Chapter 8 : Lines & Angles

Exercise 8.1

1.) Write the complement of each of the following angles:

i) 20°

→ We know that, the sum of an angle & it's complement is 90°.

Thus, the complement of $20^\circ = 90^\circ - 20^\circ = 70^\circ$.

ii) 35°

→ The sum of an angle & it's complement is 90°.

Thus, the complement of $35^\circ = 90^\circ - 20^\circ = 55^\circ$

iii) 90°

→ The sum of an angle & it's complement is 90°.

Thus, the complement of $90^\circ = 90^\circ - 90^\circ = 0^\circ$.

iv) 77°

→ The sum of an angle & it's complement is 77°.

Thus, the complement of $77^\circ = 90^\circ - 77^\circ = 13^\circ$

v) 30°

→ The sum of an angle & it's complement is 30°.

Thus, the complement of $30^\circ = 90^\circ - 30^\circ = 60^\circ$.

2.) Write the supplement of each of the following angles:

i) 54°

→ The sum of an angle & it's supplement is $= 180^\circ$

Thus, the supplement of angle $54^\circ = 180^\circ - 54^\circ = 126^\circ$

ii) 132°

→ The sum of an angle & it's supplement $= 180^\circ$

Thus, the supplement of angle $132^\circ = 180^\circ - 132^\circ = 48^\circ$.

iii) 138°

→ The sum of an angle & it's supplement $= 180^\circ$

Thus, the supplement of angle $138^\circ = 180^\circ - 138^\circ = 42^\circ$

3.) If an angle is 28° less than its complement, find its measure?

→ Let us consider the measure of angle is 'x°'

Then, it's complement will be $(90-x)^\circ$.

Thus, the required angle = complement of $x - 28^\circ$

$$x = (90-x) - 28$$

$$2x = 9 - 28$$

$$2x = 62$$

$$\boxed{x = 31^\circ}$$

Hence, the measure of required angle is 31°.

4.) If an angle is 30° more than one half of it's complement, find the measure of the angle?

→ Let us consider the measure of required angle is x°.

The complement of $x^\circ = (90-x)^\circ$

Hence, Required angle $= 30^\circ +$ complement/2

$$x = 30^\circ + (90-x)/2$$

$$x + x/2 = 30^\circ + 45^\circ$$

$$3x/2 = 75^\circ$$

$$\boxed{x^\circ = 50^\circ}$$

Hence, the measure of required angle is 50°.

5) Two supplementary angles are in the ratio 4:5. find the angles?

→ Given that, Two supplementary angles are in the ratio 4:5.

Let us consider, the angles are $4x$ & $5x$ (in degrees).

As the angles are supplementary angles.

$$\Rightarrow \quad 4x + 5x = 180^\circ$$
$$9x = 180^\circ$$
$$\boxed{x = 20^\circ}$$

Thus, $4x = 4(20)^\circ = 80^\circ$ & $5x = 5(20)^\circ = 100^\circ$

Hence, the required angles are 80° & 100° respectively.

6) Two supplementary angles differ by 48°. find the angles?

→ Given that, two supplementary angles differ by 48°.

Let us consider, x° be one angle then it's supplementary angle will be equal to $(180 - x)^\circ$.

From given condition,

$$(180 - x) - x = 48$$
$$180 - 2x = 48$$
$$180 - 48 = 2x$$
$$x = 132/2 \quad \boxed{x = 66^\circ} \quad \Rightarrow 180 - x = 180 - 66 = 144^\circ.$$

Hence, The required angles are 66° & 114°.

7.) An angle is equal to 8 times its complement. Determine its measure?

→ Given that, Required angle = 8 times of it's complement

Let us consider x° be the one angle then it's complementary angle will be equal to $(90 - x)^\circ$.

From given condition,

Required angle = 8 (complement of $x°$)

$$x° = 8\,(90-x)$$

$$x = 8\,(90-x)$$

$$x + 8x = 720$$

$$9x = 720$$

$$\boxed{x = 80°}$$

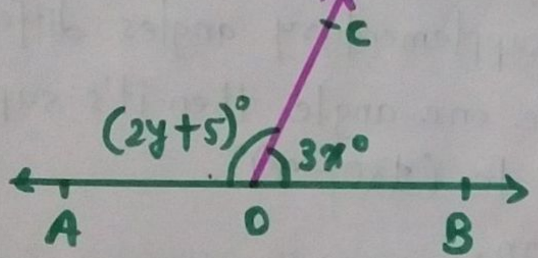
Thus, the required angle is 80°.

