Chapter 9 : Arithmetic Progressions

Exercise 9.1

1) Write the first terms of each of the following sequences whose nth term are:

i) $a_n = 3n+2$

→ Given that, $a_n = 3n+2$

To find first five terms put $n = 1, 2, 3, 4, 5$

$a_1 = 3(1)+2 = 5$

$a_2 = 3(2)+2 = 6+2 = 8$

$a_3 = 3(3)+2 = 9+2 = 11$

$a_4 = 3(4)+2 = 12+2 = 14$

$a_5 = 3(5)+2 = 15+2 = 17$

Thus, the required first five terms of given sequence are

5, 8, 11, 14, 17.

ii) $a_n = (n-2)/3$

→ Given that, $a_n = \frac{(n-2)}{3}$

put $n = 1, 2, 3, 4, 5$

$a_1 = \frac{1-2}{3} = \frac{-1}{3}$ $\qquad a_2 = \frac{2-2}{3} = 0$

$a_3 = \frac{3-2}{3} = \frac{1}{3}$ $\qquad a_4 = \frac{4-2}{3} = \frac{2}{3}$

$a_5 = \frac{5-2}{3} = 1$

The required first five terms are $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, 1$.

iii) $a_n = 3^n$

put $n = 1, 2, 3, 4, 5$

$a_1 = 3^1 = 3$

$a_2 = 3^2 = 9$

$a_3 = 3^3 = 27$

$a_4 = 3^4 = 81$

$a_5 = 3^5 = 243$

Thus, the required first five terms are 3, 9, 27, 81, 243.

iv) $a_n = (3n-2)/5$

$\rightarrow$ Given that $a_n = \frac{(3n-2)}{5}$

put $n = 1, 2, 3, 4, 5$

$a_1 = \frac{(3\times1-2)}{5} = \frac{(3-2)}{5} = \frac{1}{5}$

$a_2 = \frac{(3\times2-2)}{5} = \frac{(6-2)}{5} = \frac{4}{5}$

$a_3 = \frac{(3\times3-2)}{5} = \frac{(9-2)}{5} = \frac{7}{5}$

$a_4 = \frac{(3\times4-2)}{5} = \frac{(12-2)}{5} = \frac{10}{5} = 2$

$a_5 = \left(\frac{3\times5-2}{5}\right) = \left(\frac{15-2}{5}\right) = \frac{13}{5}$

The required first five terms are $1/5, 4/5, 7/5, 10/5, 13/5$.

v) $a_n = (-1)^n 2^n$

$\rightarrow$ Given that, $a_n = (-1)^n 2^n$

put $n = 1, 2, 3, 4, 5$

$a_1 = (-1)^1 2^1 = -2$

$a_2 = (-1)^2 2^2 = 4$

$a_3 = (-1)^3 2^3 = -8$

$a_4 = (-1)^4 2^4 = 16$

$a_5 = (-1)^5 2^5 = -32$

The required first five terms are $-2, 4, -8, 16, -32$.

vii) $a_n = n^2 - n + 1$

$\rightarrow$ put $n = 1, 2, 3, 4, 5$

$a_1 = 1^2 - 1 + 1 = +1$

$a_2 = 4 - 2 + 1 = 3$

$a_3 = 9 - 3 + 1 = 7$

$a_4 = 4^2 - 4 + 1 = 13$

$a_5 = 5^2 - 5 + 1 = 21$

Thus, the required first five terms are $+1, 3, 7, 13, 21$.

viii) $a_n = 2n^2 - 3n + 1$

$\rightarrow$ put $n = 1, 2, 3, 4, 5$

$a_1 = 2(1)^2 - 3(1) + 1 = 2 - 3 + 1 = 0$

$a_2 = 2(2)^2 - 3(2) + 1 = 8 - 6 + 1 = 3$

$a_3 = 2(3)^2 - 3(3) + 1 = 18 - 9 + 1 = 10$

$a_4 = 2(4)^2 - 3(4) + 1 = 32 - 12 + 1 = 21$

$a_5 = 2(5)^2 - 3(5) + 1 = 50 - 15 + 1 = 36$

Thus, the required first five terms are 0, 3, 10, 21, 36.

Exercise 9.2

1) Show that the sequence defined by $a_n = 5n + 7$ is an A.P. find its common difference.

→ Given sequence is $a_n = 5n - 7$ — ①

put $n = 1, 2, 3, 4$ in ①

$a_1 = 5(1) - 7 = 5 - 7 = -2$

$a_2 = 5(2) - 7 = 10 - 7 = 3$

$a_3 = 5(3) - 7 = 15 - 7 = 8$

$a_4 = 5(4) - 7 = 20 - 7 = 13$

~~$a_5 = 5(5$~~ Here, $a_2 - a_1 = 3 - (-2) = 5$

$a_3 - a_2 = 8 - 3 = 5$

$a_4 - a_3 = 13 - 8 = 5$

Thus, here difference between two successive numbers is same & hence the given sequence ① is A.P.

And the common difference is 5.

2) show that the sequence defined by $a_n = 3n^2 - 5$ is not A.P.

→ Given sequence is $a_n = 3n^2 - 5$ — ①

put $n = 1, 2, 3, 4$

$a_1 = 3(1)^2 - 5 = 3 - 5 = -2$

$a_2 = 3(2)^2 - 5 = 12 - 5 = 7$

$a_3 = 3(3)^2 - 5 = 22$

$a_4 = 3(4^2) - 5 = 48 - 5 = 43$

we can see, $a_2 - a_1 = 7 + 2 = 9$

$a_3 - a_2 = 22 - 7 = 15$

$a_4 - a_3 = 43 - 22 = 21$

Here, the difference between two successive terms is not same. And hence the given sequence ① is not A.P.

3) The general terms of a sequence is given by $a_n = -4n+15$. Is the sequence an A.P.? If so, find its 15th term & common difference?

→ Given sequence is

$$a_n = -4n+15 \quad \text{—— ①}$$

put $n = 1, 2, 3, 4$

$a_1 = -4+15 = 11$

$a_2 = -4(2)+15 = -8+15 = 7$

$a_3 = -4(3)+15 = -12+15 = 3$

$a_4 = -4(4)+15 = -16+15 = -1$

And here,

$a_2 - a_1 = 7 - 11 = -4$

$a_3 - a_2 = 3 - 7 = -4$

$a_4 - a_3 = -1 - 3 = -4$.

As, the difference between to consecutive numbers or terms is same. The given sequence ① is a an A.P.

The common difference found to be (-4).

Now, $a_{15} = -4(15)+15 = 15(-4+1) = 15(-3) = -45$

$\boxed{\text{Exercise 9.3}}$

1) For the following arithmetic progression write the first term a and the dommon difference d:

i) $-5, -1, 3, 7, \ldots$

→ we know that 'a' is the first term & 'd' is the common difference then arithmetic progression is

$a, a+d, a+2d, a+3d, \ldots$

Here, $-5, -1, 3, 7$

$a = -5, \quad a+d = -1, \quad a+2d = 3, \quad a+3d = 7$

But $a = -5$

$\Rightarrow -5+d = -1$

$\boxed{d = 4}$

First term $= a = -5$ and, common difference $= d = 4$

ii) $1/5, 3/5, 5/5, 7/5, \ldots$

→ Here, given A.P. is $1/5, 3/5, 5/5, 7/5, \ldots$

$\Rightarrow \boxed{a = 1/5}$ $\quad a+d = 3/5, \quad a+2d = 5/5, \quad a+3d = 7/5$

$\Rightarrow \quad 1/5 + d = 3/5$

$d = 3/5 - 1/5 = 2/5 \quad \Rightarrow \boxed{d = 2/5}$

Thus, first term $a = 1/5$ and common difference $d = 2/5$.

iii) $0.3, 0.55, 0.80, 1.05, \ldots$

→ Given arithmetic series is $0.3, 0.55, 0.80, 1.05, \ldots$

$\Rightarrow \boxed{a = 0.3}$ $\quad a+d = 0.55$

$a+2d = 0.80$

$a+3d = 1.05$

$\Rightarrow a+d = 0.55$

$0.3 + d = 0.55$

$d = 0.55 - 0.3 = 0.25 \quad \boxed{d = 0.25}$

Thus, first term $= 0.3$, common difference $= d = 0.25$

iv) $-1.1, -3.1, -5.1, -7.1, \ldots$

→ Given series is $-1.1, -3.1, -5.1, -7.1, \ldots$

$\Rightarrow \boxed{a = -1.1}$ $\quad a+d = -3.1 \quad \Rightarrow a+d = -3.1$

$a+2d = -5.1 \quad -1.1 + d = -3.1$

$a+3d = -7.1 \quad d = -3.1 + 1.1$

$\boxed{d = -2}$

Thus, first term $= a = -1.1$

common difference $= d = -2$

2) Write the arithmetic progression when first term a & common difference 'd' are as follows:

i) $a=4$, $d=-3$

→ we know that, if first term is 'a' and common difference is 'd' then the terms of A.P. are $a, a+d, a+2d, a+3d, \ldots$

Thus, $a=4$

$a+d = 4-3 = 1$

$a+2d = 4+2(-3) = 4-6 = -2$

$a+3d = 4+3(-3) = 4-9 = -5$

$a+4d = 4+4(-3) = 4-12 = -8$

Thus, the required arithmetic progression is $1, -2, -5, -8, \ldots$

ii) $a=-1$, $d=1/2$

→ $a=-1$

$a+d = -1+1/2 = -1/2$

$a+2d = -1+2(1/2) = -1+1 = 0$

$a+3d = -1+3(1/2) = -1+3/2 = 1/2$

Thus, the required A.P. is $-1, -1/2, 0, 1/2, \ldots$

iii) $a=-1.5$, $d=-0.5$

→ $a=-1.5$

$a+d = -1.5-0.5 = -2$

$a+2d = -1.5-2(0.5) = -1.5-1.0 = -2.5$

$a+3d = -1.5-3(0.5) = -1.5-1.5 = -3$

Thus, the required A.P. is $-1.5, -2, -2.5, -3, \ldots$

3) In which of the following situations, the sequence of numbers formed will form an A.P?

i) The cost of digging a well for the first meter is Rs. 150 and rises by Rs. 20 for each succeeding meter.

