Chapter 14. Quadrilaterals

Exercise 14.1

1.) Three angles of a quadrilateral are respectively equal to $110°$, $50°$ & $40°$. Find it's fourth angle.

→ Given that, the three angles of a quadrilateral are $110°$, $50°$ and $40°$ respectively.

Let us consider 'x' be the fourth angle of quadrilateral.

But, the sum of all angles of quadrilateral $= 360°$

$$\Rightarrow 110° + 50° + 40° + x° = 360°$$

$$x = 360° - 200°$$

$$\boxed{x = 160°}$$

Hence, the fourth angle of quadrilateral is $x = 160°$.

2.) In a quadrilateral ABCD, the angle A, B, C and D are in the ratio 1:2:4:5. Find the measure of each angles of the quadrilateral.

→ Given that, In a quadrilateral ABCD, the angle A, B, C & D are in the ratio 1:2:4:5.

Let us consider the angles are $A = x$, $B = 2x$, $C = 4x$ & $D = 5x$.

But, the sum of all four angles of quadrilateral $= 360°$

$$A + B + C + D = 360°$$

$$x + 2x + 4x + 5x = 360°$$

$$12x = 360°$$

$$\boxed{x = 30°}$$

Thus, $A = x = 30°$,
$B = 2x = 60°$
$C = 4x = 120°$ and $D = 5x = 150°$.

3) In a quadrilateral ABCD, CO & DO are the bisectors of $\angle C$ and $\angle D$ respectively. Prove that $\angle COD = 1/2 (\angle A + \angle B)$

→ Given that, in a quadrilateral ABCD,

CO & DO are the bisectors of $\angle C$ & $\angle D$ respectively.

Now, from fig. In ΔDOC,

$$\angle CDO + \angle COD + \angle DCO = 180^\circ$$

Since, the sum of three angles of a triangle is 180°.

$$\frac{1}{2}(\angle CDA) + \angle COD + \frac{1}{2}(\angle DCB) = 180^\circ$$

$$\angle COD = 180^\circ - \frac{1}{2}(\angle CDA + \angle DCB) \quad \text{—— (1)}$$

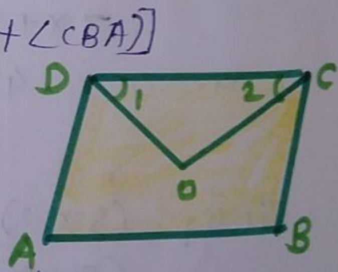
Also, sum of all four angles of a quadrilateral is 360°.

$$\Rightarrow \angle CDA + \angle DCB = 360^\circ - (\angle DAB + \angle CBA) \quad \text{—— (2)}$$

from (1) & (2) ⇒

$$\angle COD = 180^\circ - \frac{1}{2}[360^\circ - (\angle DAB + \angle CBA)]$$

Also, $\angle DAB = \angle A$ & $\angle CBA = \angle B$



$$\angle COD = 180^\circ - 180^\circ + \frac{1}{2}(\angle A + \angle B)$$

$$\boxed{\angle COD = \frac{1}{2}(\angle A + \angle B)}$$

Hence proved.

4) The angles of a quadrilateral are in the ratio 3:5:9:13. Find all the angles of the quadrilateral.

→ Here, given that the angles of a quadrilateral are in the ratio 3:5:9:13.

Let us consider the interior angles of a quadrilateral be $3x, 5x, 9x$ & $13x$ respectively.

we know that, the sum of all four interior angles of a quadrilateral is 360°.

$$3x + 5x + 9x + 13x = 360^\circ$$

$$30x = 360^\circ$$

Thus, the remaining angles are found to be

$3x = 3(12) = 36°$
$5x = 5(12) = 60°$
$9x = 9(12) = 108°$
$13x = 13(12) = 156°$

Exercise 14.2

1.) Two opposite angles of a parallelogram are $(3x-2)°$ and $(50-x)°$. Find the measure of each angle of the parallelogram.