Exercise 8.2

1.) In the below fig, OA & OB are opposite rays:

i) if $x = 25°$, what is the value of y?

ii) if $y = 35°$, what is the value of x?



i) Given that $x = 25°$

from fig, $\angle AOC$ & $\angle BOC$ are linear pair angles.

$\Rightarrow \quad \angle AOC + \angle BOC = 180°$

Again, from fig. $\angle AOC = 2y+5$ & $\angle BOC = 3x$

$$\angle AOC + \angle BOC = 180°$$

$$(2y+5) + 3x = 180$$

$$2y + 5 + 3(25) = 180$$

$$2y + 5 + 75 = 180$$

$$2y + 80 = 180$$

$$2y = 100$$

$$\boxed{y = 50°}$$

ii) Given that, $y = 35°$

from fig., $\angle AOC$ & $\angle BOC$ are linear pair angles.

$$\Rightarrow \angle AOC + \angle BOC = 180°$$

$$(2y+5) + 3x = 180$$

$$2(35)+5 + 3x = 180$$

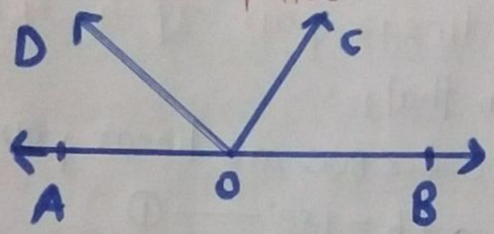
$$75 + 3x = 180$$

$$3x = 105$$

$$\boxed{x = 35°}$$

2.) In the below fig, write all pairs of adjacent angles & all the linear pairs.

→



from fig, pairs of adjacent angles are found to be:

($\angle AOC$ & $\angle COB$), ($\angle AOD$ & $\angle COD$), ($\angle AOD$ & $\angle BOD$), ($\angle BOC$ & $\angle COD$).

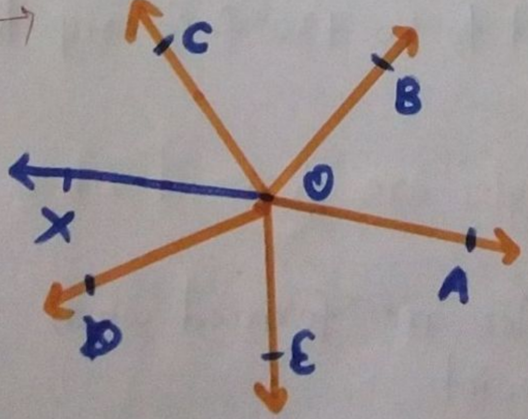
And the linear pairs of angles are found to be:

($\angle AOD$ & $\angle BOD$), ($\angle AOC$ & $\angle BOC$)

Since, ($\angle AOD + \angle BOD = 180°$) & ($\angle AOC + \angle BOC = 180°$)

3.) In fig, rays OA, OB, OC, OD & OE have the common end point 'O'. show that $\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$.

→



from fig.

$\angle AOB$ & $\angle BOX$ are linear pair angles.

Since Rays OA, OB, OC, OD & OE have the common end point 'O'.

we drawn here, an opposite ray OX to a ray OA which makes a straight line AX.

Thus, $\angle AOB + \angle BOX = 180^\circ$

(or) $\angle AOB + \angle BOC + \angle COX = 180^\circ$ —— ①

Also, $\angle AOE$ & $\angle EOX$ are also linear pair angles.

$\angle AOE + \angle EOX = 180^\circ$

(or) $\angle AOE + \angle DOE + \angle DOX = 180^\circ$ —— ②

equⁿ ① + ② ⇒

$\angle AOB + \angle BOC + \angle COF + \angle AOE + \angle DOE + \angle DOX$

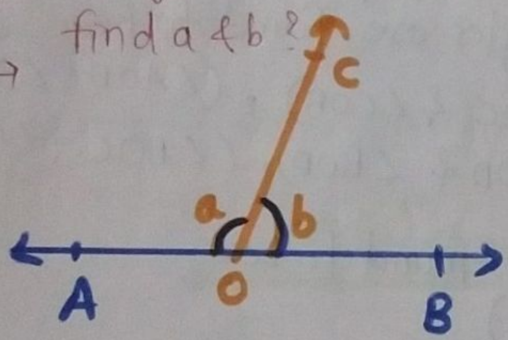
$= 180^\circ + 180^\circ$

$\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360^\circ$

Hence proved.

