°$ respectively.

6) If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right angle triangle.

→ Given that, one angle of a triangle is equal to the sum of the other two angles.

If one angle of a triangle is $x°$ then other two may be $y°$ & $z°$ respectively.

from given condition,

$$z = x + y \quad \text{—— ①}$$

But, sum of all three angles of a triangle is $180°$.

$$x + y + z = 180°$$

$$z = 180° - x - y \quad \text{—— ②}$$

from ① & ②

$$x+y = 180^\circ - x - y$$

$$2x+2y = 180^\circ$$

$$\boxed{x+y = 90^\circ}$$

$$x+y+z = 180^\circ$$

$$90^\circ + z = 180^\circ$$

$$z = 180^\circ - 90^\circ$$

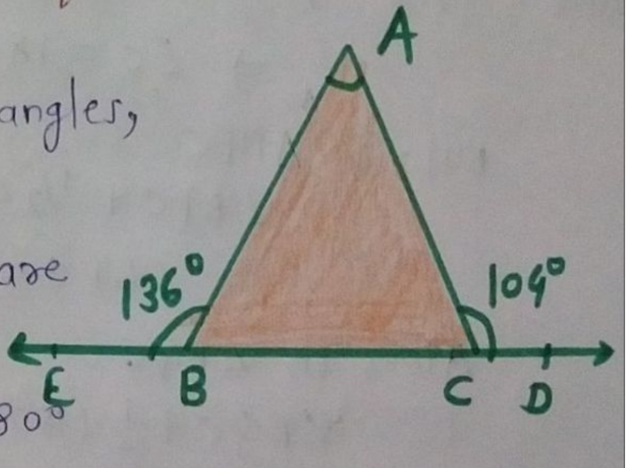
$$\boxed{z = 90^\circ}$$

Thus, the required angle is a right angle triangle.

Hence proved.

Exercise 9.2

1.) The exterior angles, obtained on producing the base of a triangle both ways are 104° & 136°. find the angles of the triangle.



→ By the property of exterior angles,

$$\angle ACD = \angle ABC + \angle BAC$$

Here, angles $\angle ABC$ & $\angle ABE$ are linear pair angles.

$$\Rightarrow \angle ABC + \angle ABE = 180^\circ$$

$$\angle ABC + 136^\circ = 180^\circ$$

$$\boxed{\angle ABC = 44^\circ}$$

Again, angles $\angle ACB$ & $\angle ACD$ are linear pair angles.

$$\angle ACB + \angle ACD = 180^\circ$$

$$\angle ACB + 104^\circ = 180^\circ$$

$$\boxed{\angle ACB = 76^\circ}$$

But, the sum of all three angles of a triangle is 180°.

$\angle A + 44° + 76° = 180°$

$\angle A° = 180° - 44° - 76°$

$\boxed{\angle A = 60°}$

Thus, the required angles are $\angle A = 60°$, $\angle B = 44°$ & $\angle C = 76°$.

2.) In a ΔABC, the internal bisectors of $\angle B$ & $\angle C$ meet at P and the external bisectors of $\angle B$ & $\angle C$ meet at Q. Prove that $\angle BPC + \angle BQC = 180°$.

→ Given that, In ΔABC,

BP & CP are internal bisectors of $\angle B$ & $\angle C$ respectively.

$\Rightarrow \angle B = 180° - \angle B$

And BQ & CQ are external bisectors of $\angle B$ & $\angle C$ respectively.

$\Rightarrow \angle C = 180° - \angle C$

But, in ΔBPC

$\angle BPC + 1/2 \angle B + 1/2 \angle C = 180°$

$\angle BPC = 180° - \frac{1}{2}(\angle B + \angle C)$ —— ①

Also, In ΔBQC

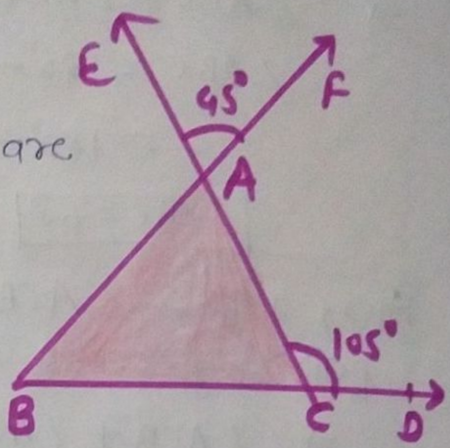
$\angle BQC + \frac{1}{2}(180° - \angle B) + \frac{1}{2}(180° - \angle C) = 180°$

$\angle BQC + 180° - 1/2(\angle B + \angle C) = 180°$

$\boxed{\angle BPC + \angle BQC = 180°}$ by ①

Hence proved

3) In fig, the sides BC, CA & AB of a ΔABC have been produced to D, E and F respectively. If $\angle ACD = 105°$ & $\angle AEAF = 45°$, find all the angles of the ΔABC.



→ $\angle BAC = \angle EAF = 45°$

→ since angle $\angle BAC$ & $\angle EAF$ are vertically opposite angles.

$\angle ACD = 180° - 105° = 75°$

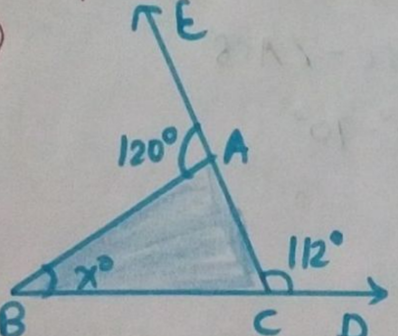
since, linear pair angles.