→ Here, given that, the two opposite angles of a parallelogram are $(3x-2)°$ and $(50-x)°$.

But, the opposite ~~sides~~ angles of a parallelogram are equal.

$(3x-2)° = (50-x)°$
$3x + x = 50 + 2$
$4x = 52$
$\boxed{x = 13°}$

Thus, $(3x-2)° = (3(13)-2) = 37°$
$(50-x)° = (50-13) = 37°$

Also, we know that the adjacent angles of a parallelogram are supplementary.

$x + 37 = 180°$
$x = 180° - 37 = 143°$.

Thus, the required angles of a parallelogram are 37°, 143°, 37° and 143° respectively.

2.) If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

→ Let us consider the angle of parallelogram is x.
Then from given condition, the measure of adjacent angle is $2x/3$.

But, we already know that
The adjacent angles of a parallelogram is supplementary.

$x + 2x/3 = 180°$

$3x + 2x = 540°$

$5x = 540°$

$\boxed{x = 108°}$

Then, adjacent angle $= \frac{2x}{3} = \frac{2}{3}(108) = 72°$

Remaining angles will be $108°$ & $72°$.

Thus, the four angles of parallelogram are 108°, 72°, 108° and 72° respectively.

3) Find the measure of all the angles of a parallelogram, if one angle is 24° less than twice the smallest angle.

→ Here, given that
One angle is 24° less than twice the smallest angle.
Let 'x' be the smallest angle.

$\Rightarrow \quad x + 2x - 24° = 180°$

~~Since, sum of~~ $3x - 24° = 180°$

$3x = 180° + 24°$

$3x = 204°$

$x = 204/3$

$\boxed{x = 68°}$

Remaing angle $= 2x - 24° = 2(68°) - 24 = 112°$

Thus, the required four angles of a parallelogram are 68°, 112°, 68°, 112° respectively.

4) The perimeter of a parallelogram is 22 cm. If the longer side measures 6.5 cm what is the measure of the shorter side?

→ Here, given that the perimeter of a parallelogram is 22 cm.
Longer side measures 6.5 cm.

Let us consider 'x' is the shorter side of parallelogram.
Perimeter = sum of all sides of parallelogram

$22 = x + 6.5 + 6.5 + x$

$22 = 2(x + 6.5)$

$x = 11 - 6.5 = 4.5$ cm

Thus, the shorter side of a parallelogram measures 4.5 cm.

Exercise 14.3

1.) In a parallelogram ABCD, determine the sum of angles $\angle C$ & $\angle D$.

→ In a parallelogram ABCD, $\angle C$ & $\angle D$ are consecutive interior angles on the same side of the traversal CD.

So, $\angle C + \angle D = 180°$

2.) In a parallelogram ABCD, if $\angle B = 135°$, determine the measures of its other angles.

→ Given that,
In a parallelogram ABCD, $\angle B = 135°$
Also, $\angle A = \angle C$, $\angle B = \angle D$ and $\angle A + \angle B = 180°$

$\angle A + 135° = 180°$

$\boxed{\angle A = 45°}$

Thus, the required angles are $\angle A = \angle C = 45°$
$\angle B = \angle D = 135°$

3.) ABCD is a square. AC & BD intersect at O. State the measure of $\angle AOB$.

→ we already know that,
The diagonals of a square bisect each other at right angle.

$\Rightarrow \boxed{\angle AOB = 90°}$

4.) ABCD is a rectangle with $\angle ABD = 40°$. Determine $\angle DBC$.

→ We know that,

The each angle of rectangle is $90°$.

$\Rightarrow \angle ABC = 90°$

Given that, $\angle ABD = 40°$

Then, $\angle ABD + \angle DBC = 90°$

$40° + \cancel{\angle B'}\angle DBC = 90°$

$\boxed{\angle DBC = 50°}$

Exercise 14.4

1.) In a ΔABC, D, E and F are respectively the midpoints of BC, CA and AB. If the lengths of the sides AB, BC and CA are 7 cm, 8 cm and 9 cm respectively, find the perimeter of ΔDEF.