5.) In fig, $\angle AOC$ & $\angle BOC$ forms a linear pair. If $a - 2b = 30^\circ$, find a & b ?

→



Given that,

$\angle AOC$ & $\angle BOC$ are linear pair angles.

$a + b = 180^\circ$ —— ①

$a - 2b = 30^\circ$ —— ②

equⁿ ① − ② ⇒ $a + b - a + 2b = 180 - 30$

$3b = 150$

$b = 150/3 \Rightarrow \boxed{b = 50^\circ}$

Since, $a - 2b = 30^\circ$

$a - 2(50) = 30^\circ$

$a = 30 + 100 \Rightarrow \boxed{a = 130^\circ}$

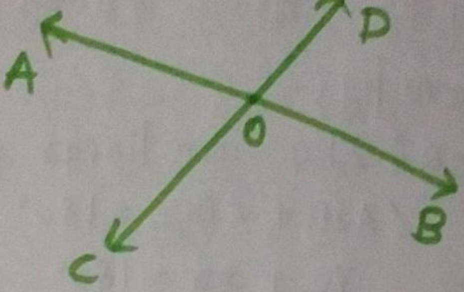
Hence, the values of a & b found to be 130° & 50° respectively.

6.) How many pairs of adjacent angles are formed when two lines intersects at a point ?

→ Four pairs of adjacent angles are formed when two lines intersects at a point.

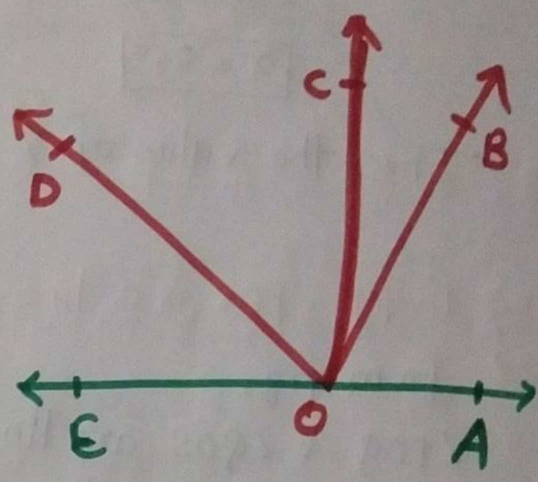
Let us consider, the two lines AB & CD intersects each other at point O.

Then, the four pair of adjacent angles are:
(∠AOD & ∠DOB), (∠DOB & ∠BOC)
(∠COA & ∠AOD), (∠BOC & ∠COA).



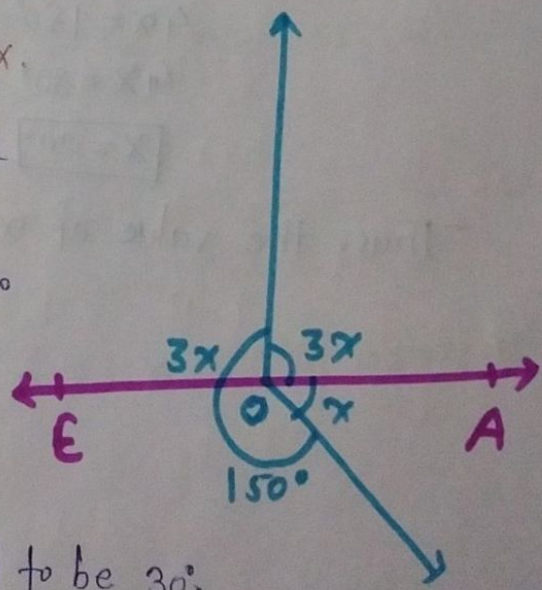
7.) How many pairs of adjacent angles, in all, can you name in figure given?

→ In given fig.
The no. of pairs of adjacent angles formed are:
(∠EOC & ∠DOC), (∠EOD & ∠DOB)
(∠DOC & ∠COB), (∠EOD & ∠DOA)
(∠BOC & ∠BOA), (∠BOA & ∠BOD)
(∠BOA & ∠BOE), (∠EOC & ∠COA)
(∠EOC & ∠COB).
Thus, total 10 pairs of adjacent angles are formed.



