Chapter 8: Quadratic Equations

Exercise 8.1

1.) Which of the following are quadratic equations?

i) $x^2+6x-4=0$

→ Let us consider, $p(x) = x^2+6x-4$ —— ①

Here, the equnⓘ is having degree 2 & it is in only one variable. So it is a quadratic equation.

ii) $\sqrt{3}x^2-2x+1/2=0$

→ Letus consider, $p(x) = \sqrt{3}x^2-2x+1/2$ —— ①

Here, the equn① is having degree 2, in only one variable & with all real coefficients only.

Hence, it is a quadratic equation.

iii) $x^2+1/x^2=5$

→ Given that, $x^2+1/x^2=5$

$$x^4+1=5x^2$$

$$\Rightarrow x^4-5x^2+1=0 \text{ —— ①}$$

Here, the degree of equn① is 4.

Hence, equn① is not the quadratic equation.

iv) $x - 3/x = x^2$

→ Given that, $x - \frac{3}{x} = x^2$

$$x^2 - 3 = x^3$$

$$\Rightarrow x^3 - x^2 + 3 = 0 \text{ ——— (1)}$$

Here, equn (1) is having degree 3.

Hence, equn (1) is not the quadratic equation.

v) $2x^2 - \sqrt{3x} + 9 = 0$

→ Here, $(2x^2 - \sqrt{3x} + 9)$ is not a polynomial. Because it contains the the term $\sqrt{x} = x^{1/2}$, where $1/2$ is not an integer. Hence, the given equation is not a quadratic equation.

vi) $3x^2 - 5x + 9 = x^2 - 7x + 3$

→ Given that, $3x^2 - 5x + 9 = x^2 - 7x + 3$

$$2x^2 + 2x + 6 = 0 \text{ — (1)}$$

Here, $2(x^2 + x + 3) = 0$ is the quadratic polynomial. And hence equn (1) is the quadratic equation.

vii) $x^2 - 2x - \sqrt{x} - 5 = 0$

→ Here, $x^2 - 2x - \sqrt{x} - 5 = 0$ ——— (1)

is not a polynomial, because it contains the term $x^{1/2}$.

Hence, equn (1) is not a quadratic polynomial.

viii) $x + 1/x = 1$

→ Given that, $(x + 1/x) = 1$

$$x^2 + 1 = x$$

$$x^2 - x + 1 = 0 \text{ —— (1)}$$

Here, equn (1) is the quadratic polynomial. Hence it is the quadratic equation.

ix) $x^2-3x=0$

→ Here, $x^2-3x=0$ — ①

The equⁿ① is the quadratic polynomial and hence equⁿ① is the quadratic equation also.

x) $(x+1/x)^2 = 3(x+1/x)+4$

→ Here, Given that $(x+1/x)^2 = 3(x+1/x)+4$

$$x^2+\frac{1}{x^2}+2 = 3x+\frac{3}{x}+4$$

$$x^4+1+2x^2 = 3x^3+3x+4x^2$$

$$x^4-3x^3-2x^2-3x+1=0 \quad — ①$$

Here, equⁿ ① is not a quadratic polynomial. And hence equⁿ① is not a quadratic equation also.

xi) $(2x+1)(3x+2) = 6(x-1)(x-2)$

→ Here, $(2x+1)(3x+2) = 6(x-1)(x-2)$.

$$6x^2+4x+3x+2 = 6(x^2-2x-x+2)$$

$$6x^2+7x+2 = 6x^2-12x-6x+12$$

$$18x+7x+2-12=0$$

$$25x-10=0 \quad — ①$$

equⁿ① is a linear polynomial & not a quadratic polynomial. And hence equⁿ ① is not a quadratic equⁿ also.