→ Given that,

Cost of digging a well for first meter = 150 Rs.

And cost is increasing by Rs. 20 for each succeeding meter.

Then, cost of digging a well for second meter = 170 Rs.

cost of digging a well for third meter = 170 + 20
= 190 Rs.

Thus, here the cost of digging a well for different lengths are 150, 170, 190, 210, ...

And hence, this is an A.P.

$\boxed{\text{first term} = a = 170}$ $\boxed{\text{common difference} = d = 20}$

ii) The amount of air present in the cylinder when a vacuum pump removes each time 1/4 of their remaining in the cylinder.

→ Here given that

Let initial volume of air in a cylinder is 'V'.

The air remaining in the cylinder at each time is 3/4.

first time, air in cylinder is V.

Second time, the air in the cylinder is $\frac{3}{4}V$.

Third time, the air in the cylinder is $(3/4)^2 V$.

Thus, the series formed here is $V, (3/4)V, (\frac{3}{4})^2 V, \ldots$

Hence, the series formed is not A.P.

iii) Divya deposited 1000 Rs. at compound interest at the rate of 10% per annum. The amount at the end of first year, second year, third year, ... & so on.

→ Given that,

Divya deposited Rs. 1000 at compound interest of 10% p.a.

Thus, the amount at the end of first year

$= 1000 + 0.1(1000) = \text{Rs. } 1100$

The amount at the end of second year is

$= 1100 + 0.1(1100) = \text{Rs. } 1210$

The amount at the end of third year is

$= 1210 + 0.1(1210) = \text{Rs. } 1331$

As the difference betn the successive terms is not same. The ~~fom~~ series formed is not an A.P.

Exercise 9.4

1. find i) 10th term of AP 1, 4, 7, 10, ...

→ Given A.P. is 1, 4, 7, 10, ...

Here, first term is $a=1$

Common difference $(d) = 4-1=3$

The nth term of an A.P. is given by

$$t_n = a+(n-1)d$$

To find 10th term of an A.P.

put $n=10$, $t_{10} = 1+(10-1)\times 3$

$$= 1+9\times 3 = 28$$

∴ The tenth term of given A.P. is 28.

ii) nth term of the A.P. 13, 8, 3, -2, ...

→ Given A.P. is 13, 8, 3, -2, ...

Here, the first term is $a=13$

Common difference $= 8-13 = -5$

The nth term of an A.P. is given by

$$t_n = a+(n-1)d$$
$$= 13+(n-1)(-5)$$
$$= 13-5n+5$$

$$\boxed{t_n = 18-5n}$$

is the required nth term of an given A.P.

iii) 18th term of the AP $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$

→ Given AP is $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, \ldots$

Here, first term is $a=\sqrt{2}$

Common difference $= 3\sqrt{2}-\sqrt{2} = 2\sqrt{2}$

nth term of an AP is given by

$$t_n = a+(n-1)d$$

$$t_{18} = \sqrt{2} + (18-1)(2\sqrt{2})$$
$$= \sqrt{2} + 17(2\sqrt{2})$$
$$\boxed{t_{18} = 35\sqrt{2}}$$

Here, the 18th term of an A.P. is $35\sqrt{2}$.

iv) 10th term of the AP $-40, -15, 10, 35, \ldots$

→ Given AP is $-40, -15, 10, 35, \ldots$

The first term is $a = -40$

Common difference $= -15 + 40 = 25$

The nth term of an AP is given by

$$t_n = a + (n-1)d$$
$$= -40 + (10-1)(25)$$
$$= -40 + 9(25) = 185$$

Thus, 10th term of an A.P. is 185.

v) 8th term of the AP $117, 104, 91, 78, \ldots$

→ Given AP is $117, 104, 91, 78, \ldots$

first term $= a = 117$

Common difference $= d = 104 - 117 = -13$

The nth term of an AP is given by

$$t_n = a + (n-1)d$$
$$t_8 = 117 + (8-1)(-13)$$
$$= 117 - 91$$
$$= 26$$

Thus, 8th term of an AP is 26.

vi) 11th term of the AP 10.0, 10.5, 11.0, 11.2, ...

→ Given AP is 10.0, 10.5, 11.0, 11.2, ...

first term = a = 10.0

Common difference = d = 10.5 − 10.0 = 0.5

The nth term of an AP is given by

$$t_n = a + (n-1)d$$

$$t_{11} = 10 + (11-1)(0.5)$$

$$= 10 + 10 \times 0.5$$

$$= 10 + 5$$

$$\boxed{t_{11} = 15}$$

Thus, here 11th term is 15.

2. i) Which term of the AP 3, 8, 13, ... is 248?

→ Here, given AP is 3, 8, 13, ...

first term = a = 3

Common difference = 8 − 3 = 5

The nth term is given by,

$$t_n = a + (n-1)d$$

$$248 = 3 + (n-1)5$$

$$248 = -2 + 5n$$

$$5n = 250$$

$$n = 250/5 = 50$$

Thus, here 50th term of given A.P. is 248.

ii) Which term of the A.P. 84, 80, 76, ... is 0?

→ Here, given A.P. is 84, 80, 76, ...

first term = a = 84

Common difference = 80 − 84 = −4

The nth term of an AP is given by

$t_n = a + (n-1)d$

$\cancel{84} \cdot 0 = 84 + (n-1)(-4)$

$84 = 4(n-1)$

$(n-1) = 84/4 = 21$

$n = 21+1 = 22$

Thus, 22^{nd} term of given A.P is 0.

iii) Which term of the AP 4, 9, 14, ... is 254?

→ Here, the given AP is 4, 9, 14, ...

first term = a = 4

Common difference = 9 − 4 = 5

The n^{th} term of an AP is given by,

$t_n = a + (n-1)d$

$254 = 4 + (n-1)(5)$

$250 = 5(n-1)$

$250 + 5 = 5n$

$\frac{255}{5} = n \Rightarrow \boxed{n = 51}$

Thus, 51^{st} term of given AP is 254.

iv) Which term of the AP 21, 42, 63, 84, ... is 420?

→ Given AP is 21, 42, 63, 84, ...

Here, first term = a = 21

Common difference = 42 − 21 = 21

The n^{th} term of an AP is given by

$t_n = a + (n-1)d$

$420 = 21 + (n-1)(21)$

$399 = 21n - \cancel{21}$

$\frac{399}{21} = n$

$\boxed{n = 19}$

Thus, 19^{th} term of given AP is 420.

3. i) Is 68 a term of the A.P. 7, 10, 13, ...?

→ Given AP is 7, 10, 13, ...

first term = $a = 7$, $d = 10 - 7 = 3$

The nth term of an A.P. is given by,

$$t_n = a + (n-1)d$$

$$68 = 7 + (n-1)3$$

$$68 = 7 + 3n - 3$$

$$68 = 4 + 3n$$

$$3n = 64 \Rightarrow n = 64/3$$

$\boxed{n = 64/3}$ is the number which is not a whole number.

Thus, 68 is not the term of given A.P.

ii) Is 302 a term of the A.P. 3, 8, 13, ...?

→ Here, the given A P is 3, 8, 13, ...

first term = $a = 3$,

Common difference = $d = 8 - 3 = 5$

The nth term of an AP is given by,

$$t_n = a + (n-1)d$$

$$302 = 3 + (n-1)5$$

$$302 = 3 + 5n - 5$$

$$304 = 5n$$

$\boxed{n = \frac{304}{5}}$ is not a whole number.

Thus, the term 302 is not a term of given A.P.

iii) Is -150 a term of the A.P. $11, 8, 5, 2, \ldots$?

→ Here, the given A.P. is $11, 8, 5, 2, \ldots$

first term $= a = 11$

Common difference $= 8 - 11 = -3$

The nth term of an A.P. is given by

$$t_n = a + (n-1)d$$

$$-150 = 11 + (n-1)(-3)$$

$$-150 = 11 - 3n + 3$$

$$-150 - 14 = -3n$$

$$164 = 3n$$

$\boxed{n = \frac{164}{3}}$ is not a whole number.

Thus, -150 is not a term of given A.P.

4. How many terms are there in A.P.

i) $7, 10, 13, \ldots, 43$

→ Given AP is $7, 10, 13, \ldots 43$

$a = 7$, $d = 13 - 10 = 3$

The nth term of an AP is given by

$$t_n = a + (n-1)d$$

$$43 = 7 + (n-1)3$$

$$43 = 7 + 3n - 3$$

$$43 = 4 + 3n$$

$$39 = 3n$$

$$n = 39/3 = 13$$

Thus, there are total 13 terms in given A.P.

ii) $-1, -5/6, -2/3, -1/2, \ldots, 10/3$.