8.) In fig. determine the value of x.

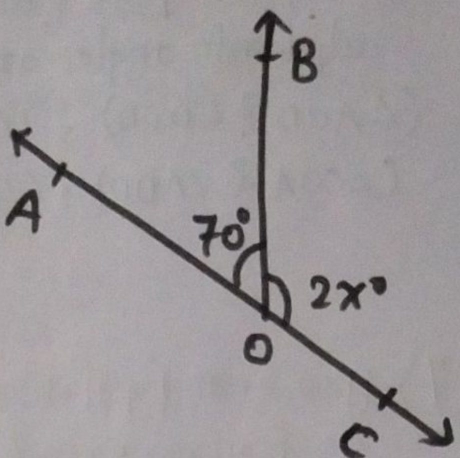
→ The sum of all angles around a point 'O' is equal to 360°.
Thus, $3x + 3x + 150 + x = 360°$
$7x = 360° - 150°$
$7x = 210°$
$\boxed{x = 30°}$
Hence, value of x is found to be 30°.



9.) In fig., AOC is a line, find x.

→ from fig,

$\angle AOB$ & $\angle BOC$ are linear pairs angles.



$$\angle AOB + \angle BOC = 180°$$

$$70 + 2x = 180°$$

$$2x = 180° - 70°$$

$$2x = 110°$$

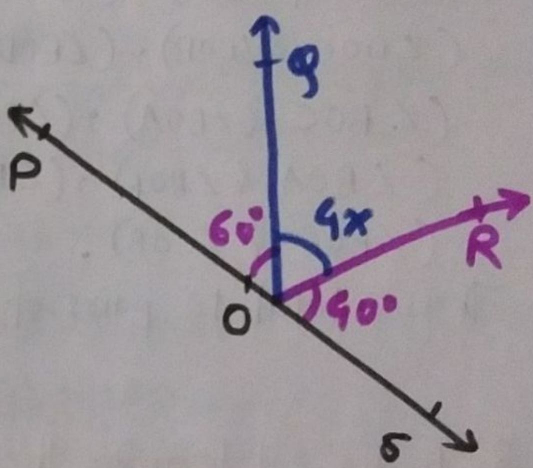
$$\boxed{x = 55°}$$

Thus, the value of x is found to be 55°.

10.) In fig., POS is a line, find x.

→ from fig,

$\angle POQ$ & $\angle QOS$ are linear pairs.



Thus, $\angle POQ + \angle QOS = 180°$

$$\angle POQ + \angle QOR + \angle SOR = 180°$$

$$60° + 4x + 40° = 180°$$

$$4x = 180° - 100°$$

$$4x = 80°$$

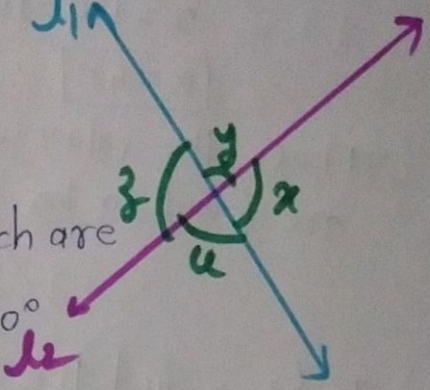
$$\boxed{x = 20°}$$

Thus, the value of x is found to be 20°.

Exercise 8.3

1.) In fig, lines l_1 & l_2 intersect at O, forming angles as shown in the fig. If $x = 45$. Find the values of y, z & u

→ Given that, $x = 45^\circ$



As vertically opposite angles are equal. $\Rightarrow \boxed{z = x = 45^\circ}$

Here, z & u are the angles which are linear pair angles. $\therefore z + u = 180^\circ$

$u = 180^\circ - z$

$= 180^\circ - 45^\circ$

$\boxed{u = 135^\circ}$

Also, x & y are the linear pair angles.

$\therefore x + y = 180^\circ$

$y = 180^\circ - x$

$y = 180 - 45$

$\boxed{y = 135^\circ}$

Thus, the remaining angles found to be $y = 135^\circ$, $u = 135^\circ$, $z = 45^\circ$

2.) In fig. three coplanar lines intersect at a point 'O', forming angles as shown in fig. Find the values of x, y, z & u.

