Chapter 12. Heron's Formula

Exercise 12.1

1.) Find the area of a triangle whose sides are respectively 180cm, 120cm and 200cm.

→ Given that, the sides of a triangle are 150cm, 120cm, 200cm

According to Heron's formula,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

where semiperimeter ⇒ $s = \frac{(a+b+c)}{2}$

and a, b, c are the sides of a triangle respectively.

Here, $a = 150cm$, $b = 120cm$, $c = 200cm$

Now, $s = \frac{150+120+200}{2} = 235$ cm

Area of triangle $= \sqrt{235(235-150)(235-120)(235-200)}$

$= \sqrt{235(85)(115)(35)}$

$= \sqrt{80399375}$

$\boxed{\text{Area} = 8966.56 \text{ sq. cm}}$

2.) Find the area of a triangle whose sides are respectively 9cm, 12cm and 15cm.

→ According to Heron's formula,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

where, $s = \frac{(a+b+c)}{2}$ and a, b, c are the three sides of triangle.

Here, $a = 9cm$, $b = 12cm$, $c = 15cm$

Now, semiperimeter $= s = \frac{(a+b+c)}{2} = \frac{(9+12+15)}{2}$

$\boxed{s = 18 cm}$

And Area of triangle $= \sqrt{18(18-9)(18-12)(18-15)}$

$= \sqrt{18(9)(6)(3)}$

$= \sqrt{2916}$

$\boxed{A = 54 \text{ sq. cm}}$

3.) Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42 cm.

→ Given that, the two sides of a triangle are 18 cm and 10 cm.

The perimeter of a triangle is 42 cm.

That means, $a = 18$ cm, $b = 10$ cm and $2s = 42$ cm

$s = 21$ cm

we have, $s = \frac{a+b+c}{2}$

$2s = 18 + 10 + c$

$2s = 28 + c$

$\cancel{c = 2(42) - 28 = 84 - 28}$

$42 = 28 + c$

$c = 42 - 28 = 14$ cm. $\boxed{c = 14 cm}$

Now, Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-18)(21-10)(21-14)}$

$= \sqrt{21 \times 3 \times 11 \times 7}$

$= \sqrt{4851}$

$= 21\sqrt{11}$

$\boxed{Area = 21\sqrt{11} \text{ sq. cm}}$

4.) In a triangle ABC, AB = 15 cm, BC = 13 cm and AC = 14 cm find the area of a triangle ABC and its altitude on AC.

→ Let us consider the sides of a triangle ABC are

$AB = a$, $BC = b$, $AC = c$ respectively.

Given that, $a = 15$ cm, $b = 13$ cm, $c = 14$ cm

According to Heron's formula,

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad \& \text{ Semiperimeter} = \frac{(a+b+c)}{2}$$

where a, b, c are sides of triangle respectively.

$$\text{Now, } s = \frac{(15+13+14)}{2} = 21$$

$$A = \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21 \times 8 \times 7 \times 6}$$

$$= \sqrt{7056}$$

$$\boxed{A = 84 \text{ cm}^2}$$

Let us consider, BE is the perpendicular on AC.

Then, Area of triangle $= \frac{1}{2}$ (Base) (Height)

$$84 = \frac{1}{2}(BE)(AC)$$

$$\boxed{BE = 12 \text{ cm}}$$

Thus, altitude on AC is found to be $BE = 12$ cm.

5.) The perimeter of a triangular field is 540m and its sides are in the ratio 25 : 17 : 12. Find the area of the triangle.

→ Let us consider, the sides of a given triangle be respectively.

$a = 25x$, $b = 17x$ and $c = 12x$ respectively.

Given that, Perimeter of triangle = 540 cm

$$\Rightarrow 2s = a + b + c$$

$$a + b + c = 540$$

$$25x + 17x + 12x = 540$$
$$54x = 540 \text{ cm}$$
$$\boxed{x = 10 \text{ cm}}$$

Thus, the sides of a triangle are

$a = 250$ cm , $b = 170$ cm , $c = 120$ cm.

