Chapter 18. Surface Area & Volume of Cuboid & Cube

Exercise 18.1

1.) Find the lateral surface area & total surface area of a cuboid of length 80cm, breadth 40 cm & ~~length~~ height 20cm.

→ Given that,

(l) Length of cuboid = 80 cm

(b) breadth of cuboid = 40cm

(h) height of cuboid = 20 cm

We have, (Total surface area of cuboid) $= 2(lb + bh + lh)$

$= 2[(80)(40) + (40)(20) + (20)(80)]$

$= 2[3200 + 800 + 1600]$

$= 2[5600]$

$= 11200$

Thus, the total surface area of cuboid = $11200\ cm^2$

Lateral surface area of cuboid $= 2(l+b)h$

$= 2(80+40)20$

$= 40(120)$

$= 4800$

Thus, lateral surface area of cuboid = $4800\ cm^2$.

2) find the lateral surface area & total surface area of a cube of edge 10cm.

→ Here, given that

Cube of side (a) = 10 cm

(The lateral surface area of cube) = 4 (side)^2

$= 4 (10)^2$

$= 400 \text{ cm}^2$

(The Total surface area of cube) = 6 (side)^2

$= 6 (10)^2$

$= 600 \text{ cm}^2$

Thus, Lateral surface area of cube $= 400 \text{ cm}^2$

Total surface area of cube $= 600 \text{ cm}^2$.

3) find the ratio of the total surface area & lateral surface area of cube.

→ We know that,

Total surface area of a cube = 6 (side)^2

Lateral surface area of a cube = 4 (side)^2

Then, $\frac{\text{(Total surface area of cube)}}{\text{(Lateral surface area of cube)}} = \frac{6 (\text{side})^2}{4 (\text{side})^2}$

$= \frac{6}{4}$

$= 3/2$

Hence,

TSA of cube : LSA of cube = 3:2.

9) Mary wants to decorate her Christmas tree. She wants to place the tree on a wooden block covered with colored paper with a picture of santa Claus on it. She must know the exact quantity of paper to buy for this purpose. If the box has length, breadth & height as 80cm, 40cm & 20cm respectively. How many square sheets of paper of side 40cm would she require?

→ Here given that,

Length of the wooden block (l) = 80cm

Breadth of the wooden block (b) = 40cm

Height of the wooden block (h) = 20cm

Then, Surface area of wooden box = $2[lb + bh + hl]$

$= 2[(80 \times 40) + (40 \times 20) + (20 \times 80)]$

$= 2[5600]$

$= 11200$

Thus, the surface area of wooden box is $11200\ cm^2$.

Hence, Area of each sheet of paper $= 40 \times 40 = 1600\ cm^2$

Now, The total no. of sheets required $= \frac{\text{(surface area of box)}}{\text{(Area of one sheet of paper)}}$

$= \frac{11200}{1600}$

$= 7$

Thus, the Mary would require 7 sheets.

5) The length, breadth & height of a room are 5m, 4m & 3m respectively. find the cost of white washing the walls of the room & the ceilling at the rate of Rs. 7.5 cm^2.

→ Here, given that

Length (l) = 5m

Breadth (b) = 4m

Height (h) = 3m

Here, the total area to be washed = $lb + 2(l+b)h$ — ①

$= (5 \times 4) + 2(5+4)3$

$= 74$

Then, total area to be washed here $74m^2$.

The cost of white washing $1m^2$ area is 7.5 Rs.

Thus, the cost of white washing $74m^2$ = (74×7.5)

= Rs. 555.

6) Three equal cubes are placed adjacently in a row. Find the ratio of a total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.

→ Here, we can consider

Breadth of cuboid = a

Length of new cuboid = $3a$

Height of new cuboid = a

Then, TSA of new cuboid = $2(lb + bh + hl)$

$= 2(3a \times a + a \times a + a \times 3a)$

$= 14a^2$

Now, TSA of three cubes = 3 (6 side^2)

$= 3(6a^2) = 18a^2$

Thus, the ratio of a total surface area of new cuboid to that of the sum of surface areas of the three cubes $= \frac{14a^2}{18a^2} = \frac{7}{9}$ or 7:9

Thus, the required ratio is 7:9.