→ Here, ($\angle BOZ$ & z) & ($\angle DOF$ & y) are the pairs of vertically opposite angles.

Hence, $\angle BOD = z = 90^\circ$

$\angle DOF = y = 50^\circ$

Since, vertically opposite angles are equal.

Also, angles x, y & z are linear pair angles.

$\Rightarrow x + y + z = 180$

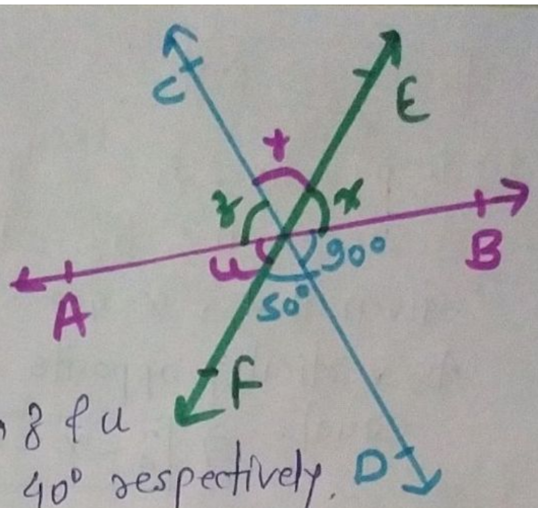
Since AB is a straight line.

$\Rightarrow x + y + z = 180°$

$x + 50 + 90 = 180$

$x = 180 - 140$

$\boxed{x = 40°}$



Thus, the values of angles x, y, z & u are found to be $40°, 50°, 90°$ & $40°$ respectively.

3.) In fig., find the values of x, y & z.

→ From fig, $y = 25°$

Since, vertically opposite angles are equal.

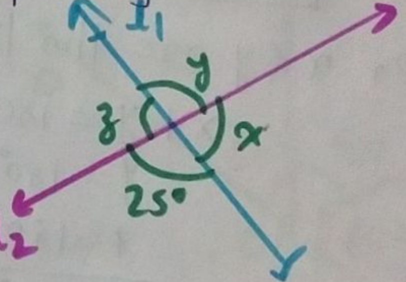
Again, angles x & y are linear pair angles.

$\Rightarrow x + y = 180°$

$x = 180 - y$

$x = 180 - 25$

$\boxed{x = 155°}$

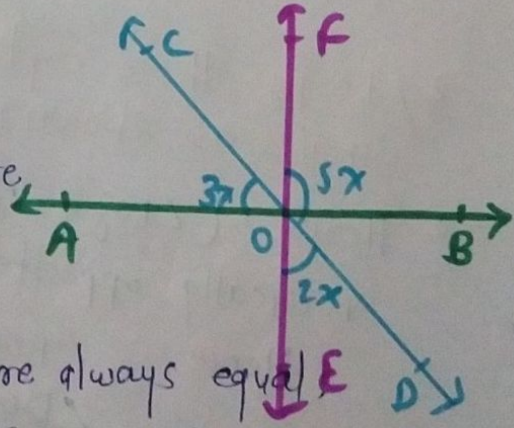


Also, $x = z = 155°$

Since, vertically opposite angles are always equal.

Thus, here $\boxed{y = 25°}$ & $\boxed{z = 155°}$

4.) In fig. find the value of x.



→ From fig, angles $\angle AOE$ & $\angle BOF$ are vertically opposite angles.

$\Rightarrow \angle AOE = \angle BOF = 5x$

Since, vertically opposite angles are always equal.

Also, ~~$\angle AO$~~ $\angle COA + \angle AOE + \angle EOD = 180°$

Since, these angles are linear pair angles.

$\Rightarrow 3x + 5x + 2x = 180$

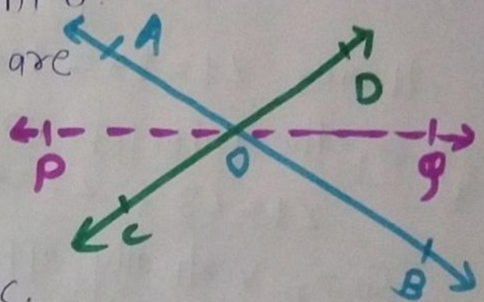
$$10x = 180$$
$$x = 180/10$$
$$\boxed{x = 18^\circ}$$

Thus, here angle $x = 18^\circ$.