xii) $x+1/x = x^2$, $x \neq 0$

→ Here, $x+\frac{1}{x} = x^2$, $x \neq 0$

$$x^2+1 = x^3$$

$$x^3-x^2-1=0 \quad — ①$$

Here, equⁿ① is a cubic polynomial & not a quadratic polynomial. And hence equⁿ① is ~~an~~ not quadratic polynomial

xiii) $16x^2 - 3 = (2x+5)(5x-3)$

$\longrightarrow$ Given that, $16x^2-3 = (10x^2 - 6x + 25x - 15)$

$$16x^2 - 3 = 10x^2 - 19x - 15$$

$$6x^2 - 19x + 12 = 0 \quad \text{--- (1)}$$

Here, equⁿ (1) is a quadratic polynomial. And hence equⁿ (1) is a quadratic equⁿ also.

xiv) $(x+2)^3 = x^3 - 4$

$\longrightarrow$ Here, $(x+2)^3 = x^3 - 4$

$$x^3 + 6x^2 + 8x + 8 = x^3 - 4$$

$$6x^2 + 8x + 12 = 0 \quad \text{--- (1)}$$

Here, equⁿ (1) is a quadratic polynomial. And hence equⁿ (1) is a quadratic equation also.

xv) $x(x+1) + 8 = (x+2)(x-2)$

$\longrightarrow$ Here, $x(x+1) + 8 = (x+2)(x-2)$

$$x^2 + x + 8 = x^2 - 2x + 2x - 4$$

$$x + 12 = 0 \quad \text{--- (1)}$$

Here, equⁿ (1) is not a quadratic polynomial. Hence equⁿ (1) is not a quadratic equation also.

2- In each of the following, determine whether the given value are solution of the given equation or not.

i) $x^2 - 3x + 2 = 0$, $x = 2$, $x = -1$

$\longrightarrow$ Here, The quadratic equⁿ is $x^2 - 3x + 2 = 0$ --- (1)

put $x = 2$ in (1) $\Rightarrow (2)^2 - 3(2) + 2$

$$= 4 - 6 + 2$$

$$= 6 - 6$$

$$= 0$$

Hence, $\boxed{x=2}$ is the solution of equⁿ (1).

put $x=-1$ in equ^n ①.

$\Rightarrow (-1)^2 - 3(-1) + 2$
$= 1+3+2$
$= 6$
$\neq 0$

Hence, $\boxed{x=-1}$ is not the solution of equ^n ①.

ii) $x^2+x+1=0$, $x=0$, $x=1$

$\rightarrow$ Here, given quadratic equ^n is $x^2+x+1=0$ — ①

put $x=0$ in ①
LHS $= 0+0+1$
$= 1$
$\neq 0$

put $x=1$ in equ^n ①
LHS $= 1^2+1+1$
$= 3$
$\neq 0$

Thus, $\boxed{x=0}$ and $\boxed{x=1}$ are not the solution of equ^n ①.

iii) $x^2 - 3\sqrt{3}x + 6 = 0$, $x=\sqrt{3}$ and $x=-2\sqrt{3}$

$\rightarrow$ Here, the given quadratic equ^n is $x^2-3\sqrt{3}x+6=0$ — ①

put $x=\sqrt{3}$ in equ^n ①
LHS $= (\sqrt{3})^2 - 3\sqrt{3}(\sqrt{3}) + 6$
$= 3-9+6$
$= 9-9$
$= 0$

put $x=-2\sqrt{3}$ in equ^n ①
LHS $= (-2\sqrt{3})^2 - 3\sqrt{3}(-2\sqrt{3}) + 6$
$= 12+18+6$
$= 36$
$\neq 0$

Hence, $\boxed{x=\sqrt{3}}$ is the solution of equ^n ① but $\boxed{x=-2\sqrt{3}}$ is not the solution of equ^n ①.

iv) $x+1/x = 13/6$, $x=5/6$, $x=4/3$

$\rightarrow$ Here, given that $x+1/x = 13/6$ — ①

$x^2 + \cancel{x} =$

put $x=5/6$ in equ^n ①
LHS $= 5/6 + 6/5$
$= 61/30$
$\neq 13/6$

put $x=4/3$ in equ^n ①
LHS $= 4/3 + 3/4$
$= 25/12$
$\neq 13/6$

Thus, $\boxed{x=5/6}$ and $\boxed{x=4/3}$ are not the solutions of a given equation ①.