7.) A 4cm cube is cut into 1cm cubes. Calculate the total surface area of all the small cubes.

→ Here, given that

Edge of the cube = 4cm

Then, Volume of the cube = $side^3 = 4^3 = 64$

The edge of the small cube = 1cm

Volume of small cube = $1\ cm^3$

Then, total no. of small cubes = $\frac{64\ cm^3}{1\ cm^3} = 64$

The required total no. of small cubes are 64.

Then, the total surface area of all the cubes = $64 \times 6 \times 1$

$= 384\ cm^2$

8.) The length of a hall is 18m & the width 12m. The sum of the areas of the floor & the flat roof is equal to the sum of the areas of the four walls. Find the height of the hall.

→ Here, given that

Length of hall = (l) = 18m

Width of hall = (b) = 12m

From given condition,

(Area of the floor & the flat roof) = (sum of the areas of four walls)

Then, (Area of the floor & the flat roof) $= 2lb$

$= 2(18)(12)$

$= 432$ sqft.

Now, (sum of the areas of four walls) $= (2 \times 18h + 2 \times 12h)$.

$432 = 2 \times 18h + 2 \times 12h$

$18h + 12h = 216$

$\boxed{h = 7.2 \text{ m}}$

Thus, the height of the wall is found to be 7.2 m.

10.) Each edge of a cube is increased by 50%. find the percentage increase in the surface area of the cube.

→ Let us consider the edge of a cube be 'a'

Then, Surface area of cube $= 6a^2$

When we increased the edge by 50% then, the new edge formed is $= a + \frac{50}{100} \times a$.

$= a + \frac{a}{2}$

$= 3a/2$

Now, surface area of cube with edge $(3a/2) = 6\left(\frac{3a}{2}\right)^2 = \frac{27}{2}a^2$

Now, Increase in surface area $= \left(\frac{27}{2}\right)a^2 - 6a^2$

$= (15/2)a^2$

Again, (The surface area increased in %) $= \left[\left(\frac{15}{2}\right)a^2 / 6a^2\right] \times 100$

$= \left(\frac{15}{12}\right) \times 100$

$= 125\%$

The percentage increase in the surface area of a cube is found to be 125%.

Exercise 18.2

1.) A cuboidal water tank is 6m long, 5m wide & 4.5m deep. How many liters of water can it hold.

→ Here, for cuboidal water tank

Length (l) = 6m

Breadth (b) = 5m

Height (h) = 4.5m.

Then, Volume of the cuboidal water tank $\} = l \times b \times h$

Volume = $6 \times 5 \times 4.5$

$= 135\ m^3$.

Now, $1m^3 = 1000$ liters.

Then, $135m^3 = (135 \times 1000)$ liters

$= 135000$ liters

Thus, water holding capacity of water tank is found to be 135,000 liters of water.

2.) A cuboidal vessel is 10m long & 8m wide. How high must it be made to hold 380 cubic meters of a liquid?

→ Here, for cuboidal vessel,

Length (l) = 10m

Breadth (b) = 8m

Volume of vessel = $380\ m^3$

Let us consider 'h' be the height of cuboidal vessel.

Then, volume of cuboidal vessel = $l \times b \times h$

$lbh = 380m^3$

$10 \times 8 \times h = 380$

$\boxed{h = 4.75\ m}$

Thus, the height of the vessel is found to be 4.75m.

3) Find the cost of digging a cuboidal pit 8m long, 6m broad & 3m deep at the rate of Rs. 30 per m^3

→ Here, for cuboidal pit

length (l) = 8m

breadth (b) = 6m

height (h) = 3m

Then, volume of a cuboidal pit = $l \times b \times h$

$= 8 \times 6 \times 3$

$= 144\ m^3$.

The cost of digging $1\ m^3$ ⇒ Rs. 30

Then, the total cost of digging $144\ m^3$ = 144×30 = Rs 4320.

Thus, the total cost of digging $144\ m^3$ is Rs. 4320.