And Semiperimeter $= s = \frac{(a+b+c)}{2}$

$$= \frac{540}{2}$$
$$\boxed{s = 270 \text{ cm}}$$

According to Heron's formula,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{270(270-250)(270-170)(270-120)}$$
$$= \sqrt{270(20)(100)(150)}$$
$$= \sqrt{81000000}$$
$$\boxed{A = 9000 \text{ cm}^2}$$

Thus, the area of the triangle is found to be 9000 cm^2.

Exercise 12.2

1.) Find the area of the quadrilateral ABCD in which $AB = 3$ cm , $BC = 4$ cm , $CD = 4$ cm , $DA = 5$ cm , $AC = 5$ cm.

→ Here, for quadrilateral ABCD

$AB = 3$ cm, $BC = 4$ cm , $CD = 4$ cm , $DA = 5$ cm, $AC = 5$ cm

Then, $\left(\begin{array}{c}\text{Area of quadrilateral} \\ ABCD\end{array}\right) = A(\Delta ABC) + (\Delta ADC)$ —— (1)

Also, ΔABC is a right-angled triangle with $\angle B = 90°$.

$$A(\Delta ABC) = \frac{1}{2}(\text{Base})(\text{Height})$$
$$= \frac{1}{2}(AB)(BC)$$

$$A(\Delta ABC) = \frac{1}{2}(3\times 4) = 6$$

$$\boxed{A(\Delta ABC) = 6\,cm^2} \quad \text{--- (2)}$$

Now, In ΔCAD

According to Heron's formula

$A = \sqrt{s(s-a)(s-b)(s-c)}$ & semiperimeter $= \frac{(a+b+c)}{2}$

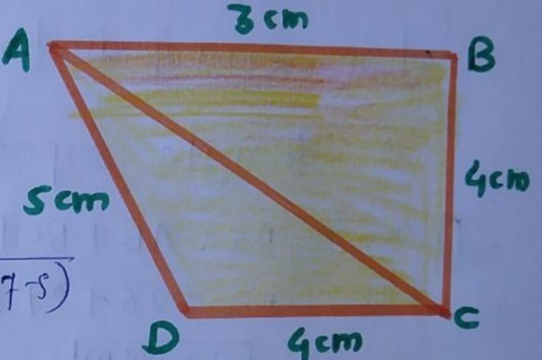
where, a, b, c are sides of triangle respectively.

Perimeter $= 2s = AC + CD + DA$

$2s = 5 + 4 + 5$

$2s = 14\,cm$

$\boxed{s = 7\,cm}$



Now, $A(\Delta CAD) = \sqrt{7(7-5)(7-4)(7-5)}$

$= \sqrt{7\times 2\times 3\times 2}$

$= 2\sqrt{21}$

$\boxed{A(\Delta CAD) = 9.16\ cm^2}$

Now, $A(\Delta CAD) = 9.16\ cm^2$ --- (3)

from (1), (2) and (3),

(Area of quadrilateral ABCD = 6 + 9.16)

$\boxed{A(\square ABCD) = 15.16\,cm^2}$

2.) The sides of a quadrilateral field, taken in order are 26 m, 27 m, 7m, 24m respectively. The angle contained by the last two sides is a right angle. Find its area.

→ Here, given that sides of a quaddrilateral field.

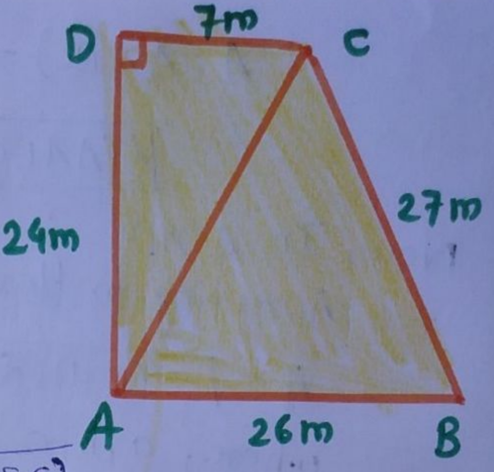
Let us consider a quadrilateral with ABCD with sides

$AB = 26m$, $BC = 27m$, $CD = 7m$, $DA = 24m$

Let AC is the diagonal formed after joining point A to point C.