$\rightarrow$ Here, the given A.P is $-1, -5/6, -2/3, -1/2, \ldots, 10/3$.

first term $= a = -1$

Common difference $= \frac{-5}{6} + 1 = \frac{-5+6}{6} = \frac{1}{6}$

The nth term of an AP is given by

$$t_n = a + (n-1)d$$

$$\frac{10}{3} = -1 + (n-1)\frac{1}{6}$$

$$\frac{10}{3} = -1 + \frac{n}{6} - \frac{1}{6}$$

$$\frac{10}{3} + \frac{1}{6} + 1 = \frac{n}{6} \Rightarrow \frac{2(10) + 1 + 6}{6} = \frac{n}{6}$$

$$20 + 7 = n$$

$$\boxed{n = 27}$$

Thus, there are total 27 terms present in given A.P.

iii) $7, 13, 19, \ldots, 205$

$\rightarrow$ Here, the given A.P is $7, 13, 19, \ldots, 205$

first term $= a = 7$

Common difference $= d = 13 - 7 = 6$

The nth term of an AP is given by

$$t_n = a + (n-1)d$$

$$205 = 7 + (n-1)6$$

$$205 = 7 + 6n - 6 = 1 + 6n$$

$$204 = 6n$$

$$n = 204/6 = 34 \quad \boxed{n = 34.}$$

Thus, there are total 34 terms in the given A.P.

iv) 18, 15½, 13, ..., −47.

→ Here, the given A.P. is 18, 15½, 13, ..., −47.

First term = $a = 18$

Common difference = $d = 15\frac{1}{2} - 18 = -5/2$

The nth term of an A.P. is given by

$$t_n = a + (n-1)d$$

$$-47 = 18 + (n-1)(-5/2)$$

$$-47 = 18 - \frac{5n}{2} + \frac{5}{2}$$

$$\Rightarrow \quad 36 - 5n + 5 = -94$$

$$5n = 94 + 36 + 5$$

$$5n = 135$$

$$\boxed{n = 27}$$

Thus, there are total 27 terms in given A.P.

5. The first term of an AP is 5, the common difference is 3 and the last term is 80; find the number of terms.

→ Given that, $a = 5$, $d = 3$, last term = 80

The nth term of an AP is given by

$$t_n = a + (n-1)d$$

$$80 = 5 + (n-1)3$$

$$80 = 5 + 3n - 3$$

$$80 - 2 = 3n$$

$$78 = 3n \quad \Rightarrow \quad n = 78/3 = 26$$

Thus, the total number of terms in given A.P. are 26.

6. The 6th & 17th terms of an A.P. are 19 & 41 respectively, find the 40th term.

→ Here, given that, $a_6 = 19$ and $a_{17} = 41$

The nth term of an A.P. is given by,

$$t_n = a + (n-1)d$$

for $t_6 \Rightarrow$ $t_6 = a + (6-1)d = a + 5d$

and $a + 5d = 19$ —— ①

for $t_{19} \Rightarrow$ $t_{17} = a + (17-1)d = a + 16d$

$a + 16d = 41$ —— ②

② − ① $\Rightarrow$ $a + 16d - (a + 5d) = 41 - 19$

$11d = 22$

$\boxed{d = 2}$

put in ① $\Rightarrow$ $a + 5(2) = 19$

$a = 19 - 10$

$\boxed{a = 9}$ → first term of A.P.

The 40th term should be $\Rightarrow t_{40} = a + (40-1)d$

$= 9 + 39(2)$

$= 9 + 78 = 87$.

7. If 9th term of an A.P. is zero, prove it's 29th term is double the 19th term.

→ Here, given that, $a_9 = 0$ or $t_9 = 0$

The nth term of an A.P. is given by

$$t_n = a + (n-1)d$$

for $n = 9$, $t_9 = a + (9-1)d$

$0 = a + 8d$

$a + 8d = 0$ —— ①

Now, $t_{29} = a+(29-1)d$

$t_{29} = a+28d$

$= a+8d+20d$

$t_{29} = (a+8d)+20d$

$\boxed{t_{29} = 20d}$ $\because$ from 1 ——— 2

Similarly, $t_{19} = a+(19-1)d$

$t_{19} = a+18d = a+8d+10d$

$\boxed{t_{19} = 10d}$ $\because$ from 1 ——— 3

Now, compairing 2 & 3,

$\boxed{t_{29} = 2t_{19}}$

Thus, 29^{th} term is double of the 19^{th} term.

8. If 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term, show that 25^{th} term of the A.P. is zero.

→ Given that, 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term.

The n^{th} term of an A.P. is given by,

$t_n = a+(n-1)d$

But, given that $10(t_{10}) = 15(t_{15})$

$10[a+(10-1)d] = 15[a+(15-1)d]$

$10[a+9d] = 15[a+14d]$

$10a+90d = 15a+210d$

$5a+120d = 0$

$5(a+24d) = 0$

$a+24d = 0$

$a+(25-1)d = 0$

$\Rightarrow a+(25-1)d = t_{25}$ $\Rightarrow$ $\boxed{t_{25} = 0}$

Thus, 25^{th} term of an A.P. is 0.

9. The 10th & 18th term of an A.P. are 41 & 73 respectively. Find 26th term.

→ Given that, $t_{10} = 41$, $t_{18} = 73$

we know that, the nth term of an A.P. is given by

$$t_n = a + (n-1)d$$

$t_{10} = a + (10-1)d$ and $t_{18} = a + (18-1)d$

$41 = a + 9d$ —— ① and $73 = a + 17d$ —— ②

② − ① ⇒ $a + 17d - a - 9d = 73 - 41$

$8d = 32$

$\boxed{d = 4}$

⇒ $a + 9(4) = 41$

$a = 41 - 36 = 5$ $\boxed{a = 5}$ → first term

Now, 26th term should be $t_{26} = 5 + (26-1)4$

$= 5 + 25(4) = 5 + 100$

$= 105$

Thus, 26th term is found to be 105.

10. In a certain A.P. the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.

→ Given term is 10th term. $2t_{10} = t_{24}$.

But, nth term of an AP is given by

$$t_n = a + (n-1)d$$

$2[a + (10-1)d] = a + (24-1)d$

$2(a + 9d) = a + 23d$

$2a - a = 23d - 18d$

$\boxed{a = 5d}$ —— ①

Now, 72nd term should be

$t_{72} = a + (72-1)d$

$= a + 71d$

$$t_{72} = a + 5d + 66d$$
$$= a + a + 66d \quad \text{from } ①$$
$$= 2(a + 33d)$$
$$= 2[a + (34-1)d]$$
$$\boxed{t_{72} = 2t_{34}}$$

Thus, the 72^{nd} term is twice the 34^{th} term of the given A.P.

11. The 26^{th}, 11^{th} & the last term of an A.P. are 0, 3 and $-1/5$ respectively. Find the common difference and the number of terms.

→ Given that, $t_{26} = 0$, $t_{11} = 3$ and $t_n = -1/5$.

The nth term of an A.P. is given by

$$t_n = a + (n-1)d$$

$$t_{26} = a + (26-1)d$$
$$0 = a + 25d$$
$$a + 25d = 0 \quad ①$$

$$t_{11} = a + (11-1)d$$
$$3 = a + 10d$$
$$a + 10d = 3 \quad ②$$

$$① - ② \Rightarrow \quad a + 25d - a - 10d = 0 - 3$$
$$15d = -3$$
$$\boxed{d = -1/5} \quad \text{put in } ①$$

$$a + 25\left(\frac{-1}{5}\right) = 0$$
$$a - 5 = 0$$
$$\boxed{a = 5}$$

$$t_n = -1/5$$
$$a + (n-1)d = -1/5$$
$$5 + (n-1)(-1/5) = -1/5$$
$$5 - \frac{n}{5} + \frac{1}{5} = -\frac{1}{5}$$
$$25 - n + 1 = -1$$
$$\boxed{n = 27}$$

Thus, the given A.P. has 27 terms and the common difference is found to be $-1/5$.

13. Find the 12th term from the end of the following A.P.

i) 3, 5, 7, 9, ... 201

→ The nth term of an A.P. is given by

$t_n = a + (n-1)d$, $a = 3$, $d = 7 - 5 = 2$

Now, $201 = a + (n-1)d$

$201 = 3 + (n-1)2$

$201 = 3 + 2n - 2$

$200 = 2n$

$\boxed{n = 100}$ Thus, given AP has 100 terms total.

Thus, $t_{89} = 3 + (89-1)2$

$= 3 + 88(2)$

$= 3 + 176$

$\boxed{t_{89} = 179}$

Thus, 12th term from the end of the A.P. is 179.

ii) 3, 8, 13, ..., 253

→ Given A.P. is 3, 8, 13, ..., 253

Thus, $a = 3$, $d = 8 - 3 = 5$

The nth term of an A.P. is given by,

$t_n = a + (n-1)d$

$253 = 3 + (n-1)5$

$253 = 3 + 5n - 5$

$255 = 5n$

$\boxed{n = 51}$

Now,

$t_{40} = 3 + (40-1)5$

$= 3 + 39(5)$

$= 3 + 195$

$\boxed{t_{40} = 198}$

Thus, the given A.P. has 51 terms.

Thus, 40th terms is the required 12th term from the end of the given A.P., and which is 198.