4) If 'V' is the volume of a cuboid of diemensions a, b, c & S is the surface area then poove that $\frac{1}{V} = \frac{2}{S}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

→ Here, the diemensions of cuboid given are

length (l) = a

breadth (b) = b

height (h) = c

Then, Volume of the cube = $V = l \times b \times h$

$V = a \times b \times c$

$$\boxed{V = abc}$$

And Surface Area of cube (S) = $2(lb + bh + hl)$

$$\boxed{S = 2(ab + bc + ca)}$$

Now, $\frac{2}{S}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = \frac{2}{S}\left(\frac{ab+bc+ca}{abc}\right)$

$= \frac{2}{2(ab+bc+ca)}\left(\frac{ab+bc+ca}{abc}\right)$

$= \frac{1}{abc}$

$= \frac{1}{V}$

Thus, $\frac{1}{V} = \frac{2}{S}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

Hence Proved.

5.) The areas of three adjacent faces of a cuboid are x, y & z. If the volume is V, Prove that $V^2 = xyz$.

→ Here, given that

The areas of three adjacent faces of a cuboid are x, y & z.

Let us consider, a, b & c be the length, breadth & height of the cuboid respectively.

Then, $x = ab$, $y = bc$ & $z = ca$

Also, $xyz = (ab)(bc)(ca) = (abc)^2$

Then, Volume of Cuboid $(V) = a \times b \times c$

$\Rightarrow V^2 = (abc)^2$

$\boxed{V^2 = xyz}$ Hence proved.

6.) If the areas of three adjacent face of a cuboid are $18\,cm^2$, $18\,cm^2$ & $25\,cm^2$. Find the volume of the cuboid.

→ Let us consider the areas of adjacent faces of cuboid be x, y & z.

Let 'a', 'b' & c be the length, breadth & height of cuboid respectively.

Then, $x = ab$, $y = bc$, $z = ca$

$xyz = (ab)(bc)(ca) = (abc)^2$

But Here, $x = 8\,cm^2$, $y = 18\,cm^2$, $z = 25\,cm^2$

$\Rightarrow (8)(18)(25) = (abc)^2$

$3600 = (abc)^2$

$(abc) = 60$ ——①

But ; Volume of a cuboid $= a \times b \times c = abc = 60\,cm^3$. $\because$ by ①

Thus, volume of a cuboid is found to be $60\,cm^3$.

7) The breadth of a room is twice its height, one half of its length & the volume of the room is 512 cu.dm. find its dimensions.

→ Let us consider, l - length of a room
b - breadth of a room
h - height of a room

from given condition,

$b = 2h$ and $b = l/2$

$l/2 = 2h$

$\boxed{l = 4h}$

Also, $l = 4h$ & $b = 2h$

But, Volume of the room $= l \times b \times h = 512\,(dm)^3$

$\Rightarrow 4h \times 2h \times h = 512$

$8h^3 = 512$

$h^3 = 64$

$\boxed{h = 4\,dm}$

Then, length of room $= l = 4h = 4 \times 4 = 16\,dm$

Height of room $= h = 4\,dm$

9.) A river 3m deep and 40m wide is flowing at the rate of 2km/hour. How much water fall into the sea in a minute?

→ Given that, A river 3m deep & 40m wide is flowing at the rate of 2km/hour.

Then, River flow = 2 km/hr
= (2000/60) m/min

Depth/height of river = 3 m (h)
width/breadth of river = 40m (b)

Then, Volume of water flowing in 1 min = $\frac{100}{3} \times 40 \times 3$
= 4000 m^3
= 4000000 liters.

Thus, the amount of water which will fall in the sea in 1 minute is found to be 4000000 liters

10.) Three metal cubes with edges 6cm, 8cm, 10cm respectively are melted together & formed into a single cube. Find the volume, surface area & diagonal of the new cube.

→ Let us consider 'x' is the length of the each edge of the new cube formed.

Then, Volume of cube = x^3

$x^3 = (6^3 + 8^3 + 10^3)\ cm^3$

$x^3 = 1728$

$\boxed{x = 12}$

Volume of the cube = 1728 cm^3

Surface area of new cube = $6(side)^2 = 6(12)^2 = 864\ cm^2$

Diagonal of the newly formed cube = $\sqrt{3}a = 12\sqrt{3}$ cm.