Now, In ΔADC,

By Pythagoras Theorem,



$$AC^2 = AD^2 + CD^2$$

$$AC^2 = 14^2 + 7^2$$

$$\boxed{AC = 25}$$

According to Heron's formula,

$$A(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$

where s = semiperimeter $= \dfrac{(a+b+c)}{2}$ & a, b, c are the sides of triangle respectively.

Then, $2s = AB + BC + CA$

$$2s = 26 + 27 + 25$$

$$\boxed{s = 39\,m}$$

$$A(\Delta ABC) = \sqrt{39(39-25)(39-26)(39-27)}$$

$$= \sqrt{39(14)(13)(12)}$$

$$= \sqrt{85176}$$

$$\boxed{A(\Delta ABC) = 291.84\ m^2}$$

for $A(\Delta ADC) = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times 7 \times 24$$

$$= 84\ m^2$$

$$\Rightarrow \boxed{A(\Delta ADC) = 84 m^2}$$

Now, $\left(\begin{array}{c}\text{Area of rectangular} \\ \text{field } ABCD\end{array}\right) = A(\Delta ABC) + A(\Delta ADC)$

$$= 291.84 + 84$$

$$\boxed{A(\square ABCD) = 375.8\ m^2}$$

3.) The sides of a quadrilateral, taken in order as 5, 12, 14, 15 meters respectively, and the angle contained by first two side is right angle. find its area.

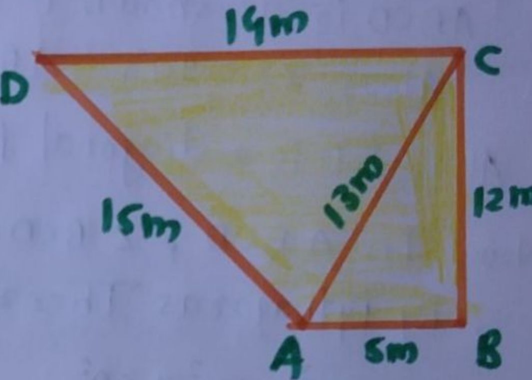
→ Here, given that the sides of a quadrilateral are 5, 12, 14, 15 m respectively.

Let us consider a quadrilateral ABCD as shown in fig.

Here, $AB = 5m$, $BC = 12m$, $CD = 14m$

and $DA = 15m$

Now, we joined diagonal AC Here.



$$A(\triangle ABC) = \frac{1}{2}(AB)(BC)$$

$$= \frac{1}{2}(5)(12)$$

$$\boxed{A(\triangle ABC) = 30}\ m^2$$

In $\triangle ABC$, $\angle ABC = 90°$

Then, Pythagoras Theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 5^2 + 12^2$$

$$AC^2 = 25 + 144 = 169$$

$$\boxed{AC = 13m}$$

Now, In $\triangle ADC$ $AD = 15m$, $DC = 14m$, $AC = 13m$

Then By Heron's Formula,

$$A(\triangle ADC) = \sqrt{s(s-a)(s-b)(s-c)}$$

where, $s = \frac{(a+b+c)}{2}$ and a, b, c are sides of a triangle.

Perimeter of $\triangle ADC = 2s = AD + AC + DC$

$$2s = 15 + 13 + 14$$

$$2s = 42m$$

$$\boxed{s = 21m}$$

$$A(\triangle ADC) = \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21 \times 8 \times 7 \times 6} = 84\ m^2$$