14. The 4th term of an A.P. is three times the first and the 7th term exceeds twice the third term by 1. Find the first term and the common difference.

→ Let us consider for an A.P.

'a' be the first term and

'd' be the common difference.

Then, nth term of an A.P. is given by

$$t_n = a + (n-1)d$$

But, given that $t_4 = 3(a)$

$$\Rightarrow a + (4-1)d = 3a$$

$$a + 3d = 3a$$

$$3d = 2a$$

$$\boxed{a = 3d/2} \quad \text{——— (1)}$$

And 7th term exceeds twice the third term by 1.

$$\Rightarrow t_7 = 2t_3 + 1$$

$$a + (7-1)d = 2[a + (3-1)d] + 1$$

$$a + 6d = 2[a + 2d] + 1$$

$$a + 6d = 2a + 4d + 1$$

$$2d = a + 1$$

$$a - 2d + 1 = 0 \quad \text{——— (2)}$$

from (1) & (2) $\Rightarrow$

$$\frac{3d}{2} - 2d + 1 = 0$$

$$3d - 4d + 2 = 0$$

$$-d + 2 = 0$$

$$\boxed{d = 2} \quad \text{put in (1)}$$

$$\Rightarrow a = \frac{3(2)}{2}$$

$$\boxed{a = 3}$$

Thus, the first term of an A.P. is 3.
And common difference is 2.

15. How many numbers of two digit are divisible by 3?

→ The first two digit number which is divisible by 3 is 12.

And the last two digit number which is divisible by 3 is 99.

Thus, the A.P. formed here is 12, 15, 18, 21, ..., 99.

$a = 12$, $d = 3$

The nth term of an A.P. is given by

$$t_n = a + (n-1)d$$

$$99 = 12 + (n-1)3$$

$$99 = 12 + 3n - 3$$

$$99 = 9 + 3n$$

$$3n = 90$$

$$\boxed{n = 30}$$

Thus, there are total 30 two digit numbers which are divisible by 3.

16. An A.P. consists of 60 terms. If the first & the last terms be 7 and 125 respectively, find 32nd term.

→ Given that, an A.P. consists of 60 terms.

$a = 7$ and $t_{60} = 125$

The nth term of an A.P. is given by

$$t_n = a + (n-1)d$$

$$125 = 7 + (60-1)d$$

$$125 = 7 + 59d$$

$$59d = 118$$

$$\boxed{d = 2}$$

Thus, $t_{32} = 7 + (32-1)2 = 7 + 62 = 69$

$$\boxed{t_{32} = 69}$$

18. The sum of 4th & 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P.

→ In given A.P.

The sum of 4th & 8th terms of an A.P. is 24.

$\Rightarrow t_4 + t_8 = 24$

The nth term of an A.P. is given by

$t_n = a + (n-1)d$

$\Rightarrow [a+(4-1)d] + [a+(8-1)d] = 24$

$a + 3d + a + 7d = 24$

$2a + 10d = 24$

$\boxed{a + 5d = 12}$ —— ①

Also, given that $t_6 + t_{10} = 34$

$\Rightarrow [a+(6-1)d] + [a+(10-1)d] = 34$

$a + 5d + a + 9d = 34$

$2a + 14d = 34$

$\boxed{a + 7d = 17}$ —— ②

② − ① $\Rightarrow$ $17 - 12 = a + 7d - a - 5d$

$5 = 2d$

$\boxed{d = 5/2}$ put in ①

$a + 5(5/2) = 12$

$a + 25/2 = 12$

$2a + 25 = 24$

$2a = -1$

$\boxed{a = -1/2}$

Thus, here first term is $-1/2$ and common difference is $5/2$.

19. The first term of an A.P. is 5 and its 100th term is -292. find the 50th term of this A.P.

$\rightarrow$ For given A.P. $a=5$ and $t_{100}=-292$

The nth term of an A.P. is given by

$$t_n = a+(n-1)d$$

$$t_{100} = 5+(100-1)d$$

$$-292 = 5+99d$$

$$99d = -297$$

$$\boxed{d=-3}$$

Thus, the 50th term should be,

$$t_{50} = a+(50-1)d = a+49d$$

$$t_{50} = 5+49(-3) = 5-147 = -142$$

20. find $(t_{30}-t_{20})$ for the A.P.

i) $-9, -14, -19, -24$

$\rightarrow$ The given A.P. is

$-9, -14, -19, -24, \ldots$

$a=-9$, $d=-14+9=-5$

Thus,

$t_{30}-t_{20}$

$\Rightarrow [a+(30-1)d]-[a+(20-1)d]$

$= [a+29d]-[a+19d]$

$= 29d-19d$

$= 10d$

$= 10(-5) = -50$

Thus, $\boxed{t_{30}-t_{20}=-50}$

ii) $a, a+d, a+2d, a+3d, \ldots$

$\rightarrow$ The given A.P. is

$a, a+d, a+2d, a+3d, \ldots$

Here, $a=a$, $d=a+d-a=d$

Thus, $t_{30}-t_{20}$

$\Rightarrow [a+(30-1)d]-[a+(20-1)d]$

$= [a+29d]-[a+19d]$

$= 29d-19d$

$= 10d$

Thus, $\boxed{t_{30}-t_{20}=10d}$

21. Write the expression $t_n - t_k$ for A.P. $a, a+d, a+2d, \ldots$
Hence, find the common difference of the A.P. for which
i) 11th term is 5 and 13th term is 79.

→ Given A.P. is $a, a+d, a+2d, \ldots$
And nth term of an A.P. is given by
$$t_n = a+(n-1)d = a+nd-d$$
And $t_k = a+(k-1)d = a+kd-d$

Now, $t_n - t_k = (a+nd-d)-(a+kd-d)$
$$t_n - t_k = (n-k)d$$

i) Given that, $t_{11} = 5$ and $t_{13} = 79$
$$t_{13} - t_{11} = (13-11)d = 2d$$
$$\Rightarrow 79-5 = 2d$$
$$d = 74/2 = 37 \quad \Rightarrow \boxed{d=37}$$

ii) $t_{10} - t_5 = 200$
→ $t_{10} - t_5 = 200$
$(10-5)d = 200$
$5d = 200$
$\boxed{d=40}$

iii) 20th term is 10 more than 18th term.
→ $\cancel{t_{13} - t_{11} = (13-11)d = 2d}$
$\cancel{79-5}$
$t_{20} - t_{18} = 10$
$(20-18)d = 10$
$2d = 10$
$\boxed{d=5}$

22. The eighth term of an A.P. is half of its second term and the eleventh term exceeds one third of it's fourth term by 1. Find the 15th term.

→ For given A.P. $t_8 = \frac{1}{2}(t_2)$
$$t_{11} = \frac{1}{3}(t_4) + 1$$
The nth term of an A.P. is given by
$$t_n = a+(n-1)d$$

$\Rightarrow t_8 = \frac{1}{2}(t_2)$

$a+7d = \frac{1}{2}(a+d)$

$2a+14d = a+d$

$a+13d = 0$ —— ①

$t_{11} = \frac{1}{3}(t_4)+1$

$(a+10d) = \frac{1}{3}(a+3d)+1$

$a+10d - 1 = \frac{1}{3}(a+3d)$

$3a+30d-3 = a+3d$

$2a+27d = 3$ —— ②

② $- 2\times$ ① $\Rightarrow$

$$\begin{array}{r} 2a+27d=3 \\ 2a+26d=0 \\ - \quad - \quad - \\ \hline d=3 \end{array}$$

put $d=3$ in ①

$\Rightarrow a+13(3)=0$

$a+39=0$

$\boxed{a=-39}$

Thus, the 15th term of given A.P. $= -39+14(3) = -39+42 = 3$.

23. Find the arithmetic progression whose third term is 16 and the seventh term exceeds it's fifth term by 12.

→ for given A.P. $t_3 = 16$, $t_7 = t_5 + 12$

We know that, The nth term of an A.P. is given by

$t_n = a+(n-1)d$

$t_3 = a+(3-1)d$

$16 = a+2d$ —— ①

And, $a+6d = a+4d+12$

$2d = 12$

$\boxed{d=6}$ put in ①

$16 = a+12$

$\boxed{a=4}$

Thus, the given A.P is 4, 10, 16, 22, ...

24. The 7^{th} term of an A.P. is 32 and it's 13^{th} term is 62. Find the A.P.

$\longrightarrow$ Given that, $t_7 = 32$ and $t_{13} = 62$

Then, $t_n - t_k = (a+nd-d) - (a+kd-d)$

$t_n - t_k = (n-k)d$

But, $t_{13} - t_7 = (13-7)d = 62-32 = 30$

$6d = 30$

$\boxed{d=5}$

Now, $t_7 = a+(7-1)d$

$32 = a+6d$

$32 = a+30$

$\boxed{a=2}$

Thus, the given A.P. is found to be $2, 7, 12, 17, \dots$

25. Which term of the A.P. $3, 10, 17, \dots$ will be 84 more than its 13^{th} term?

$\longrightarrow$ Given A.P. is $3, 10, 17, \dots$

So, $a=3$, $d = 10-3 = 7$

From given condition,

$t_n = t_{13} + 84$

$t_n = a+(n-1)d \Rightarrow \quad a+(n-1)d = a+(13-1)d+84$

$3+(n-1)7 = 3+(12)7+84$

$3+7n-7 = 3+84+84$

$7n = 168+7$

$n = 175/7$

$\boxed{n=25}$

Thus, the 25^{th} term which is 84 more than it's 13^{th} term.