→ Here, given that

In a ΔABC, $AB = 7cm$, $BC = 8cm$, $AC = 9cm$

Also, D, E & F are midpoints of BC, CA and AB respectively.

Then By midpoint theorem,

$EF = \frac{1}{2} BC$, $DF = \frac{1}{2} AC$ and $DE = \frac{1}{2} AB$

Perimeter of $\Delta DEF = DE + EF + DF$

$= \frac{1}{2}(AB + BC + AC)$

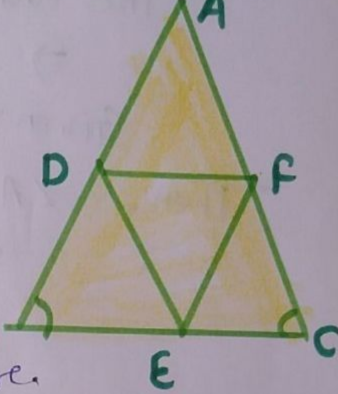
$= \frac{1}{2}(7 + 8 + 9)$

$\boxed{P(\Delta DEF) = 12 cm}$

2.) In a ΔABC, $\angle A = 50°$, $\angle B = 60°$ and $\angle C = 70°$. Find the measures of the angles of the triangle formed by joining the midpoints of the sides of this triangle.

→

Here, given that
In ΔABC, $\angle A = 50°$, $\angle B = 60°$, $\angle C = 70°$.
And points D, E and F are midpoints of AB, BC and AC respectively.



In quadrilateral DECF formed,
By midpoint formula,

$DE \parallel AC \Rightarrow DE = \frac{1}{2} AC$

and $CF = \frac{1}{2} AC \Rightarrow \boxed{DE = CF}$

Thus, DECF is a parallelogram formed here.

$\Rightarrow \angle C = \angle D = 70°$

Since, opposite angles of a parallelogram are equal.

Simillarly, ADEF is a parallelogram $\Rightarrow \angle A = \angle E = 50°$

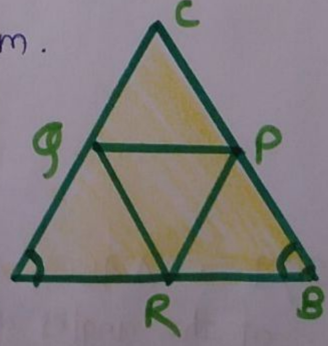
And BEFD is a parallelogram $\Rightarrow \angle B = \angle F = 60°$

Thus, the angles of ΔDEF are found to be

$\angle D = 70°$, $\angle E = 50°$, $\angle F = 60°$.

3.) In a triangle, P, Q and R are the midpoints of sides BC, CA and AB respectively. If AC = 21 cm, BC = 29 cm and AB = 30 cm, find the perimeter of the quadrilateral ARPQ.

→ Here, given that
In a triangle. P, Q and R are the midpoints of sides BC, CA & AB respectively.



AB respectively.
Also, AC = 21 cm, BC = 29 cm, AB = 30 cm.
Then, By midpoint theorem

$RP \parallel AC \Rightarrow RP = \frac{1}{2} AC$

Now, In □ARPQ,

$RP \parallel AQ \Rightarrow RP = AQ$

since, pair of parallel side is equal also.

$\Rightarrow$ ARPQ is a parallelogram.

Again, $AR = \frac{AB}{2} = \frac{30}{2} = 15cm$

and $AR = QP = 15cm$

since in parallelogram opposite sides are equal.

