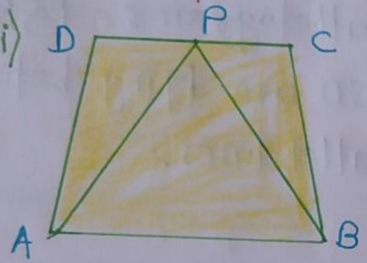
Chapter 15. Area of Parallelogram & Triangles

Exercise 15.1

1.) Which of the following fig. lie on the same base and between the same parallel. In such a case, write the common base and two parallels.

→ i)

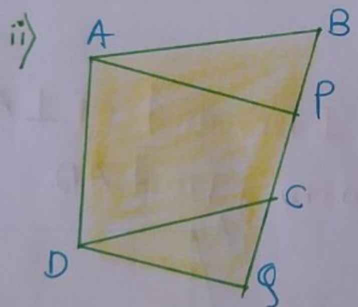


In fig ① ΔAPB and also the trapezium ABCD are on the same or common base AB and also between the same parallel lines AB & DC.

Thus, common base = AB

Parallel lines AB and DC.

ii)

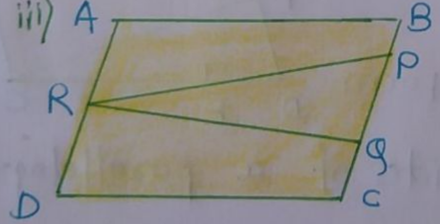


In fig.② parallelograms APQD and ABCD are on the same base AD and also Between the same parallel lines AD & BQ.

So, Common base = AD

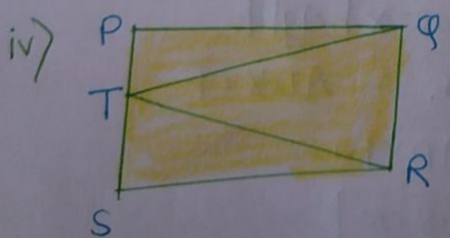
Parallel lines AD and BQ.

iii)



In fig.ⓘⓘⓘ, we consider parallelogram ABCD & ΔPQR, lies between the same parallel lines AD & BC. But they are not having common base.

iv)

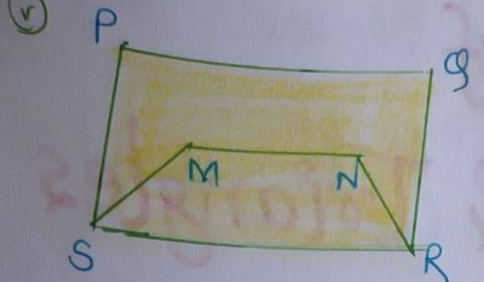


In fig. ⓘⓥ, ΔQRT & parallelogram PQRS are on the same base QR & lies between the same parallel lines QR and PS.

So, common base = QR

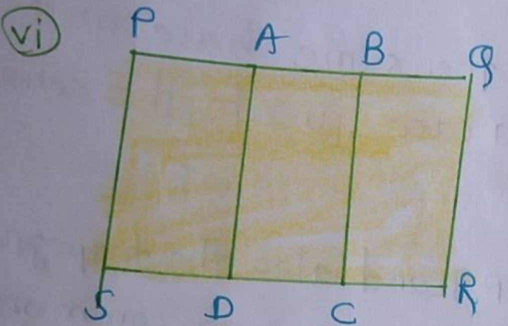
Parallel lines QR & PS.

(v)



In fig (v):
Parallelograms PQRS & trapezium SMNR are having common base SR. But they are not lying between the same parallel lines.

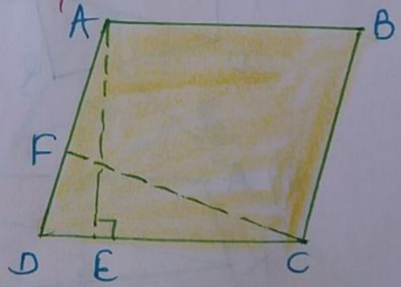
(vi)



In fig. (vi)
Parallelograms AQRD, BCQR, PQRS are lying between the same parallel lines.
And the parallelograms PQRS, BPSC & APSD are lying between the same parallel lines.

Exercise 15.2

1.) If fig. ABCD is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm, find AD.



→ Here, given that
ABCD is a parallelogram.
$AE \perp DC$ and $CF \perp AD$
Also, $AB = 16$ cm, $AE = 8$ cm & $CF = 10$ cm
But, we know that, the opposite sides of a parallelogram are equal. $\Rightarrow AB = CD = 16$ cm

And, Area of parallelogram = Base × height

$$A(\square ABCD) = CD \times AE = AD \times CF$$

$$\Rightarrow 16 \times 18 = AD \times 10$$

$$\boxed{AD = 12.8 \text{ cm}}$$

Thus $l(AD) = 12.8$ cm.