Exercise 9.5

1. Find the value of x for which $(8x+4), (6x-2)$ & $(2x+7)$ are in A.P.

$\longrightarrow$ Given that, $(8x+4)$, $(6x-2)$, $(2x+7)$ are in A.P.

The common difference is given by

$$(6x-2)-(8x+4) = (2x+7)-(6x-2)$$

$$6x-2-8x-4 = 2x+7-6x+2$$

$$-2x-6 = -4x+9$$

$$2x = 9+6$$

$$2x = 15$$

$$\boxed{x = 15/2}$$

2. If $(x+1)$, $3x$ and $(4x+2)$ are in A.P. find the value of x.

$\longrightarrow$ Given that, $(x+1)$, $3x$ and $(4x+2)$ are in A.P.

Then, the common difference is given by

$$(3x)-(x+1) = (4x+2)-3x$$

$$3x-x-1 = 4x+2-3x$$

$$2x-1 = x+2$$

$$\boxed{x = 3}$$

3. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in A.P.

$\longrightarrow$ Given that, $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in A.P.

Then, $\boxed{2b = a+c}$ is the condition satisfied by a, b, c for in A.P.

$$\Rightarrow \quad 2(a^2+b^2) = (a-b)^2 + (a+b)^2$$

$$2(a^2+b^2) = a^2+b^2-2ab+a^2+b^2+2ab$$

$$2(a^2+b^2) = 2a^2+2b^2 = 2(a^2+b^2)$$

$\therefore$ LHS = RHS

Hence proved.

4. The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find three terms.

→ Let us consider, $a-d, a, a+d$ are the three terms of an A.P.

So, the sum of three terms = 21

$a-d+a+a+d=21$

$3a=21$

$\boxed{a=7}$

And $(t_1)(t_3) = t_2+6$

$(a-d)(a+d) = a+6$

$a^2-d^2 = a+6$

$(7)^2-d^2 = 7+6$

$49-d^2=13$

$49-13 = d^2$

$36 = d^2$

$\boxed{d=6}$

Thus, the three terms of given A.P. are

$7-6, 7, 7+6 \Rightarrow 1, 7, 13.$

5. Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers.

→ Let us consider, the three terms of an A.P. be

$a-d, a, a+d.$

Sum of the numbers = 27

$a-d+a+a+d=27$

$3a=27$

$\boxed{a=9}$

product of numbers = 648

$(a-d)(a)(a+d) = 648$

$a(a^2-d^2)=648$

$a^2-648/a=d^2$

$$9^2 - (648/9) = d^2$$
$$729 - 648 = 9d^2$$
$$81 = 9d^2$$
$$d^2 = 9$$
$$\boxed{d = 3 \text{ or } -3}$$

Thus, the required three terms are: $9-3, 9, 9+3$

$6, 9, 12$

or $12, 9, 6$ (when $d = -3$)

6. Find the four numbers in A.P. whose sum is 50 and in which the greatest number is 4 times the least

→ Let us consider, the four terms of an A.P. be $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$.

From given condition,

sum of these terms = 50

$$(a-3d) + (a-d) + (a+d) + (a+3d) = 50$$
$$a - 3d + a - d + a + d + a - 3d = 50$$
$$4a = 50$$
$$\boxed{a = 25/2}$$

And, greater number = 4 × least no.

$$a + 3d = 4(a - 3d)$$
$$a + 3d = 4a - 12d$$
$$4a - a = 3d + 12d$$
$$3a = 15d$$
$$\boxed{a = 5d}$$

$$a = 5d$$
$$\frac{25}{2} = 5d \Rightarrow d = \frac{25}{10} = \frac{5}{2} \Rightarrow \boxed{d = \frac{5}{2}}$$

Thus, the required four terms are:

$(a-3d) = (25/2 - 3 \times 5/2)$, $(a-d) = (25/2 - 5/2)$

$(a+d) = (25/2 + 5/2)$ & $(a+3d) = (25/2 - 3 \times 5/2)$

↳ 5, 10, 15, 20.

Exercise 9.6

1. Find the terms of the following arithmetic progression.

i) 50, 46, 42, ... to 10 terms.

→ The sum of 'n' terms of an A.P. is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where, a - first term

d - common difference

i) Given A.P. is 50, 46, 42, ... , to 10 terms.

$n=10 \Rightarrow \quad S_{10} = \frac{10}{2}[2(50) + (10-1)(-4)]$

$\because a = 50, \quad d = 46 - 50 = -4$

$\Rightarrow \quad S_{10} = 5[100 + 9(-4)] = 5[100 - 36] = 5 \times 64$

$\boxed{S_{10} = 320}$

ii) 1, 3, 5, 7, ... to 12 terms.

→ Given A.P. is 1, 3, 5, 7, ... to 12 terms

Here, $n = 12, \quad a = 1, \quad d = 3 - 1 = 2$

$S_{12} = \frac{12}{2}[2(1) + (12-1)2]$

$= 6[2 + 22] = 6(24)$

$\boxed{S_{12} = 144}$

iii) 3, 9/2, 6, 15/2, ... to 25 terms.

→ Given A.P. is 3, 9/2, 6, 15/2, ... to 25 terms.

Here, $a = 3, \quad d = \frac{9}{2} - 3 = \frac{9-6}{2} = \frac{3}{2}, \quad n = 25$

$S_{25} = \frac{25}{2}[2(3) + (25-1)\frac{3}{2}]$

$= \frac{25}{2}[6 + 24 \times \frac{3}{2}] = \frac{25}{2}[6 + 36]$

$= \frac{25}{2}[42] = 25(21)$

$\therefore \boxed{S_{25} = 525}$

iv) $(a+b), (a-b), (a-3b), \dots$ to 22 terms

$\rightarrow$ Here, first term $= a+b$

Common difference $= a-b-a-b = -2b$, $n = 22$

$\therefore S_{22} = \frac{22}{2}[2(a+b) + (22-1)(-2b)]$

$= 11[2a+2b+21(-2b)]$

$= 11[2a+2b-42b]$

$S_{22} = 11(2a-40b)$

$\therefore \boxed{S_{22} = 22a - 440b}$

v) $(x-y)^2, (x^2+y^2), (x+y)^2$, to 22 terms

$\rightarrow$ Here, first term $= a = (x-y)^2$

common difference $= d = (x^2+y^2) - (x-y)^2 = (x^2+y^2) - (x^2-2xy+y^2)$

$= x^2+y^2-x^2+2xy-y^2$

$\boxed{d = 2xy}$ and $\boxed{n = 22}$

Thus, $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{22} = \frac{22}{2}[2(x-y)^2 + (22-1)2xy]$

$= 11[2(x^2+y^2-2xy) + (21)(2xy)]$

$= 22[x^2+y^2-2xy+21xy]$

$\boxed{S_{22} = 22[x^2+y^2+19xy]}$

vi) $-26, -24, -22, \dots$ to 36 terms.

$\rightarrow$ $a = -26$, $d = -24+26 = 2$, $n = 36$

$\therefore S_{36} = \frac{36}{2}[2(-26) + (36-1)(2)]$

$= 18[-52+70]$

$= 18 \times 18$

$= 324$

$\therefore \boxed{S_{36} = 324}$

2. Find the sum to n terms of the A.P. 5, 2, −1, −4, −7, ...

→ Given A.P. is 5, 2, −1, −4, −7, ...

$\Rightarrow a = 5,\ d = 2 - 5 = -3.$

Here, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{n}{2}[2(5) + (n-1)(-3)]$

$= \frac{n}{2}[10 - 3(n-1)]$

$= \frac{n}{2}(13 - 3n)$

$\boxed{\therefore S_n = \frac{n}{2}(13 - 3n)}$

3. Find the sum of n terms of an A.P. whose the term is given by $t_n = 5 - 6n$.

→ The nth term of given A.P. is $t_n = 5 - 6n$

put n=1 $\Rightarrow t_1 = 5 - 6 = -1$

put n=2 $\Rightarrow t_2 = 5 - 12 = -7$

Common difference $= -7 + 1 = -6$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2(-1) + (n-1)(-6)]$

$= \frac{n}{2}[-2 - 6n + 6] = \frac{n}{2}(-6n + 4)$

$S_n = n(-3n + 2)$

$\therefore \boxed{S_n = n(2 - 3n)}$

4. Find the sum of last ten terms of the A.P.: 8, 10, 12, 14, ..., 126.

→ Given terms of A.P. are 8, 10, 12, 14, ... 126.

$a = 8,\ d = 10 - 8 = 2$

$t_n = a + (n-1)d$

Thus, $126 = 8 + (n-1)2$

$126 = 8 + 2n - 2$

$2n = 120$

$\boxed{n = 60}$

Now, $t_{51} = 8 + (50)(2) = 108$

Thus, $t_{51} + t_{52} + t_{53} + \cdots + t_{60}$.

Since, $n = 10$, $a = 108$, $l = 126$

$$S = \frac{n}{2}[a + l] = \frac{10}{2}[108 + 126]$$

$$\boxed{S = 1170}$$

Thus, the sum of last ten terms of the A.P. is 1170.