Thus, $RP = \frac{1}{2} AC = \frac{1}{2}(21) = 10.5cm$

and $RP = AQ = 10.5 cm$

Now, perimeter of $ARPQ = AR + QP + RP + AQ$

$= 15 + 15 + 10.5 + 10.5$

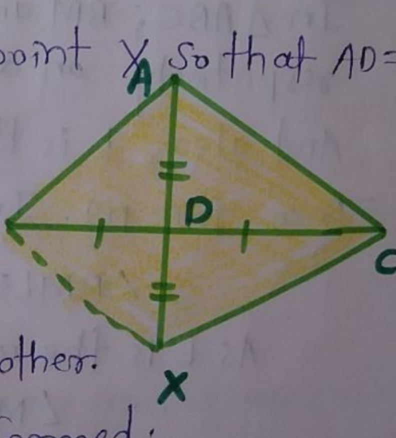
$= 51$ cm

Thus, the perimeter of a quadrilateral ARPQ is found to be 51cm.

4) In a ΔABC, median AD is produced to X such that $AD = DX$. Prove that ABXC is a parallelogram.

→ Here, given that

In ΔABC, median AD is produced to point X so that $AD = DX$.



Now, from fig.

In a quadrilateral ABXC,

$AD = DX$ and $BD = DC$

then from fig. we can say that

the diagonals AX and BC bisect each other.

Thus, ABXC is a parallelogram formed.

Hence proved.

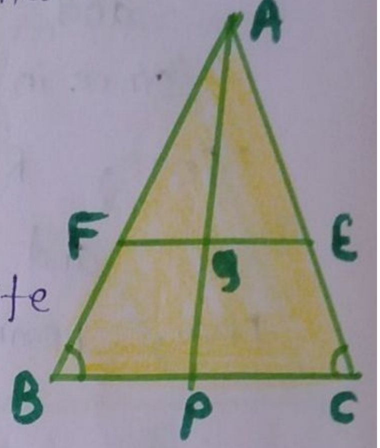
5.) In a ΔABC, E and F are the mid-points of AC and AB respectively. The altitude AP to BC intersects FE at Q. Prove that $AQ = QP$.

→ Here, given that

In ΔABC, E and F are the mid-points of AC & AB respectively.

From fig, In ΔABC

$EF \parallel BC \Rightarrow EF = \frac{1}{2} BC$ ∵ By midpoint theorem



Again In ΔABP,

By mid-point theorem,

F is the mid-point of AB $\Rightarrow$ $Fg \parallel BP$

As 'g' is the midpoint of AP, we can write

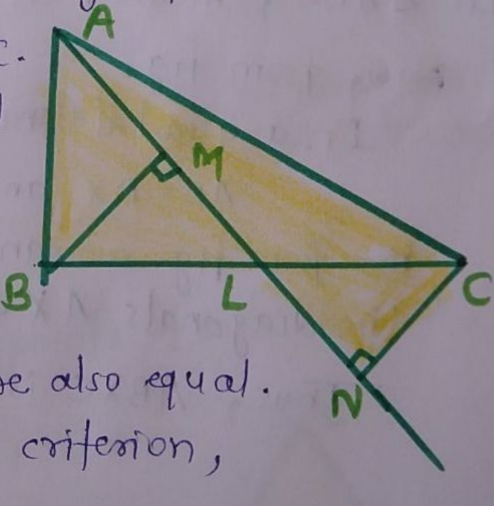
$\boxed{Ag = gP}$ Hence proved.

6) In a ΔABC, BM and CN are perpendiculars from B and C respectively on any line passing through A. If L is the mid-point of BC, prove that ML=NL.

→ Here, given that

In ΔABC, BM and CN are perpendiculars from B & C respectively on any line passing through point A.

And also, L is the midpoint of BC.



Then from fig. In ΔBLM & ΔCLN

$\angle BML = \angle CNL = 90^\circ$

As L is the midpoint $\Rightarrow BL = CL$

$\angle MLB = \angle NLC$

Since, vertically opposite angles are also equal.