6) If two straight lines intersects each other, prove that the ray opposite to the bisector of one of the angles thus formed bisects vertically opposite angle.

→ from fig,

AB, CD & PQ are the straight lines which intersects each other in point 'O'.



Here, the vertically opposite angles are

$$\angle AOP = \angle BOQ$$
$$\& \quad \angle COP = \angle DOQ$$

Again OP is the bisector of $\angle AOC$.

$$\Rightarrow \angle AOP = \angle COP$$

Thus, $\boxed{\angle BOQ = \angle DOQ}$

Hence, we can say that OQ is the bisector of $\angle BOD$.

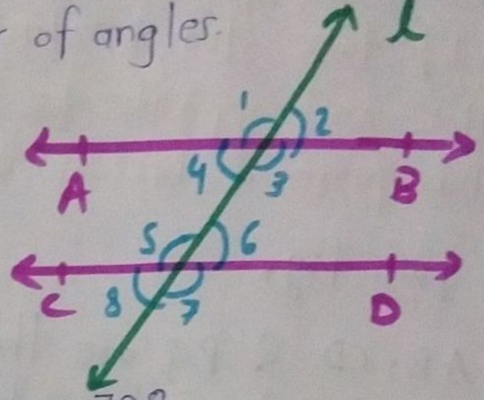
Exercise 8.4

1.) In fig. AB, CD & $\angle 1$ & $\angle 2$ are in the ratio 3:2.
Find the all angles 1 to 8.

→ Let us consider,

$\angle 1 = 3x$ & $\angle 2 = 2x$

But, here $\angle 1$ & $\angle 2$ are linear pair of angles.



$\cancel{3x+} \Rightarrow \angle 1 + \angle 2 = 180$

$\angle 1 + \angle 2 = 180°$

$3x + 2x = 180°$

$5x = 180°$

$\boxed{x = 36°}$

Hence, $\angle 1 = 3x = 108°$ & $\angle 2 = 2x = 72°$

Here, the pairs of vertically opposite angles are found to be

$\angle 1 = \angle 3$, $\angle 2 = \angle 4$, $\angle 5 = \angle 7$, $\angle 6 = \angle 8$

Since, vertically opposite angles are always equal.

$\Rightarrow \angle 1 = \angle 3 = 108°$ & $\angle 5 = \angle 7$

$\angle 2 = \angle 4 = 72°$ & $\angle 6 = \angle 8$

As we know that, if a transversal intersects any parallel lines then the corresponding angles formed are also equals.

$\Rightarrow \angle 1 = \angle 5 = \angle 7 = 108°$

$\angle 2 = \angle 6 = \angle 8 = 72°$

Thus, $\angle 1 = 108°$, $\angle 2 = 72°$

$\angle 3 = 108°$, $\angle 4 = 72°$, $\angle 5 = 108°$,

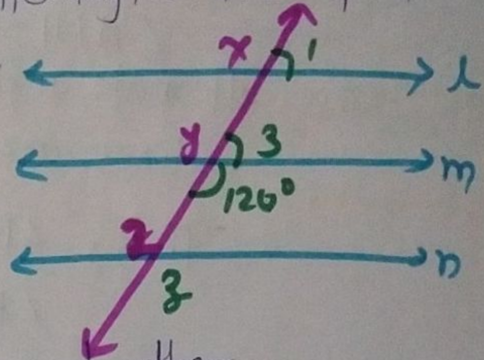
$\angle 6 = 72°$, $\angle 7 = 108°$ & $\angle 8 = 72°$.

2.) In fig. l, m & n are parallel lines intersected by transversal p at x, y & z respectively.
find $\angle 1$, $\angle 2$ & $\angle 3$.

→ Here, from fig.

$\angle y = 120°$ since, vertically opposite angles are equal.

$\angle 3$ & $\angle y$ are linear pair angles.



$\Rightarrow \angle 3 + \angle y = 180°$

$\angle 3 = 180° - \angle y$

$\angle 3 = 180° - 120°$

$\boxed{\angle 3 = 60°}$

Here, lines l & m are parallel to each other.