5. Find the sum of first 15 terms of each of the following sequences having nth term as:

i) $a_n = 3 + 4n$

$\longrightarrow$ The nth term of an A.P. is $a_n = 3 + 4n$

The sum of n terms of an A.P. is

$$S_n = \frac{n}{2}(a + l)$$

where, $a \to$ first term $l \to$ last term

$n = 1 \Rightarrow$ $t_1 = 3 + 4 = 7$

$n = 15 \Rightarrow$ $t_{15} = 3 + 4(15) = 63$

Thus, $S_n = \frac{15}{2}(17 + 63)$

$= 15 \times 35$

$$\boxed{S_{15} = 525}$$

Thus, the sum of the 15 terms of the given A.P. is found to be 525.

ii) $t_n = 5+2n$

→ Given, n^{th} term of an A.P. is $t_n = 5+2n$

And sum of n^{th} term of an A.P. is

$$S_n = \frac{n}{2}(a+l) \qquad a \to \text{first term}, \quad l \to \text{last term}$$

$n=1 \Rightarrow t_1 = 5+2 = 7$

$n=15 \Rightarrow t_{15} = 5+2(15) = 35$

Thus, $S_n = \frac{15}{2}(7+35)$

$= 15 \times 21$

$$\boxed{S_{15} = 315}$$

Thus, the sum of 15 terms of the given A.P. is 315.

iii) $t_n = 6-n$

→ Given n^{th} term of an A.P. is $t_n = 6-n$

The sum of n^{th} term of an A.P. is given by

$$S_n = \frac{n}{2}(a+l) \qquad a \to \text{first term}, \quad l \to \text{last term}$$

$n=1 \Rightarrow t_1 = 6-1 = 5$

$n=15 \Rightarrow t_{15} = 6-15 = -9$

Thus, $S_{15} = \frac{15}{2}(5-9)$

$$\boxed{S_{15} = -30}$$

Thus, the sum of 15 terms of the given A.P. is -30.

6. Find the sum of first 20 terms of the sequence whose nth term is $t_n = An + B$.

→ Given, nth term of an A.P. is

$$t_n = An + B$$

The sum of 'n' terms of an A.P. is given by,

$$S_n = \frac{n}{2}(a + l) \quad a \rightarrow \text{first term}, \ l \rightarrow \text{last term}$$

put $n = 1 \Rightarrow t_1 = A + B$

put $n = 20 \Rightarrow t_{20} = 20A + B$

$$\Rightarrow S_{20} = \frac{20}{2}[A + B + 20A + B]$$

$$= 10(21A + 2B)$$

$$\boxed{S_{20} = 210A + 20B}$$

7. Find the sum of first 25 terms of an A.P. whose nth term is given by $t_n = 2 - 3n$.

→ Given, nth term of an A.P. is $t_n = 2 - 3n$

The sum of nth term of an A.P. is given by

$$S_n = \frac{n}{2}(a + l) \quad a \rightarrow \text{first term}, \ l \rightarrow \text{last term}$$

put $n = 1 \Rightarrow t_1 = 2 - 3 = -1$

put $n = 25 \Rightarrow t_{25} = 2 - 75 = -73$

Thus, $S_n = \frac{25}{2}(-1 - 73)$

$$S_{25} = 25(-37)$$

$$\boxed{S_{25} = -925}$$

Thus, the sum of 25 terms of the given A.P. is -925.

8. Find the sum of first 25 terms of an A.P. whose nth term is given by $t_n = 7-3n$.

→ Given nth term of an A.P. is $t_n = 7-3n$.

The sum of 'n' terms of an A.P. is given by

$$S_n = \frac{n}{2}(a+l)$$

a → first term

l → last term

put $n=1 \Rightarrow t_1 = 7-3 = 4$

put $n=25 \Rightarrow t_{25} = 7-75 = -68$

Thus, $S_{25} = \frac{25}{2}(4-68)$

$= 25 \times (-32)$

$$\boxed{S_{25} = -800}$$

9. i) How many terms of the sequence 18, 16, 14, ... should be taken so that their sum is zero.

→ Given A.P. is 18, 16, 14, ...

$a = 18$, $d = 16-18 = -2$

we know that, The sum of 'n' terms of an A.P.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_n = \frac{n}{2}[36+(n-1)(-2)]$$

$$S_n = n[18-(n-1)]$$

$$0 = n(18-n+1)$$

$0 = n(19-n) \Rightarrow n=0$ or $19-n=0$

$\boxed{n=0}$ or $\boxed{n=19}$

But, n=0 should not be possible.

Thus, the total number of terms present are 19.

ii) How many terms are there in the A.P. whose first and fifth terms are -14 and 2 respectively & the sum of the terms is 40?

→ Given that, $a=-14$, $t_5=2$, $S_n=40$.

Let 'd' be the common difference, then

$$t_5 = a+4d$$
$$2 = -14+4d$$
$$16 = 4d$$
$$\boxed{d=4}$$

But, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$40 = \frac{n}{2}[2(-14)+(n-1)(4)]$$
$$= \frac{n}{2}[-28+4n-4]$$
$$= \frac{n}{2}[-32+4n]$$
$$40(2) = -32n+4n^2$$
$$\Rightarrow 4n^2-32n-80=0$$
$$n^2-8n-20=0$$
$$n^2-10n+2n-20=0$$
$$n(n-10)+2(n-10)=0$$
$$(n-10)(n+2)=0$$
$$\boxed{n=10} \text{ or } \boxed{n=-2}$$

But, $n=-2$ is not possible.

Thus, the sum of 10 terms is 40.

iii) How many terms of the A.P. is 27, 24, 21, ... should be taken that their sum is zero?

→ Given A.P. is 27, 24, 21, ...

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$a=27$, $d=24-27=-3$

$0 = \frac{n}{2}[2(27)+(n-1)(-3)]$

$0 = n[54+(n-1)(-3)]$

$0 = n[54-3n+3]$

$0 = n(57-3n)$

$\boxed{n=0}$ or $57-3n=0$

$57 = 3n$

$\boxed{n=19}$

But, $n=0$ is not possible.

11. Find the sum of the first

i) 11 terms of the A.P: 2, 6, 10, 14, ...

→ The sum of 'n' terms of an A.P. is given by

$S_n = \frac{n}{2}[2a+(n-1)d]$

$a \to$ first term $d \to$ common difference

Here, $a=2$, $d=6-2=4$, $n=11$

Thus, $S_{11} = \frac{11}{2}[2(2)+(11-1)4]$

$= \frac{11}{2}[4+40] = \frac{11}{2}(44)$

$= 11 \times 22$

$\boxed{S_{11} = 242}$

Thus, the sum of first 11 terms of given A.P. is 242.

ii) 13 terms of the A.P.: −6, 0, 6, 12, ...

→ Here, $a=-6$, $d=0+6=6$, $n=13$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{13} = \frac{13}{2}[2(-6)+(13-1)6]$

$= 13[-6+12\times3]$

$= 13[-6+36]$

$S_{13} = 13\times30 = 390$

$\boxed{S_{13} = 390}$

iii) 51 terms of the A.P : whose second term is 2 and fourth term is 8.

→ Here, given that $t_2 = 2$, $t_4 = 8$

$t_2 = a+d$ $\qquad$ $t_4 = a+3d$

$\boxed{2 = a+d}$ —— ① $\qquad$ $\boxed{8 = a+3d}$ —— ②

② − ① ⇒ $8-2 = a+3d-a-d$

$6 = 2d$

$\boxed{d=3}$ put in ①

$2 = a+3$

$\boxed{a=-1}$

Thus, $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{51} = \frac{51}{2}[2(-1)+(51-1)3]$

$= \frac{51}{2}[\cancel{2}+50 \times 3]$

$= 51(-1+25 \times 3)$

$= 51(-1+75)$

$= 51(74)$

$\boxed{S_{51} = 3774}$

Thus, the sum of first 51 terms of an A.P. is 3774.

12. Find the sum of

i) the first 15 multiples of 8

→ The sum of 'n' terms of an A.P. is given by

$S_n = \frac{n}{2}[2a+(n-1)d]$

a → first term

d → common difference

The first 15 multiples of 8 forms the series

8, 16, 24, ..., 120

⇒ $a = 8$, $d = 16-8 = 8$, $n = 15$,

$S_n = \frac{15}{2}[2(8)+(15-1)8]$

$= \frac{15}{2}[16 + 14\times 8]$

$= \frac{15}{2}(128)$

$\boxed{S_{15} = 960}$

Thus, the sum of first 15 multiples of 8 is found to be 960

ii) All 3-digit natural numbers which are divisible by 13.

→ The first 3-digit number which is divisible by 13 is 104 and the last 3 digit number which is divisible by 13 is 988.

Thus, $a = 104$, $d = 13$, $l = 988$.

$t_n = a + (n-1)d$

$988 = 104 + (n-1)13$

$988 = 91 + 13n$

$13n = 897$

$\boxed{n = 69}$

Now, $S_{69} = \frac{69}{2}[2(104) + (69-1)13]$

$= \frac{69}{2}[208 + 884]$

$= \frac{69}{2}(1092) = 69(546)$

$\boxed{S_{69} = 37674}$

Thus, the sum of all 3-digit 69 numbers is 37674.