v) $2x^2-x+9=x^2+4x+3$, $x=2$, $x=3$

→ Here, $2x^2-x+9=x^2+4x+3$

$x^2-5x+6=0$ —— ①

put $x=2$ in equⁿ ①

$LHS=(2)^2-5(2)+6$

$=4-10+6$

$=10-10$

$=0$

put $x=3$ in equⁿ ①

$LHS=(3)^2-5(3)+6$

$=9-15+6$

$=15-15$

$=0$

Thus, $\boxed{x=2}$ and $\boxed{x=3}$ are the solutions of equⁿ ①.

vi) $x^2-\sqrt{2}x-4=0$, $x=-\sqrt{2}$, $x=-2\sqrt{2}$.

→ Here, given quadratic equⁿ is $x^2-\sqrt{2}x-4=0$ —— ①

put $x=-\sqrt{2}$ in equⁿ ①

$LHS=(-\sqrt{2})^2-\sqrt{2}(-\sqrt{2})-4$

$=2+2-4$

$=4-4$

$=0$

put $x=-2\sqrt{2}$ in equⁿ ①

$LHS=(-2\sqrt{2})^2-\sqrt{2}(-2\sqrt{2})-4$

$=8+4-4$

$=8$

$\neq 0$

Thus, $\boxed{x=-\sqrt{2}}$ is the solution of equⁿ ① and $\boxed{x=-2\sqrt{2}}$ is not the solution of equⁿ ①.

vii) $a^2x^2-3abx+2b^2=0$, $x=a/b$, $x=b/a$

→ Here, given equⁿ is $a^2x^2-3abx+2b^2=0$ —— ①

put $x=a/b$ in ①

$LHS=a^2\left(\frac{a^2}{b^2}\right)-3ab\left(\frac{a}{b}\right)+2b^2$

$=\frac{a^4}{b^2}-3a^2+2b^2$

$\neq 0$

put $x=b/a$ in ①

$LHS=a^2\left(\frac{b^2}{a^2}\right)-3ab\left(b/a\right)+2b^2$

$=b^2-3b^2+2b^2$

$=3b^2-3b^2$

$=0$

Thus, $\boxed{x = a/b}$ is not the solution of equⁿ① but $\boxed{x = b/a}$ is the solution of equⁿ①.

Exercise 8.2

1. The product of two consecutive positive integers is 306. Form the quadratic equn to find the integers, if x denotes the smaller integer.

→ Let us consider, x and $(x+1)$ are the two integers. where 'x' is the smaller integer.

Thus, $$x(x+1) = 306$$
$$x^2 + x - 306 = 0$$

This is the required quadratic equation.

2. John and Jivani together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 128. Form the quadratic equation to find how many marbles they to start with, if John had x marbles.

→ Here, given that John and Jivani together have a total 45 marbles.

Let us consider, John have 'x' marbles.

So, Jivani will be having $(45-x)$ marbles.

The no. of marbles John had after losing 5 marbles $= (x-5)$

The no. of marbles Jivani had after losing 5 marbles $= (45-x)-5 = 40-x$

Now, The product of marbles that they are having now is 128.

Thus, $(x-5)(40-x)=128$

$$40x-x^2-200=128$$

$$x^2-45x+128+200=0$$

$$x^2-45x+328=0$$

This is the required quadratic equation.

3. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of articles produced in a day. On a particular day, the total cost of production was Rs. 750. If x denotes the number of toys produced that day, form the quadratic equ^n to find x.

→ Given that, 'x' denotes the no. of toys produced per day.

Hence, the cost of production of each toy $= (55-x)$

$$\left.\begin{array}{r}\text{The total cost of production} \\ \text{of toys per day}\end{array}\right\} = x(55-x)$$

But, given that, the total cost of production on that particular day is 750 Rs.