2.) In question 1, if $AD = 6cm$, $CF = 10cm$ & $AE = 8cm$ find AB.

→ from fig. Area of parallelogram ABCD is given by

$$AD \times CF = CD \times AE$$

$$6 \times 10 = CD \times 8$$

$$\boxed{CD = 7.5cm}$$

But, the opposite sides of parallelogram are equal.

$$\Rightarrow \boxed{AB = DC = 7.5cm}$$

3.) Let ABCD be a parallelogram of area $124cm^2$. If E and F are the midpoints of sides AB and CD respectively, then find the area of parallelogram AEFD.

→ Given that, ABCD is a parallelogram of area $124cm^2$. and point E and F are the midpoints of sides AB and CD respectively.

Let us consider the point 'p' and after joining AP is formed which is perpendicular to DC.

$$A(\square EBCF) = FC \times AP$$

$$A(\square AFED) = DF \times AP$$

As E is the midpoint of DC $\Rightarrow$ $\boxed{DF = FC}$

Then, $A(\square AFED) = A(\square EPCF) = \frac{1}{2}[A(\square ABCD)]$

$$= \frac{1}{2}(124)$$

$$\boxed{A(\square AFED) = 62cm^2}$$

4.) If ABCD is a parallelogram then prove that

$$A(\Delta ABD) = A(\Delta BCD) = A(\Delta ABC) = A(\Delta ACD)$$

$$= \frac{1}{2} A(\text{paralle. } ABCD)$$

→ Given that,

ABCD is a parallelogram.

After joining the diagonals of a parallelogram ABCD it get divided into two quadrilaterals.

Let us consider the diagonal AC,

$$A(\Delta ABC) = A(\Delta ACD) = \frac{1}{2}(\text{Area of parallelo. } ABCD) \quad \text{——— (1)}$$

Let us consider the diagonal BD,

$$A(\Delta ABD) = A(\Delta BCD) = \frac{1}{2}(\text{Area of parallelo. } ABCD) \quad \text{——— (2)}$$

Then, from (1) & (2)

$$A(\Delta ABC) = A(\Delta ACD) = A(\Delta ABD) = A(\Delta BCD)$$

$$= \frac{1}{2}(\text{Area of parallelogram } ABCD).$$

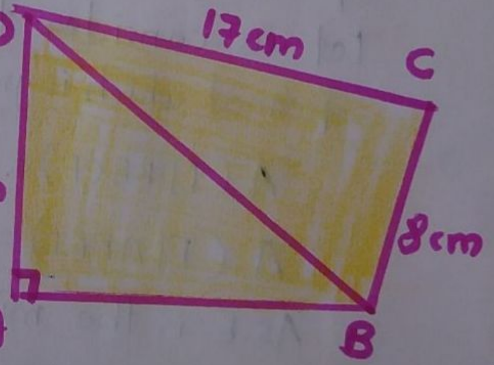
Hence proved.

Exercise 15.3

1.) In fig. compute the area of quadrilateral ABCD.

→ From fig, In quadrilateral ABCD

DC = 17 cm, AD = 9 cm, BC = 8 cm



Now, In ΔABD $\angle DAB = 90°$

Then, By Pythagoras Theorem,

$AB^2 + AD^2 = BD^2$

$15^2 = AB^2 + 9^2$

$AB^2 = 225 - 81 = 144$

$\boxed{AB = 12 cm}$

$A(\Delta ABD) = \frac{1}{2}(12 \times 9) = 54 \text{ cm}^2$

Now, In ΔBCD $\angle DBC = 90°$

Then, By Pythagoras theorem,

$CD^2 = BD^2 + BC^2$

$17^2 = BD^2 + 8^2$

$BD^2 = 289 - 64 = 225$

$\boxed{BD = 15 cm}$

And $A(\triangle BCD) = \frac{1}{2}(8 \times 17)\, cm^2 = 68\, cm^2$

And $A(\square ABCD) = A(\triangle ABD) + A(\triangle BCD)$

$= 54\, cm^2 + 68\, cm^2$

$\boxed{A(\square ABCD) = 112\, cm^2}$

2.) In fig., PQRS is a square and T and U are respectively the midpoints of PS and QR. find the area of $\triangle OTS$ if $PQ = 8$ cm.

→ Given that,

PQRS is a square as shown in fig.