13. Find the ~~te~~ sum:

i) $2+4+6+\cdots+200$

$\longrightarrow$ $S_n = \frac{n}{2}[2a+(n-1)d]$

$a=2,\quad d=4-2=2,\quad t_n=200$

$t_n = a+(n-1)d$

$200 = 2+(n-1)2$

$200 = 2+2n-2$

$200 = 2n$

$\boxed{n=100}$

And

$S_{100} = \frac{100}{2}(a+l)$

$= \frac{100}{2}(2+200)$

$= 50(202)$

$\boxed{S_{100} = 10100}$

Thus, the sum of terms of the given series is 10100.

ii) $3+11+19+\cdots+803$

$\longrightarrow$ Here, $a=3,\quad d=11-3=8,\quad t_n=803$

$t_n = a+(n-1)d$

$803 = 3+(n-1)8$

$803 = 3+8n-8$

$n = \frac{808}{8} = 101$

$\boxed{n=101}$

Now,

$S_{101} = \frac{101}{2}[3+803]$

$= \frac{101}{2}(806)$

$= 101 \times 403$

$\boxed{S_{101} = 40703}$

Thus, the sum of terms of the given series is 40703.

iii) $(-5)+(-8)+(-11)+\cdots+(-230)$.

$\longrightarrow$ Here, $a=-5,\quad d=-8-(-5)=-3,\quad t_n=-230$

$t_n = a+(n-1)d$

$-230 = -5+(n-1)(-3)$

$-230 = -5-3n+3$

$3n = -2+230$

$n = \frac{228}{3} = 76$

$\boxed{n=76}$

Now,

$S_{76} = \frac{76}{2}(-5-230)$

$= 38(-235)$

$\boxed{S_{76} = -8930}$

Thus, the sum of terms of the given series is -8930.

iv) $1+3+5+7+\cdots+199$

→ Here, $a=1$, $d=3-1=2$, $t_n=199$

$t_n = a + (n-1)d$

$199 = 1+(n-1)2$

$199 = 1+2n-2$

$199 = 2n-1$

$200 = 2n$

$\boxed{n=100}$

$S_{100} = \frac{100}{2}(a+l)$

$= 50(1+199)$

$= 50 \times 200$

$\boxed{S_{100} = 10000}$

Thus, the sum of given terms of series is 10000.

v) $25+28+31+\cdots+100$

→ Here, $a=25$, $d=28-25=3$, $t_n=100$

$t_n = a+(n-1)d$

$100 = 25+(n-1)3$

$100 = 25+3n-3$

$100 = 25+3n-3$

$n = \frac{(100-22)}{3}$

$\boxed{n=26}$

Now, $S_{26} = \frac{26}{2}(a+l)$

$= 13(25+100)$

$= 13(125)$

$\boxed{S_{26} = 1625}$

Thus, the sum of given terms of series is 1625.

14. The first & the last terms of an A. P. are 17 and 350 respectively. If the common difference is 9, how many terms are there & what is their sum?

→ Given that, $a=17$ and $l=350$

The common difference $(d) = 350$

$t_n = a+(n-1)d$

$350 = 17+(n-1)9$

$350 = 17+9n-9$

$350 = 8+9n$

$350-8 = 9n$

$\boxed{n=38}$

Now,

$S_{38} = \frac{38}{2}(17+350)$

$= 19 \times 367$

$\boxed{S_{38} = 6973}$

16. The first term of an A.P. is 2 and the last term is 50. The sum of all these terms is 442. find the common difference.

Given that, $a = 2$, $l = 50$, $S_n = 442$.

Let 'd' be the common difference here.

$S_n = \frac{n}{2}(a+l)$

$442 = \frac{n}{2}(2+50)$

$442 = \frac{n}{2}(52)$

$26n = 442$

$\boxed{n = 17}$

$t_n = a + (n-1)d$

$t_{17} = 2 + (17-1)d$

$50 = 2 + 16d$

$48 = 16d$

$\boxed{d = 3}$

$\therefore$ Common difference $= 3$.

17. If 12^{th} term of an A.P. is -13 and the sum of first four terms is 24, what is the sum of first 10 terms?

Given that, $t_{12} = -13$, $S_4 = 24$

we have, $t_n = a + (n-1)d$

$t_{12} = a + (12-1)d$

$-13 = a + 11d$ — ①

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_4 = \frac{4}{2}[2(a) + (4-1)d]$

$24 = 2[2a + 3d]$

$12 = 2a + 3d$ — ②

② $- 2\times$① $\Rightarrow$

$12 + 26$ ~~$12 + 13$~~ $= 2a + 3d - 2a - 22d$

38 ~~25~~ $= -19d$

$\Rightarrow \boxed{d = -2}$

$\therefore$ $a = -13 - 11(-2)$

$a = -13 + 22$

$\boxed{a = 9}$

And

$S_{10} = \frac{10}{2}[2(9) + (10-1)(-2)]$

$= 5[18 + 9(-2)]$

$= 5(18 - 18)$

$\boxed{S_{10} = 0}$

Thus, the sum of first 10 terms of an A.P. is $S_{10} = 0$.

18. Find the sum of n terms of the series

$\rightarrow$ $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\cdots$

$\Rightarrow$ Here, $a = 4-\frac{1}{n}$, $d = 4-\frac{2}{n}-4+\frac{1}{n} = \frac{-1}{n}$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{n}{2}\left[2\left(4-\frac{1}{n}\right)+(n-1)\left(\frac{-1}{n}\right)\right]$$

$$= \frac{n}{2}\left[8-\frac{2}{n}+\left(-1+\frac{1}{n}\right)\right]$$

$$= \frac{n}{2}\left[\frac{8n-2-n+1}{n}\right]$$

$$\boxed{S_n = \frac{1}{2}(7n-1)}$$

Thus, the sum of 'n' terms of the series is $\frac{1}{2}(7n-1)$.

19. In an A.P., if the 5^{th} and 12^{th} terms are 30 & 65 respe. what is the sum of first 20 terms?

$\longrightarrow$ Here, $t_5 = 30$, $t_{12} = 65$

$t_n = a+(n-1)d$

$t_5 = a+4d$

$a = 30-4d$ —— ①

$t_{12} = a+(12-1)d$

$65 = a+11d$

$a = 65-11d$ —— ②

from ① & ② $\Rightarrow$ $30-4d = 65-11d$

$65-30 = +11d-4d$

$35 = 7d$

$\boxed{d=5}$

$\Rightarrow a = 30-20$

$\boxed{a=10}$.

$S_n = \frac{n}{2}\left[2a+(n-1).d\right]$

$S_{20} = \frac{20}{2}\left[2(10)+(20-1)5\right]$

$= 10\left[20+19\times 5\right]$

$= 10(115)$

$\boxed{S_{20} = 1150}$

20. Find the sum of first 51 terms of an A.P. whose second & third terms are 14 & 18 respectively.

$\rightarrow$ Let 'a' be first term & 'd' be common difference

$t_2 = 14$ & $t_3 = 18$

$t_n = a + (n-1)d$ $\qquad\qquad$ $t_3 = a + 2d$

$14 = a + d$ —— ① $\qquad\qquad$ $18 = a + 2d$ —— ②

② − ① $\Rightarrow$ $18 - 14 = a + 2d - a - d$

$\boxed{4 = d}$

$\Rightarrow 14 = a + 4$

$\boxed{a = 10}$

$n = 51$

Now,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{51} = \frac{51}{2}[2(10) + (51-1)4]$

$= \frac{51}{2}[20 + 50 \times 4]$

$= \frac{51}{2}[20 + 200]$

$= 51(10 + 100) = 51(110)$

$\boxed{S_{51} = 5610}$

Thus, the sum of first 51 terms of the given A.P. is 5610.

21. If the sum of 7 terms of an A.P. is 49 and that of 17 terms is 289, find the sum of n terms.

$\rightarrow$ Given that, the sum of 7 terms of an A.P. 49.

$S_7 = 49$

And sum of 17 terms of an A.P. is 289.

$S_{17} = 289$

$\Rightarrow S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = 49 = \frac{7}{2}[2a+(7-1)d]$

$49 = \frac{7}{2}(2a+6d)$

$49 = 7(a+3d)$

$7(a+3d) = 49$

$a+3d = 7$ —— ①

$S_{17} = 289 = \frac{17}{2}[2a+16d]$

$= \frac{17}{2}[2a+16d]$

$289 = 17(a+8d)$

$a+8d = 17$ —— ②

② − ① ⟹ $17-7 = a+8d-a-3d$

$10 = 5d$

$\boxed{d=2}$

⟹ $a+6=7$

$\boxed{a=1}$

Now,

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{n}{2}[2(1)+(n-1)2]$

$= \frac{n}{2}[2+2n-2]$

$= \frac{n}{2}(2n)$

$\boxed{S_n = n^2}$

Thus, the sum of n terms of the A.P. is n^2.

23. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and common difference.

→ Sum of first 'n' terms is $S_n = \frac{n}{2}[2a+(n-1)d]$

Here, $a=5$, $t_n = 45$ & $S_n = 400$

$t_n = a+(n-1)d$

$45 = 5+(n-1)d$

$nd-d = 40$ —— ①

$S_n = \frac{n}{2}[2a+(n-1)d]$

$400 = \frac{n}{2}[2(5)+(n-1)d]$

$800 = n(10+nd-d)$

$800 = n(10+40)$ ∵ from ①

$\boxed{n=16}$

put $n = 16$ in ①

$\Rightarrow nd - d = 40$

$16d - d = 40$

$15d = 40$

$\boxed{d = 8/3}$

Thus, the common difference for given A.P. is found to be 8/3.

24. In an A.P. the first term is 8, nth term is 33 and the sum of first n term is 123. Find n & the d.

→ Here, first term $= a = 8$

Last term $= l = 33$

Sum of all the terms $= S_n = 123$

Let 'd' be the common difference.