Now, $A(\square ABCD) = A(\Delta ABC) + A(\Delta ADC)$

$= (30 + 84)$

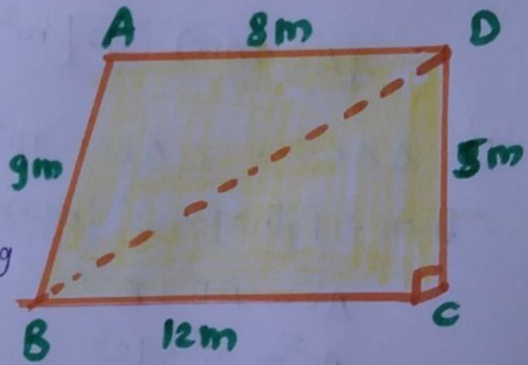
$\boxed{A(\square ABCD) = 114 m^2}$

4) A park in the shape of a quadrilateral ABCD, has $\angle C = 90^\circ$, $AB = 9m$, $BC = 12m$, $CD = 5m$, $AD = 8m$. How much area does it occupy?

→ Here given that, the shape of park is like a quadrilateral ABCD is as shown in fig.

Also, $\angle C = 90^\circ$, $AB = 9m$, $BC = 12m$, $CD = 5m$, $AD = 8m$.

Also, BD is a diagonal formed here.



Now, In ΔBCD, $\angle BCD = 90^\circ$

By Pythagoras Theorem

$BD^2 = BC^2 + CD^2$

$BD^2 = 12^2 + 5^2 = 144 + 25 = 169$

$\boxed{BD = 13m}$

$A(\Delta BCD) = \frac{1}{2}(BC)(CD)$

$= \frac{1}{2}(12)(5)$

$\boxed{A(\Delta BCD) = 30 m^2}$

Now, In ΔABD, By Heron's formula

$A(\Delta ABD) = \sqrt{s(s-a)(s-b)(s-c)}$

where semiperimeter $= 2s = \frac{(a+b+c)}{2}$ & a, b, c are sides of triangle respectively.

Perimeter of $\Delta ABD = 2s = 9m + 8m + 13m = 15m$

$A(\Delta ABD) = \sqrt{15(15-9)(15-8)(15-13)}$

$= \sqrt{15(6)(7)(2)}$

$= 6\sqrt{35}$

$\boxed{A(\Delta ABD) = 35.49 m^2}$

$A(\square ABCD) = A(\triangle ABD) + A(\triangle BCD)$

$= (35.496 + 30)$

$\boxed{A(\square ABCD) = 65.5 m^2}$

5.) Two parallel sides of a trapezium are 60m & 77 m and the other sides are 25m and 26m. Find the area of the trapezium.

→ Given that, two parallel sides of a trapezium are 60m & 77m and the other sides are 25m and 26m.

Here, Let us consider a trapezium ABCD as shown in fig.

From fig. $AB = 77m$, $CD = 60m$

$BC = 26m$, $AD = 25m$

Here, AE and CF are the diagonals.

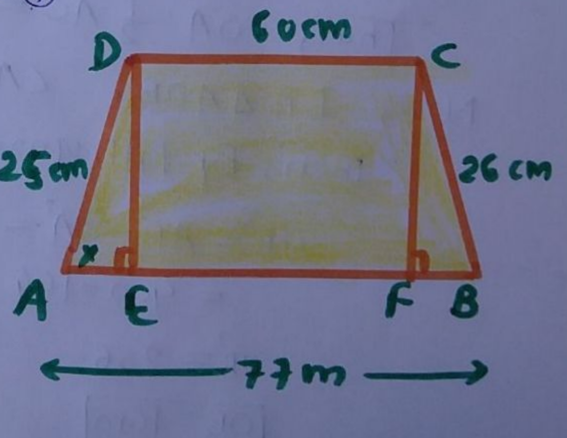
Let DE & CF are two perpendiculars on AB.