Thus, $x(55-x)=750$

$$55x-x^2=750$$

$$x^2-55x+750=0$$

This is the required quadratic equ^n.

Exercise 8.3

Solve the following quadratic equation by factorization.

1) $(x-4)(x+2)=0$

$\rightarrow$ Given that, $(x-4)(x+2)=0$ —— ①

$\Rightarrow x-4=0$ or $x+2=0$

$\Rightarrow \boxed{x=4}$ or $\boxed{x=-2}$

Thus, $x=4$ and $x=-2$ are the roots of quadratic equation ①.

2) $(2x+3)(3x-7)=0$

$\rightarrow$ Given that, $(2x+3)(3x-7)=0$ —— ①

$\Rightarrow (2x+3)=0$ or $(3x-7)=0$

$\Rightarrow \boxed{x=-3/2}$ or $\boxed{x=7/3}$

Thus, $x=-3/2$ and $x=7/3$ are the roots of quadratic equn ①.

3) $3x^2-14x-5=0$

$\rightarrow$ Given equn is $3x^2-14x-5=0$ —— ①

$3x^2-15x+x-5=0$

$3x(x-5)+1(x-5)=0$

$(3x+1)(x-5)=0$

$\boxed{x=-1/3}$ or $\boxed{x=5}$

Thus, $x=-1/3$ and $x=5$ are the roots of quadratic equn ①.

4) Find the roots of the equation $9x^2-3x-2=0$

→ Given that, $9x^2-3x-2=0$ —— ①

$$9x^2-6x+3x-2=0$$

$$3x(3x-2)+1(3x-2)=0$$

$$(3x-2)(3x+1)=0$$

$$(3x-2)=0 \quad \text{or} \quad (3x+1)=0$$

$$\boxed{x=2/3} \quad \text{or} \quad \boxed{x=-1/3}$$

Thus, $x=2/3$ and $x=-1/3$ are the roots of quadratic equn ①.

5) $\frac{1}{(x-1)} - \frac{1}{(x+5)} = \frac{6}{7}$

→ Given that, $\frac{1}{(x-1)} - \frac{1}{(x+5)} = \frac{6}{7}$

$$\frac{(x+5)-(x-1)}{(x-1)(x+5)} = \frac{6}{7}$$

$$\frac{6}{(x-1)(x+5)} = \frac{6}{7}$$

$$\Rightarrow (x-1)(x+5) = 7$$

$$x^2+5x-x-5=7$$

$$x^2+4x-12=0 \text{ —— ①}$$

$$\Rightarrow \cancel{x(}\ x^2+6x-2x-12=0$$

$$x(x+6)-2(x+6)=0$$

$$(x+6)(x-2)=0$$

$$x+6=0 \quad \text{or} \quad x-2=0$$

$$\boxed{x=-6} \quad \text{or} \quad \boxed{x=2}$$

Thus, roots of quadratic equ^n ① are $x=-6$ & $x=2$.

6) $6x^2+11x+3=0$

$\rightarrow$ Given equation is, $6x^2+11x+3=0$ —— ①

$$6x^2+9x+2x+3=0$$

$$3x(2x+3)+1(2x+3)=0$$

$$(2x+3)(3x+1)=0$$

$$(2x+3)=0 \quad \text{or} \quad (3x+1)=0$$

$$\boxed{x=-3/2} \quad \text{or} \quad \boxed{x=-1/3}$$

Thus, $x=-3/2$ and $x=-1/3$ are the roots of quadratic equ^n ①.

7) $5x^2-3x-2=0$

$\rightarrow$ Given that, $5x^2-3x-2=0$ —— ①

$$5x^2-5x+2x-2=0$$

$$5x(x-1)+2(x-1)=0$$

$$(5x+2)(x-1)=0$$

$$(5x+2)=0 \quad \text{or} \quad (x-1)=0$$

$$\boxed{x=-2/5} \quad \text{or} \quad \boxed{x=1}$$

Thus, $x=-2/5$ and $x=1$ are the roots of quadratic equ^n ①.