$\Rightarrow 123 = \frac{n}{2}(8+33)$

$\cancel{123 = \frac{(123 \times 2)}{41}}$ $\quad n = \frac{(123 \times 2)}{41}$

$\cancel{n =}$ $\quad n = \frac{246}{41}$

$\boxed{n = 6}$

Now, $t_n = a + (n-1)d$

$33 = 8 + (6-1)d$

$33 = 8 + 5d$

$5d = 25$

$\boxed{d = 5}$

Thus, the common difference for given A.P. is found to be 5.

25. In an A.P. the first term is 22, n^{th} term is -11 and the sum of first 'n' term is 66. Find 'n' and the 'd'.

→ Given that, first term $= a = 22$

Last term $= \cancel{66} = -11$

Sum of all terms $= S_n = 66$

Let us consider 'd' be the common difference.

$\Rightarrow 66 = \frac{n}{2}[22+(-11)]$

$\because S_n = \frac{n}{2}(a+l)$ a → first term
l → last term

$\Rightarrow 66 = \frac{n}{2}(22-11)$

$66 \times 2 = n \times 11$

$\boxed{n=12}$

we have, $t_n = a+(n-1)d$

$t_{12} = 22+(12-1)d$

$-11 = 22+11d$

$11d = -33$

$\boxed{d=-3}$

Thus, the number of terms $n=12$ & common difference $d=-3$

26. The first and the last terms of an A.P. are 7 and 49 respe. If sum of all its terms is 420. Find the common difference.

→ Given that, first term $= a = 7$

Last term $= l = 49$

Sum of all terms $= 420$

$t_n = a+(n-1)d$

$49 = 7+(n-1)d$

$42 = nd-d$

$nd-d = 42$ — ①

$S_n = \frac{n}{2}[2a+(n-1)d]$

$420 = \frac{n}{2}[14+nd-d]$

$840 = n[14+42]$ ∵ from ①

$840 = 54n$

$\boxed{n=15}$ — ②

∵ from ① & ② ⇒ $nd - d = 42$

$15d - d = 42$

$14d = 42$

$\boxed{d = 3}$

Thus, the common difference $= d = 3$.

27. The first & last term of an A.P. are 5 and 45 respectively. If the sum of all its terms is 400. Find common difference.

→ Given that, first term $= a = 5$

Last term $= l = 45$

Sum of all terms $= S_n = 400$

$t_n = a + (n-1)d$

$45 = 5 + (n-1)d$

$40 = nd - d$ —— ①

$S_n = \frac{n}{2}[2a + (n-1)d]$

$400 = \frac{n}{2}[10 + nd - d]$

$800 = n[10 + 40]$ ∵ from ①

$800 = 50n$

$\boxed{n = 16}$ put in ①

$\Rightarrow 40 = 16d - d$

$40 = 15d$

$\boxed{d = 8/3}$

Thus, the common difference is found to be $d = 8/3$.

28. If the 10th term of an A.P. is 21 and the sum of its first 10 terms is 120, find its nth term.

→ Let us consider 'a' is the first term & 'd' is the common difference for given A.P.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$120 = \frac{10}{2}[2a + (10-1)d]$

$120 = 5(2a + 9d)$

$24 = 2a + 9d$ —— ①

$t_n = a + (n-1)d$

$21 = a + (10-1)d$

$21 = a + 9d$ —— ②

② - ① ⇒ $24-21 = 2a+9d-a-9d$

$\boxed{a=3}$

put in ② ⇒ $3+9d=21$

$9d=18$

$\boxed{d=2}$

Thus, first term $= a = 3$, common difference $= d = 2$

The nth term is given by

$t_n = a+(n-1)d$

$t_n = 3+(n-1)2$

$t_n = 3+2n-2$

$\boxed{t_n = 2n+1}$ is the required nth term of given A.P.

31. The sum of first seven terms of an A.P. is 182. If its 4th and 17th terms are in the ratio 1:5, find the A.P.

→ Given that, $S_7 = 182$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_7 = \frac{7}{2}[2a+(7-1)d]$

$182 = \frac{7}{2}(2a+6d)$

$364 = 14a+42d$

$26 = a+3d$ —— ①

Also,

$5t_4 = 1t_{17}$

$5(a+3d) = 1(a+16d)$

$5a+15d = a+16d$

$\boxed{4a=d}$ —— ②

put in ①

⇒ $26 = a+3(4a)$

$26 = a+12a$

$26 = 13a$

$\boxed{a=2}$

⇒ $4a = d$

$4(2) = d$

$\boxed{d=8}$

Thus, here first term $= a = 2$ and

common difference $= d = 8$

Hence, the required A.P. is 2, 10, 18, 26, ...

32. The nth term of an A.P. is given by $(-4n+15)$. find the sum of first 20 terms of this A.P.

→ Given that, $t_n = -4n+15$

put $n=1 \Rightarrow t_1 = -4+15 = 11$

put $n=20 \Rightarrow t_{20} = -4(20)+15 = -80+15 = -65$

Thus, first term $= a = 11$

Last term $= l = -65$

The sum of all 20 terms is given by

$$S_n = \frac{n}{2}(a+l)$$

$$S_{20} = \frac{20}{2}(11-65)$$

$$= 10(-54)$$

$$\boxed{S_{20} = -540}$$

Thus, the sum of first 20 terms is -540.

33. In an A.P. the sum of first ten terms is -150 and the sum of its next 10 terms is -550. find the A.P.

→ Let 'a' be first term & 'd' be the common difference for given A.P.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$S_{10} = -150$ & sum of next 10 terms $= -550$

Sum of first 20 terms $= \begin{pmatrix}\text{sum of first} \\ \text{10 terms}\end{pmatrix} + \begin{pmatrix}\text{sum of next} \\ \text{10 terms}\end{pmatrix}$

$$S_{20} = -150 + (-550) = -700$$

$$S_{10} = \frac{10}{2}[2a+(10-1)d]$$

$$-150 = 5(2a+9d)$$

$$-30 = 2a+9d$$

$$2a+9d = -30 \quad \text{—— ①}$$

And, $S_{20} = \frac{20}{2}[2a + (20-1)d]$

$-700 = 10(2a + 19d)$

$-70 = 2a + 19d$ —— ②

① − ② ⇒ $19d - 9d = -70 - (-30)$

$10d = -40$

$\boxed{d = -4}$

put in ① ⇒ $2a + 9(-4) = -30$

$2a = -30 + 36$

$a = 6/2 = 3$ $\boxed{a = 3}$

Thus, common difference $= d = -4$, first term $= a = 3$

Thus, the required A.P. is $3, -1, -5, -9, -13, \dots$

34. Sum of the first 14 terms of an A.P. is 1505 and it's first term is 10. Find its 25th term.

→ Here, given that first term $= a = 10$

sum of first 14 terms $= S_{14} = 1505$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{14} = \frac{14}{2}[2(10) + (14-1)d]$

$1505 = 7(20 + 13d)$

$20 + 13d = 215$

$13d = 215 - 20$

$d = 195/13$ ⇒ $\boxed{d = 15}$

Then, 25th term is given by

$t_{25} = 10 + (25-1)15$

$= 10 + 24(15)$

$= 10 + 360$

$\boxed{t_{25} = 370}$

Thus, the 25th term of given A.P. is 370.

35. In an A.P. the first term is 2, the last term is 29 & the ~~&~~ sum of the terms is 155. find the common difference of an A.P.

→ Given that, first term $= a = 2$

Last term $= l = 29$

sum of all terms $= S_n = 155$

we have, $S_n = \frac{n}{2}(a+l)$

$155 = \frac{n}{2}(2+29)$

$155(2) = n(31)$

$31n = 310$

$\boxed{n=10}$

Now, $t_n = a+(n-1)d$

$29 = 2+(10-1)d$

$29-2 = 9d$

$9d = 27$

$\boxed{d=3}$

Thus, the common difference is found to be $\boxed{d=3}$.

36. The first & the last term of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there & what is their sum?

→ Given that, first term $= a = 17$

Last term $= l = 350$

Common difference $= d = \cancel{350}\ 9$

we have, $t_n = a+(n-1)d$

$350 = 17+(n-1)9$

$17+9n-9 = 350$

$9n = 350-8$

$n = 342/9$

$\boxed{n=38}$

The sum of all terms of an A.P. is given by

$$S_n = \frac{n}{2}(a+l)$$

$$= \frac{38}{2}(17+350)$$

$$= 19(367)$$

$$\boxed{S_n = 6973}$$

Thus, the sum of the all terms is 6973.

37. Find the number of terms of the A.P. $-12, -9, -6, \ldots 21$. If 1 is added to each term of this A.P. then find the sum of all terms of the A.P. obtained.

→ Given that, first term $= a = -12$

Common difference $= d = -9-(-12) = 3$

$$t_n = a+(n-1)d$$

$$21 = -12+(n-1)3$$

$$21 = -12+3n-3$$

$$21 = 3n-15$$

$$36 = 3n$$

$$\boxed{n=12}$$

The no. of terms are $n=12$.

Now, if 1 is added to each of the 12 terms, the sum will be increased by 12.

Thus, $S_{12}+12 = \frac{12}{2}(a+l)+12$

$$= 6(-12+21)+12$$

$$= 6 \times 9 + 12$$

$$= 66$$

Thus, the sum after adding 1 to each of the term of an given A.P. is 66.