$\angle 1 = \angle 3$ {since, corresponding angles}

$\angle 1 = 60°$

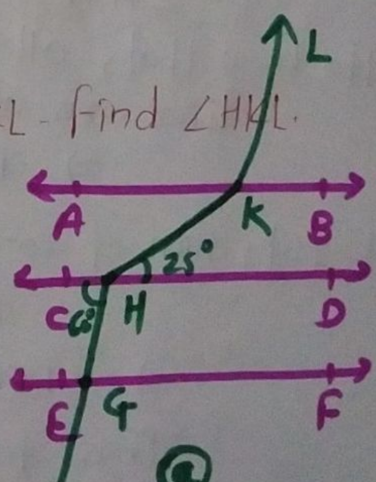
Also, line m & line n are parallel to each other.

$\angle 2 = \angle y$ {since, alternate interior angles are always equal.}

$\angle 2 = 120°$

Thus, $\angle 1 = 60°$, $\angle 2 = 120°$, $\angle 3 = 60°$

3.) In fig., AB ∥ CD ∥ EF and GH ∥ KL. Find $\angle HKL$.



→ Here, we are extending LK so that so that it meets line GF at point 'P' as shown in fig.

from fig, CD ∥ GF & hence alternate angles formed are also equal.

$\angle CHG = \angle HGP = 60°$

$\angle HGP = \angle KPF = 60°$

∵ The corresponding angles formed due to parallel lines are equal.

~~∠A+~~ Hence, $\angle KPG = 180 - 60 = 120°$

$\Rightarrow \angle GPK = \angle AKL = 120°$

Since, corresponding angles formed due to parallel lines are always equals.

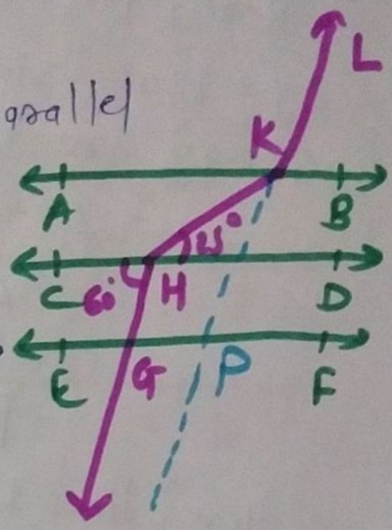
$\angle AKH = \angle KHD = 25°$

Since, alternate angles formed due to parallel lines are always equal.

Thus, $\angle HKL = \angle AKH + \angle AKL$

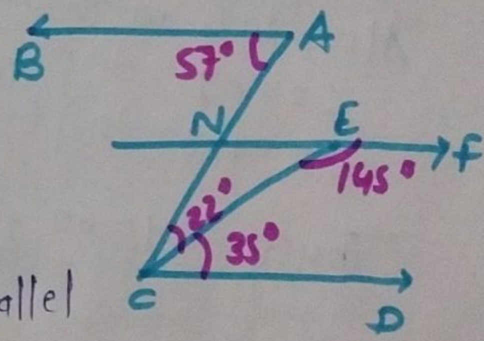
$\angle HKL = 25 + 120 = 145°$

$\boxed{\angle HKL = 145°}$



4.) In fig. Show that AB ∥ EF.

→ Here, we are producing EF so that it intersects AC at point N.



Now, from fig. ⓑ $\angle BAC = 57°$ &

$\angle ACD = 22° + 35° = 57°$

We know that, alternate angles of parallel lines are also equal.

$\Rightarrow BA \parallel EF$ —— ①

But, sum of co-interior angles of parallel lines is 180°

$EF \parallel CD$

$\Rightarrow \angle DCE + \angle CEF = 35° + 145° = 180°$ —— ②

Thus, from ① & ② $\Rightarrow AB \parallel EF$

Since, lines parallel to same lines are parallel to each other.

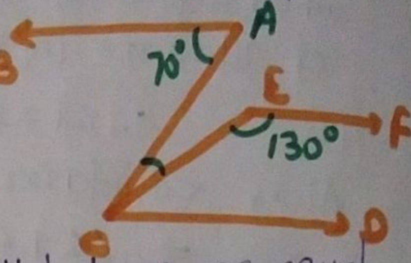
Hence proved.

5) In fig., if $AB \parallel CD$ & $CD \parallel EF$, find $\angle ACE$

→ Given that, $CD \parallel EF$

$\angle FEC + \angle ECD = 180°$

Since, sum of cointerior angles is supplementary to each other.