8) $48x^2-13x-1=0$

$\rightarrow$ Given that, $48x^2-13x-1=0$ —— ①

$$\cancel{16x(3x-} \; 48x^2-16x+3x-1=0$$

$$16x(3x-1)+1(3x-1)=0$$

$$(16x+1)(3x-1)=0$$

$$(16x+1)=0 \quad \text{or} \quad (3x-1)=0$$

$$\boxed{x=-1/16} \quad \text{or} \quad \boxed{x=1/3}$$

Thus, the roots of given equ^n ① are $x=-1/16$ & $x=1/3$.

9) $3x^2 = -11x - 10$

$\longrightarrow$ Given equ^n is $3x^2 = -11x - 10$ —— ①

$\Rightarrow 3x^2 + 11x + 10 = 0$

$3x^2 + 6x + 5x + 10 = 0$

$3x(x+2) + 5(x+2) = 0$

$(3x+5)(x+2) = 0$

$(3x+5) = 0$ or $x+2 = 0$

$\boxed{x = -5/3}$ or $\boxed{x = -2}$

Thus, the roots of quadratic equ^n ① are $x = -5/3$ & $x = -2$.

10) $25x(x+1) = -4$

$\longrightarrow$ Given equ^n is $25x(x+1) = -4$ —— ①

$25x^2 + 25x = -4$

$25x^2 + 25x + 4 = 0$

$25x^2 + 20x + 5x + 4 = 0$

$5x(5x+4) + 1(5x+4) = 0$

$(5x+4)(5x+1) = 0$

$(5x+4) = 0$ or $(5x+1) = 0$

$\boxed{x = -4/5}$ or $\boxed{x = -1/5}$

Thus, the roots of equ^n ① are $x = -4/5$ & $x = -1/5$.

11) $16x - 10/x = 27$

$\rightarrow$ Given equ^n is, $16x - 10/x = 27$

$$16x^2 - 10 = 27x$$

$$16x^2 - 27x - 10 = 0 \quad \text{--- (1)}$$

$$16x^2 - 32x + 5x - 10 = 0$$

$$16x(x-2) + 5(x-2) = 0$$

$$(x-2)(16x+5) = 0$$

$$(x-2) = 0 \quad \text{or} \quad (16x+5) = 0$$

$$\boxed{x = 2} \quad \text{or} \quad \boxed{x = -5/16}$$

The roots of quadratic equ^n (1) are $x = 2$, $x = -5/16$.

12) $\frac{1}{x} - \frac{1}{x-2} = 3$

$\rightarrow$ Given equ^n is, $\frac{1}{x} - \frac{1}{(x+2)} = 3$.

$$\frac{(x-2) - x}{x(x+2)} = 3$$

$$\frac{2}{x(x-2)} = 3$$

$$2 = 3x(x-2)$$

$$2 = 3x^2 - 6x$$

$$3x^2 - 6x - 2 = 0 \quad \text{--- (1)}$$

$$3x^2 - 3x - 3x - 2 = 0$$

$$3x^2 - (3+\sqrt{3})x - (3-\sqrt{3})x + [(\sqrt{3})^2 - 1^2] = 0$$

$$3x^2 - (3+\sqrt{3})x - (3-\sqrt{3})x + [(\sqrt{3})^2 - 1^2] = 0$$

$$(\sqrt{3})^2 x^2 - \sqrt{3}(\sqrt{3}+1)x - \sqrt{3}(\sqrt{3}-1)x + (\sqrt{3}+1)(\sqrt{3}-1) = 0$$

$$\sqrt{3}x(\sqrt{3}+1)x - (\sqrt{3}x - (\sqrt{3}+1))(\sqrt{3}-1) = 0$$

$$(\sqrt{3}x - \sqrt{3} - 1)(\sqrt{3}x - \sqrt{3} + 1)(\sqrt{3}-1) = 0$$

$(\sqrt{3}x-\sqrt{3}-1)=0$ or $(\sqrt{3}x-\sqrt{3}+1)=0$

$\boxed{x=\frac{(\sqrt{3}+1)}{\sqrt{3}}}$ or $\boxed{x=\frac{(\sqrt{3}-1)}{\sqrt{3}}}$