Thus, $DC = EF = 60m$

Let us consider, $AE = x$

Then, $BF = 77 - (60 + x)$

$BF = 17 - x$ —— ①



In $\triangle ADE$, $\angle AED = 90°$

By Pythagoras theorem,

$DE^2 = AD^2 - AE^2$

$DE^2 = 25^2 - x^2$ —— ②

Also, In $\triangle BCF$, $\angle BFC = 90°$

By Pythagoras Theorem,

$CF^2 = BC^2 - BF^2$

$CF^2 = 26^2 - (17-x)^2$ $\quad \because DE = CF$

$DE^2 = 26^2 - (17-x)^2$

$(25^2 - x^2) = 26^2 - (17-x)^2$

$625 - x^2 = 676 - (289 - 34x + x^2)$

$625 - x^2 = 676 - 289 + 34x - x^2$

$238 = 34x$

$\boxed{x = 7}$

② ⇒ $DE^2 = 25^2 - (7)^2$

$DE^2 = 625 - 49$

$\boxed{DE = 24}$

A(trapezium ABCD) $= \frac{1}{2}(60+77)\,24$

$= 1644\ m^2$

6.) Find the area of a rhombus whose perimeter is 80m & one of whose diagonal is 24m.

→ Here, given that perimeter of rhombus = 80 m and one of its diagonal is 24m.

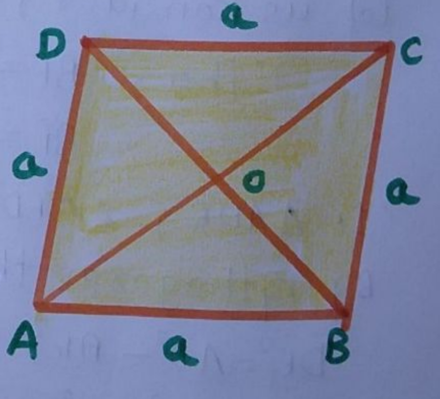
Let us consider a rhombus ABCD as shown in fig.

Perimeter of rhombus = 80 m

$4 \times (\text{side}) = 80\ m$

$4(a) = 80$ Let 'a' be side of rhombus

$\boxed{a = 20}$



And $AC = 24\ m$

Thus, $OA = \frac{1}{2} AC = 12\ m$

Now, In $\triangle AOB$, $\angle AOB = 90°$

Then, By Pythagoras Theorem,

$OB^2 = AB^2 - OA^2 = 20^2 - 12^2$

$= 400 - 144$

$OB^2 = 256$

$\boxed{OB = 16m}$

As, diagonals of rhombus bisects each other at 90°.

$\Rightarrow OB = OD$

Thus $BD = 2OB = 2(16) = 32\ m$

A(Rhombus) $= \frac{1}{2}(BD)(AC) = \frac{1}{2}(32)(24) = 384\ m^2$

7) A rhombus sheet, whose perimeter is 32m and whose diagonal is 10m long, is painted on both the sides at the rate of Rs. 5 per m^2. Find the cost of painting.

→ Here, given that

Perimeter of rhombus = 32m

But, Perimeter of rhombus = 4 (side)

Let 'a' be the side of a rhombus.

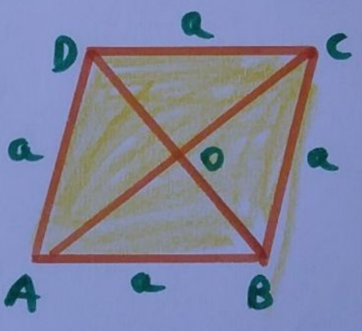
Then $4a = 32$

$\boxed{a = 8m}$

⇒ $AC = 10m$

Then, from fig. $OA = \frac{1}{2}(AC) = \frac{1}{2}(10)$

$\boxed{OA = 5m}$



Now, In ΔAOB, $\angle AOB = 90°$

By Pythagoras Theorem,

$OB^2 = AB^2 - OA^2$

$= 8^2 - 5^2$

$OB^2 = 64 - 25$

$OB^2 = 39 \Rightarrow \boxed{OB = \sqrt{39}\, m}$

And $BD = 2(OB) = 2\sqrt{39}\, m$

Thus, Area of Sheet $= \frac{1}{2}(BD)(AC) = \frac{1}{2}(2\sqrt{39})(10) = 10\sqrt{39}$

Thus the area of sheet is found to be $10\sqrt{39}\, m^2$.

Hence, the cost of painting on both sides of the sheet, at the rate of rs. 5 per m^2 $= 2(10\sqrt{39} \times 5)$

$= 625$ Rs.