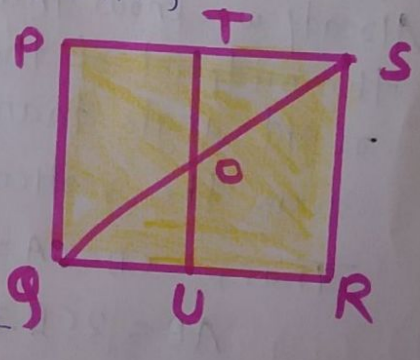
T and U are the midpoints of PS & QR respectively.

Also, $PQ = 8$ cm

$\Rightarrow TU \parallel PQ \Rightarrow TO \parallel PQ$

In $\triangle PQS$, T is the mid point of PS.

Then, By midpoint theorem $\Rightarrow TO \parallel PQ$



Thus, $TO = \frac{1}{2} PQ = 4$ cm

Also, $TS = \frac{1}{2} PS = 4$ cm

Now, $A(\triangle OTS) = \frac{1}{2}(TO \times TS)$

$= \frac{1}{2}(4 \times 4) = 8\, cm^2$

Thus, $\boxed{A(\triangle OTS) = 8\, cm^2}$

3.) Compute the area of trapezium PQRS in fig.

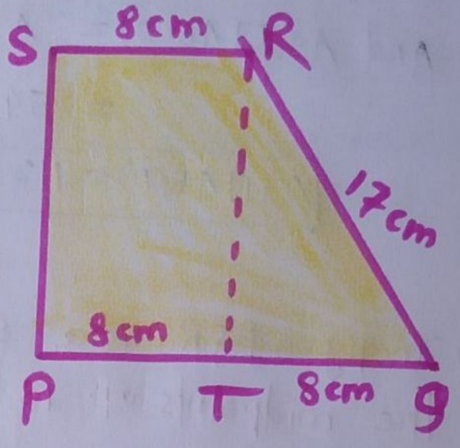
→ In fig. trapezium PQRS is given.

A (trapezium PQRS) = (Area of rectangle PSRT) + $A(\triangle QRT)$

$= PT \times RT + \frac{1}{2}(QT \times RT)$

$A(\text{trapezium } PQRS) = 8(RT) + \frac{1}{2}(8RT)$

$= 12RT$



In ΔQRT, $\angle RTQ = 90°$

Then, By Pythagoras Theorem,

$QR^2 = QT^2 + RT^2$

$RT^2 = QR^2 - QT^2$

$RT^2 = 17^2 - 8^2 = 225$

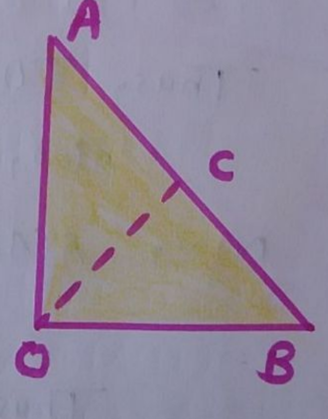
$\boxed{RT = 15}$

Thus, $A(\text{trapezium}) = 12 \times 15 = 180 \text{ cm}^2$

4.) In fig. $\angle AOB = 90°$, $AC = BC$, $OA = 12$ cm and $OC = 6.5$ cm. Find the area of ΔAOB.

→ Here, given that $\angle AOB = 90°$, $AC = BC$, $OA = 12$ cm, $OC = 6.5$ cm

Already we know that, the midpoint of the hypotenuse of a right angle triangle is equidistant from the vertices.



Thus, $CB = CA = OC = 6.5$ cm

$AB = 2CB = 2 \times 6.5 = 13$ cm

Now, In ΔOAB $\angle AOB = 90°$

By Pythagoras Theorem,

$AB^2 = OB^2 + OA^2$

$13^2 = OB^2 + 12^2$

$OB^2 = 169 - 144 = 25$

$\boxed{OB = 5 \text{ cm}}$

$A(\Delta AOB) = \frac{1}{2}(\text{Base} \times \text{height}) = \frac{1}{2}(12 \times 5)$

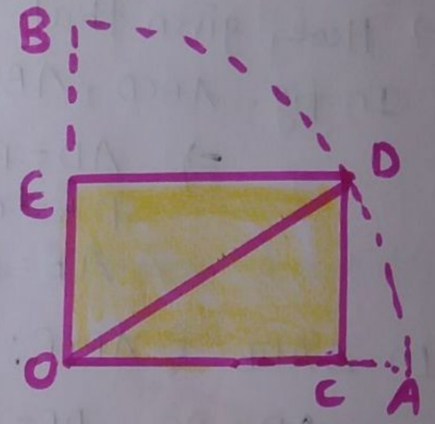
$\boxed{A(\Delta AOB) = 30 \text{ cm}^2}$

6) In fig., OCDE is a rectangle inscribed in a quadrant quadrilateral of a circle of radius 10 cm. If OE = $2\sqrt{5}$ cm find the area of the rectangle

→ Here, given that

OCDE is a rectangle inscribed in a quadrant of a circle of radius 10cm.