$\angle ABC = 105° - 45° = 60°$

Since, by the exterior angle property.

4) Compute the value of 'x' in each of the following fig.

i)



from fig,

$\angle EAB$ & $\angle BAC$ are linear pair angles.

→ $\angle BAC = 180° - 120° = 60°$

Also, angle $\angle ACB$ & $\angle ACD$ are linear pair angles.

→ $\angle ACB = 180° - 112° = 68°$

But, sum of all three angles of a triangle is 180°.

$$x = 180° - \angle BAC - \angle ACB$$

$$= 180° - 60° - 68°$$

$$\boxed{x = 52°}$$

ii) from fig.

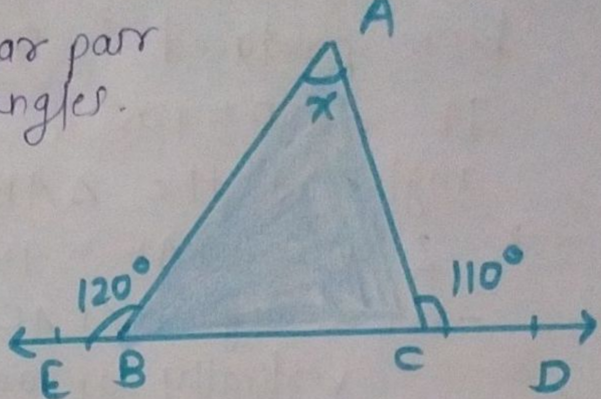
Angles $\angle ABC$ & $\angle ABE$ are linear pair angles.

$\Rightarrow \angle ABC + \angle ABE = 180^\circ$

$\angle ABC = 180^\circ - \angle ABE$

$= 180^\circ - 120^\circ$

$\boxed{\angle ABC = 60^\circ}$



Also, angles $\angle ACB$ & $\angle ACD$ are linear pair angles.

$\Rightarrow \angle ACB + \angle ACD = 180^\circ$

$\angle ACB = 180^\circ - \angle ACD$

$= 180^\circ - 110^\circ$

$\boxed{\angle ACB = 70^\circ}$

But, the sum of all three angles of a triangle is 180°.

$x = \angle BAC = 180^\circ - \angle ABC - \angle ACB$

$x = 180^\circ - 60^\circ - 70^\circ$

$\boxed{x = 50^\circ}$

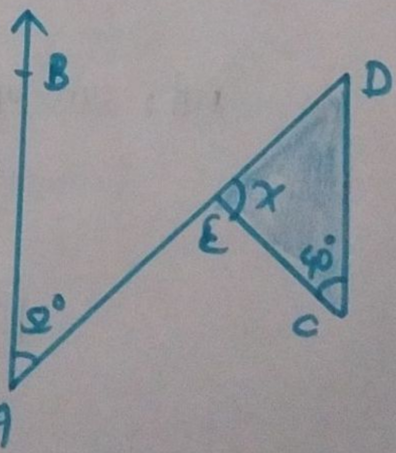
iii) from fig. angles $\angle BAE$ & $\angle EDC$ are alternate angles.

$\therefore \angle BAE = \angle EDC = 52^\circ$

But, the sum of all three angles of a triangle is 180°.

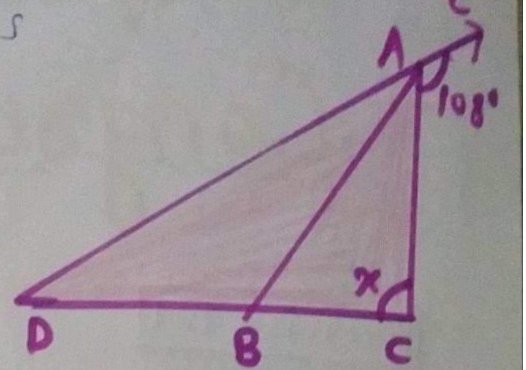
$x = 180^\circ - 40^\circ - 52^\circ$

$\boxed{x = 88^\circ}$



5.) In fig, AB divides $\angle DAC$ in the ratio 1:3 & AB = DB. determine the value of x.

→ Here from fig, we can write as



$$\angle DAC = 180° - 108° = 72°$$

Given that, $\dfrac{\angle BAC}{\angle DAB} = \dfrac{1}{3}$

$$\Rightarrow \boxed{\angle DAB = 3\angle BAC}$$

Also, $\angle BAC + \angle DAB = \angle DAC = 72°$

$$\Rightarrow \angle BAC + 3\angle BAC = 72°$$

$$4\angle BAC = 72°$$

$$\angle BAC = 72/4 = 18°$$

$$\boxed{\angle BAC = 18°}$$

Thus, $\angle DAB = 3(18) = 54°$

$\angle DAB = \angle BDA = 54°$ as AB = DB

Then, $\angle ABD = 180° - (108°)$

$$\boxed{\angle ABD = 72°}$$

finally, $\angle DBA = 72° = \angle BAC + \angle x$
(since exterior angles)

Then,
$$x = 72° - 18° = 54°$$

$$\boxed{x = 54°}$$