$\Rightarrow \angle ECD = 180° - 130° = 50°$

Also, $BA \parallel CD$

$\Rightarrow \angle BAC = \angle ACD = 70°$

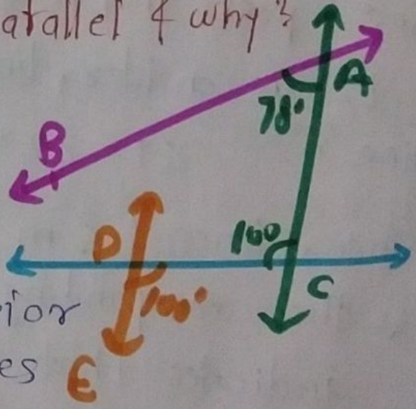
Since, alternative angles of parallel lines are equal.

But, $\angle ACE + \angle ECD = 70°$

$\Rightarrow \angle ACE = 70° - 50° = 20°$

$\boxed{\angle ACE = 20°}$

6) In fig, state which lines are parallel & why?



→

From fig,

We can say that,

If transversal intersects two lines such that a pair of alternate interior angles are equal then that two lines are also parallel.

From fig, $\angle EDC = \angle DCA = 100°$

lines DE & AC are intersected by a transversal DC such that the pair of alternate angles are equal.

Thus, $DE \parallel AC$.

8.) In fig, if $l \parallel m$ & $n \parallel p$ & $\angle 1 = 85°$, find $\angle 2$.

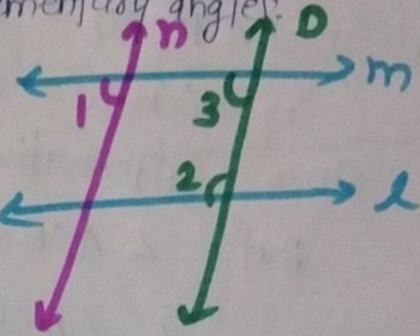
→ Here given that, $\angle 1 = 85°$

we already know that,

when a line cuts the parallel lines, the pair of alternate interior angles are equal.

$\therefore \angle 1 = \angle 3 = 85°$

Also, co-interior angles are supplementary angles.



$\Rightarrow \angle 2 + \angle 3 = 180°$

$\angle 2 + 55° = 180°$

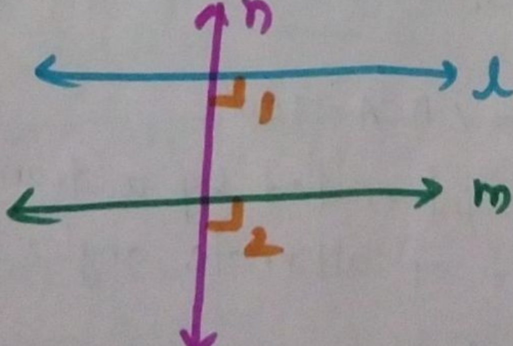
$\angle 2 = 180° - 85°$

$\boxed{\angle 2 = 95°}$

9.) If two straight lines are perpendicular to the same line, prove that they are parallel to each other.

→ Let us consider, lines l & m are perpendicular to line 'n'. $\angle 1 = \angle 2 = 90°$

Since, lines l & m are cut by a transversal line n & the corresponding angles are also equal which indicates that the line l is parallel to line m.



10.)

10) P.T. if the two arms of an angle are perpendicular to the two arms of another angle, then the angles are either equal or supplementary.

→ Let us consider two angles be $\angle ACB$ & $\angle ABD$.

Let, AC is perpendicular to AB &

CD is perpendicular to BD.

Now, we have to prove that :

$$\angle ACD \cong \angle ABD \text{ or } \angle ACD + \angle ABD = 180^\circ$$

For that, we consider a quadrilateral ABCD.

$$\therefore \angle A + \angle C + \angle D + \angle B = 360^\circ$$

Since, sum of all angles of a quadrilateral is 360°.

$$\Rightarrow \angle C + \angle B = 360^\circ - 180^\circ$$

Thus, $\angle ACD + \angle ABD = 180^\circ$

& $\angle ABD = \angle ACD = 90^\circ$

Thus, the angles $\angle ACD$ & $\angle ABD$ are equal angles & supplementary angles also.

Hence proved.