This is the solution of given quadratic equⁿ ①

14) $\frac{1}{(x+4)}-\frac{1}{(x-7)}=\frac{11}{30}$

→ Given that, $\frac{1}{(x+4)}-\frac{1}{(x-7)}=\frac{11}{30}$

$$\frac{(x-7)-(x+4)}{(x+4)(x-7)}=\frac{11}{30}$$

$$\frac{-11}{(x+4)(x-7)}=\frac{11}{30}$$

$$-30=(x+4)(x-7)$$
$$x^2-3x-2=0 \quad \text{——} ①$$
$$x^2-2x-x-2=0$$
$$x(x-2)-1(x-2)=0$$
$$(x-1)(x-2)=0$$

$\boxed{x=1}$ or $\boxed{x=2}$ are the solution of given quadratic equⁿ ①

15) $\frac{1}{(x-3)}+\frac{2}{(x-2)}=\frac{8}{x}$

→ Given that, $\frac{1}{(x-3)}+\frac{2}{(x-2)}=\frac{8}{x}$

$$\frac{(x-2)+2(x-3)}{(x-3)(x-2)}=\frac{8}{x}$$

$$\frac{3x-8}{(x-3)(x-2)}=\frac{8}{x}$$

$$x(3x-8)=8(x-3)(x-2)$$
$$3x^2-8x=8(x^2-5x+6)$$

$$8x^2 - 40x + 48 - (3x^2 - 8x) = 0$$
$$5x^2 - 32x + 48 = 0$$
$$5x^2 - 20x + 12x + 48 = 0$$
$$5x(x-4) - 12(x-4) = 0$$
$$(x-4)(5x-12) = 0$$
$$x - 4 = 0 \quad \text{or} \quad 5x - 12 = 0$$
$$\boxed{x = 4} \quad \text{or} \quad \boxed{x = 12/5}$$

This are the roots of quadratic equ^n ①.

16) $a^2x^2 - 3abx + 2b^2 = 0$

$\longrightarrow$ Given equ^n is, $a^2x^2 - 3abx + 2b^2 = 0$ —— ①
$$a^2x^2 - abx - 2abx + 2b^2 = 0$$
$$ax(ax - b) - 2b(ax - b) = 0$$
$$(ax - b)(ax - 2b) = 0$$
$$(ax - b) = 0 \quad \text{or} \quad (ax - 2b) = 0$$
$$\boxed{x = b/a} \quad \text{or} \quad \boxed{x = 2b/a}$$

This are the roots of quadratic equ^n ①.

17) $9x^2 - 6b^2x - (a^4 - b^4) = 0$

$\longrightarrow$ Given that, $9x^2 - 6b^2x - (a^4 - b^4) = 0$ —— ①
$$9x^2 - 6b^2x - (a^2 - b^2)(a^2 + b^2) = 0$$
$$9x^2 + 3(a^2 - b^2)] - 3(a^2 + b^2)x - (a^2 - b^2)(a^2 + b^2) = 0$$
$$3x[3x + a^2 + b^2)] - (a^2 + b^2)[3x + (a^2 - b^2)] = 0$$
$$[3x + (a^2 + b^2)][3x + (a^2 - b^2)] = 0$$
$$3x - (a^2 + b^2) = 0 \quad \text{or} \quad 3x + (a^2 - b^2) = 0$$
$$\boxed{x = \frac{(a^2 + b^2)}{3}} \quad \text{or} \quad \boxed{x = \frac{b^2 - a^2}{3}}$$

are the roots of equ^n ①.