$R = OD = 10$ cm and $OE = 2\sqrt{5}$ cm



In ΔDEO, $\angle DEO = 90°$

By Pythagoras Theorem,

$$OD^2 = OE^2 + DE^2$$

$$(10)^2 = (2\sqrt{5})^2 + DE^2$$

$$100 - 20 = DE^2$$

$$\boxed{DE = 4\sqrt{5}\text{ cm}}$$

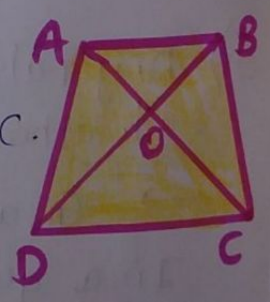
Now, $A(\square OCDE) = $ length × breadth

$= OE \times DE$

$= 2\sqrt{5} \times 4\sqrt{5}$

$$\boxed{A(\square OCDE) = 40\text{ cm}^2}$$

7) In fig, ABCD is a trapezium in which AB ∥ DC. Prove that $A(\Delta AOD) = A(\Delta BOC)$.



→ Here, given that

ABCD is a trapezium in which AB ∥ DC.

from fig. ΔADC & ΔBDC are having common base DC & also lying between same parallel lines AB & DC.

$\Rightarrow A(\Delta ADC) = A(\Delta BDC)$ —— ①

Again, ΔADC is the combination of ΔAOD & ΔDOC.

Simillarly, ΔBDC is the combination of ΔBOC & ΔDOC respectively.

from ① ⇒ $A(\triangle AOD) + A(\triangle DOC) = A(\triangle BOC) + A(\triangle DOC)$

$$\boxed{A(\triangle AOD) = A(\triangle BOC)}$$

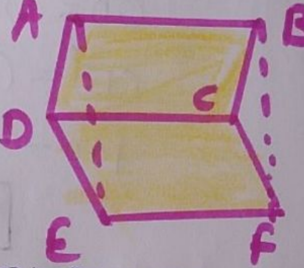
Hence proved.

8.) In fig. ABCD, ABFE & CDEF are parallelograms. Prove that $A(\triangle ADE) = A(\triangle BCF)$.

→ Here, given that

In fig. ABCD, ABFE & CDEF are parallelograms.

$\Rightarrow$ $AD = BC$, $DE = CF$, $AE = BF$ } ∵ By properties of parallelograms



In $\triangle ADE$ & $\triangle BCF$:

$AD = BC$, $DE = CF$ and $AE = BF$

Then, By Side-Side-Side criterion of congruence,

$\Rightarrow$ $\boxed{\triangle ADE \cong \triangle BCF}$

As the $\triangle ADE$ & $\triangle BCF$ are congruent triangles.

$\Rightarrow$ $\boxed{A(\triangle ADE) = A(\triangle BCF)}$

Hence proved.

9.) Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that $A(\triangle APB) \times A(\triangle CPD) = A(\triangle APD) \times A(\triangle BPC)$.

→ Given that,

In a quadrilateral ABCD, diagonals AC & BD intersects each other at point P.

We draw BQ & DR two perpendiculars on AC.

Now, Consider

$$A(\triangle APB) \times A(\triangle CPD) = \frac{1}{2}(AP \times BQ) \times \frac{1}{2}(PC \times DR)$$

$$= \frac{1}{4}(PC \times BQ) \times (AP \times DR)$$

$$= \left(\frac{1}{2} PC \times BQ\right) \times \left(\frac{1}{2} AP \times DR\right)$$

$= A(\Delta APD) \times A(\Delta BPC)$

Thus, $\boxed{A(\Delta APB) \times A(\Delta CDP) = A(\Delta APD) \times A(\Delta BPC)}$

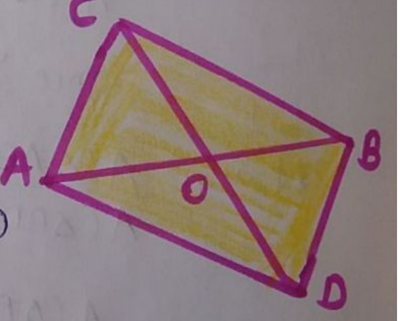
Hence proved.