Then By angle-side-angle criterion,

$\Delta BLM \cong \Delta CLN$

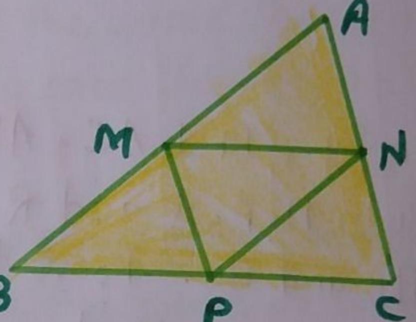
Then By congruent triangles property

$\boxed{LM = LN}$

Hence proved.

8.) In fig. M, N and P are mid-points of AB, AC and BC respectively. If MN = 3 cm, NP = 3.5 cm and MP = 2.5 cm Calculate BC, AB and AC.

→ Here given that,
The points M, N and P are midpoints of AB, AC and BC respectively.
Also, MN = 3 cm, NP = 3.5 cm and MP = 2.5 cm



Then, By midpoint theorem,

$MN \parallel BC \Rightarrow MN = \frac{1}{2} BC$

$BC = 2MN = 6$ cm

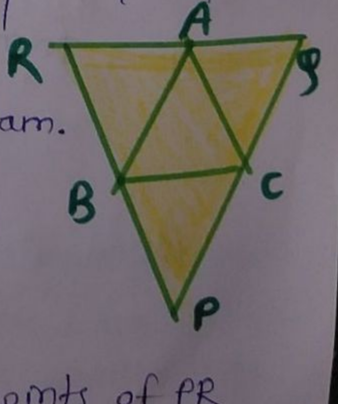
Similarly, we can write

$AC = 2MP = 2(2.5) = 5$ cm

And $AB = 2NP = 2(3.5) = 7$ cm

9.) ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC.

→ Here, given that
In ΔABC, through points A, B, C lines are drawn which are parallel to BC, CA and AB respectively and intersects at P, Q & R.



From fig. ABCQ & ARBC are parallelogram.

Thus, BC = AQ and BC = AR

$\Rightarrow \boxed{AQ = AR}$

$\Rightarrow$ A is the mid-point of QR.

Similarly, given that point B and C are midpoints of PR and PQ respectively.

Then, By midpoint theorem,

$AB = \frac{PQ}{2}$, $BC = \frac{QR}{2}$ and $CA = \frac{PR}{2}$

$\Rightarrow$ $PQ = 2AB$, $QR = 2BC$ and $PR = 2CA$

$\Rightarrow$ $PQ + QR + RP = 2(AB + BC + CA)$

Hence, perimeter of ΔPQR = 2 (Perimeter of ΔABC)

Hence proved.

10) In fig., $BE \perp AC$, AD is any line from A to BC intersecting BE in H, P, Q and R are respectively the mid-points of AH, AB and BC. Prove that $\angle PQR = 90°$.

$\rightarrow$ Here, given that

$BE \perp^{er} AC$, and AD is any line from A to BC intersecting BE in H.

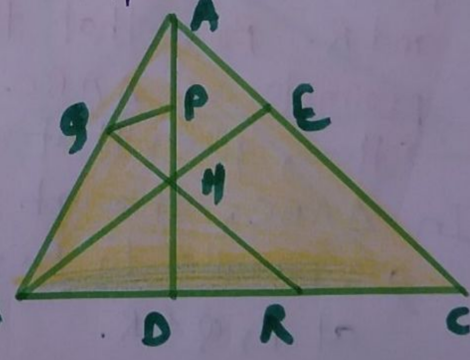
Also, P, Q, R are the midpoints of AH, AB and BC respectively.

Then By midpoint theorem,

$QR \parallel AC$ —— ①

Also, In ΔABH, Q and P are the midpoints of AB and AH respectively.

$\Rightarrow QP \parallel BH$ —— ②



But, given that $BE \perp AC$

from ① & ② $\Rightarrow$ $QP \perp QR$

$\Rightarrow$ $\boxed{\angle PQR = 90°}$

Hence proved.