11.) Two cubes, each of volume 512 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

→ Let us conside the cube with side 'a'.

Volume of the cube = 512 cm^3

But, Volume of the cube = $(side)^3 = a^3$

$\Rightarrow a^3 = 512$

$\boxed{a = 8}$

Thus, Length of cube (l) = 8 + 8 = 16 cm

Breadth of Cube (b) = 8 cm

Height of cube (h) = 8 cm

Then, Surface area = $2(lb + bh + hl)$

$= 2(16 \times 8 + 8 \times 8 + 16 \times 8)$

$= 640\ cm^2$

Hence, the surface area of the cube is found to be 640 cm^2.

12.) Half cubic meter gold-sheet is extended by hammering so as to cover an area of 1 hectare. Find the thickness of the gold-sheet.

→ Volume of the gold-sheet = $\frac{1}{2} m^3 = 0.5\ m^3$

Area of the gold-sheet = 1 hectare = 10000 m^2

Thus, thickness of gold sheet = $\frac{\text{(Volume of solid)}}{\text{(Area of gold sheet)}}$

$= \frac{0.5}{10000} = \frac{m}{20000}$

Thickness of gold sheet = 1/200 cm

Thus, the thickness of the silver sheet is found to be 1/200 cm.

13.) A metal cube of edge 12cm is melted & formed into three smaller cubes. If the edges of the two smaller cubes are 6cm & 8cm, find the edge of the third smaller cube.

→ From given condition,

Volume of the bigger cube = $V_1 + V_2 + V_3$

Let us consider 'x' be the edge of the third cube.

$\Rightarrow 12^3 = 6^3 + 8^3 + x^3$

$1728 = 216 + 512 + x^3$

$1000 = x^3$

$\boxed{x = 10\text{ cm}}$

Thus, the length of the third cube is 10cm.

Exercise VSAQs

1.) If two cubes each of side 6cm are joined face to face, then find the volume of the resulting cuboid.

→ Here, Given that

The side of the two equal cubes = 6cm

Then, length of cuboid (l) = 6cm + 6cm = 12cm

Breadth of cuboid (b) = 6cm

Height of Cuboid (h) = 6cm

Thus, the volume of the cuboid = $l \times b \times h = 12 \times 6 \times 6$

$= 432\text{ cm}^3$

2.) Three cubes of metal whose edges are in the ratio 3:4:5 are melted down into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the three edges of the cube.

→ Here, given that

The ratio of three edges of cube = 3:4:5

The corresponding edges would be

$3x, 4x$ & $5x$ respectively.

Then, diagonal of the cube $= 12\sqrt{3}$ cm.

Volume of the cube = Volume of the fig. obtained after combining three cubes

$= (3x)^3 + (4x)^3 + (5x)^3$

$= 216x^3$ — ①

Thus, New diagonal of the cube $= \sqrt{3}a = 12\sqrt{3}$

$\Rightarrow \boxed{a = 12 \text{ cm}}$

Volume of the new cube $= a^3 = 12^3$ — ②

Then, $(12)^3 = 216x^3$ ∵ by ① & ②

$x^3 = \frac{216}{12 \times 12 \times 12} = 8$

$\boxed{x = 2}$

Thus, the required edges of the cube are

$3x = 3 \times 2 = 6$ cm

$4x = 4 \times 2 = 8$ cm

$5x = 5 \times 2 = 10$ cm

3) If the perimeter of each of face of a cube is 32 cm, find its lateral surface area. Note that four faces which meet the base of a cube are called its lateral faces.

→ Here, given that

Perimeter of each face of cube = 32 cm

Let 'a' be the edge of the cube.

Then, Perimeter of each face of cube $= 4a$

$\Rightarrow 4a = 32$

$\boxed{a = 8}$

Then, Lateral surface area of cube $= 4(\text{side})^2$

$= 4a^2$

$= 4(8)^2$

$= 256 \text{ cm}^2$