10.) In fig. ABC & ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, show that $A(\Delta ABC) = A(\Delta ABD)$.

→ Here, given that

ABC & ABD are two triangles having common base AB as shown in fig.

We drawn here two perpendiculars CP and DQ on AB as shown.



Now, $\left.\begin{array}{l} A(\Delta ABC) = \frac{1}{2}(AB \times CP) \\ \& \ A(\Delta ABD) = \frac{1}{2}(AB \times DQ) \end{array}\right\} — ①$

In ΔCPO & ΔDQO,

$\angle CPO = \angle DQO = 90°$

$CO = OD$

and $\angle COP = \angle DOQ$ (since vertically opposite angles are also equal.)

Then, By Angle-Angle-Side criterion of congruence,

$\boxed{\Delta CPO \cong \Delta DQO}$

$\Rightarrow \quad CP = DQ$ —— ②

Since, corresponding parts of congruent triangles are equal.

from ① & ② $\Rightarrow$ $A(\Delta ABC) = A(\Delta ABD)$

Hence proved.

Exercise VSAQs

1.) If ABC & BDE are two equilateral triangle such that D is the mid-point of BC, then find A(△ABC) : A(△BDE).

→ Here, given that

△ABC and △BDE are two equilateral triangle.

And point 'D' is the midpoint of BC.

But $A(\text{equilateral triangle}) = \frac{\sqrt{3}}{4}(\text{side})^2$

Let us consider 'a' is the length of side of equilateral triangle.

Then, $A(\triangle ABC) = \frac{\sqrt{3}}{4}(a)^2$

$A(\triangle BDE) = \frac{\sqrt{3}}{4}(a/2)^2$

Since 'D' is the midpoint of BC.

Now, $\frac{A(\triangle ABC)}{A(\triangle BDE)} = \frac{\sqrt{3}/4\, a^2}{\sqrt{3}/4\,(a/2)^2}$

$\frac{A(\triangle ABC)}{A(\triangle BDE)} = \frac{4}{1}$

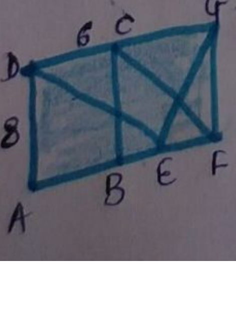
Thus, $A(\triangle ABC) : A(\triangle BDE) = 4:1$.

2.) In fig., ABCD is a rectangle in which CD = 6cm, AD = 8cm. Find the area of the parallelogram CDEF.

→ Here, given that

ABCD is a rectangle in which CD = 6cm, AD = 8cm.

From fig, Rectangle ABCD and parallelogram CDEF are on the same base & between the same parallel lines & hence both has equal areas.



A(parallelogram CDEF) = A(Rectangle ABCD) —— ①

$A(\square ABCD) = CD \times AD = 6 \times 8 = 48 \text{ cm}^2$

from ① ⇒ Area of parallelogram CDEF = 48 cm².

3.) In fig, find the area of $\triangle GEF$.

→ In fig,

Rectangle ABCD & parallelogram CDEF are on the same base & between the same parallel lines & hence they have equal areas.

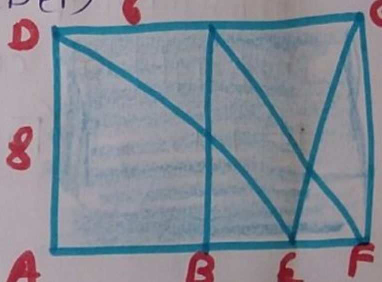
$\rightarrow A(CDEF) = A(\square ABCD) = 8 \times 6 = 48 cm^2$.

Also, triangle EFG & parallelogram CDEF are on the same base & lying between same parallel lines & hence they have equal areas.

$\Rightarrow A(\triangle EFG) = \frac{1}{2} A(\text{parallelogram } CDEF)$

$A(\triangle EFG) = \frac{1}{2}(48) = 24 cm^2$

$\boxed{A(\triangle EFG) = 24 cm^2}$



5.) PQRS is a rectangle inscribed in a quadrant of a circle of radius 13cm. A is any point on PQ. If PS = 5cm then find $A(\triangle RAS)$.

→ Given that, PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. PS = 5cm.

In $\triangle PSR$, By Pythagoras Theorem

$SR^2 = PR^2 - PS^2 = (13)^2 - (5)^2 = 169 - 25 = 144$

$\boxed{SR = 12 cm}$

$A(\triangle RAS) = \frac{1}{2}(SR)(PS) = \frac{1}{2}(12)(5) = 30$

Thus, $\boxed{A(\triangle RAS) = 30 cm^2}$