18) $4x^2+4bx-(a^2-b^2)=0$

$\rightarrow$ Given that, $4x^2+4bx-(a^2-b^2)=0$ —— ①

$4(a^2-b^2)=-4(a-b)(a+b)$

$4(a^2-b^2)=[-2(a-b)][2(a+b)]$

$\Rightarrow \Rightarrow 2(b-a)\times 2(b+a)$

$4x^2+[2(b-a)+2(b+a)]x-(a-b)(a+b)=0$

$4x^2+2(b-a)x+2(b+a)x+(b-a)(a+b)=0$

$2x[2x+(b-a)]+(a+b)[2x+(b-a)]=0$

$[2x+(b-a)][2x+(b+a)]=0$

$2x+(b-a)=0$ or $2x+(b+a)=0$

$\boxed{x=\frac{(a-b)}{2}}$ or $\boxed{x=-\frac{(a+b)}{2}}$

This are the roots of quadratic equⁿ①.

19) $ax^2+(4a^2-3b)x-12ab=0$

$\rightarrow$ Given equⁿ is $ax^2+(4a^2-3b)x-12ab=0$ —— ①

$\Rightarrow ax^2+4a^2x-3bx-12ab=0$

$ax(x+4a)-3b(x+4a)=0$

$(x+4a)(ax-3b)=0$

$x+4a=0$ or $ax-3b=0$

$\boxed{x=-4a}$ or $\boxed{x=3b/a}$

This are the roots of equⁿ①.

20) $2x^2+ax-a^2=0$

$\rightarrow$ Given that, $2x^2+ax-a^2=0$ —— ①

$$2x^2+2ax-ax-a^2=0$$

$$2x(x+a)-a(x+a)=0$$

$$(x+a)(2x-a)=0$$

$$2x-a=0 \quad \text{or} \quad x+a=0$$

$$\boxed{x=a/2} \quad \text{or} \quad \boxed{x=-a}$$

This are the roots of quadratic eqn①

21) $16/x-1=15/(x+1)$, $x\neq 0,-1$

$\Rightarrow$ Given that, $\frac{16}{x}-1=\frac{15}{(x+1)}$

$$\Rightarrow \frac{16-x}{x}=\frac{15}{(x+1)}$$

$$\Rightarrow (16-x)(x+1)=15x$$

$$-x^2+16+15x=15x$$

$$\Rightarrow -x^2+16=0 \text{ —— ①}$$

$$(x^2-16)=0$$

$$(x^2-4^2)=0 \Rightarrow (x-4)(x+4)=0$$

$$\boxed{x=4} \quad \text{or} \quad \boxed{x=-4}$$

This are the roots of eqn①

22) $\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$, $x\neq -2, 3/2$

$\rightarrow$ The given eqn is

$$\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$$

$$(x+3)(2x-3) = (x+2)(3x-7)$$

$$2x^2-3x+6x-9 = 3x^2-x-14$$

$$2x^2+3x-9 = 3x^2-x-14$$

$$x^2-3x-x-14+9=0$$

$$x^2-5x+x-5=0 \quad \text{—— ①}$$

$$x(x-5)+1(x-5)=0$$

$$(x-5)(x+1)=0$$

$$\boxed{x=5} \quad \text{or} \quad \boxed{x=-1}$$

This are the roots of quadratic equⁿ ①.

23) $\dfrac{2x}{(x-4)} + \dfrac{(2x-5)}{(x-3)} = \dfrac{25}{3}$, $x \neq 3, 4$

→ Given equⁿ is,

$$\frac{2x}{(x-4)} + \frac{(2x-5)}{(x-3)} = \frac{25}{3}$$

$$\frac{2x(x-3)+(2x-5)(x-4)}{(x-4)(x-3)} = \frac{25}{3}$$

$$\frac{2x^2-6x+2x^2-5x-8x+20}{x^2-4x-3x+12} = \frac{25}{3}$$

$$\frac{4x^2-19x+20}{x^2-7x+12} = \frac{25}{3}$$

$$3(4x^2-19x+20) = 25(x^2-7x+12)$$

$$12x^2-57x+60 = 25x^2-175x+300$$

$$13x^2-78x-40x+240=0$$

$$13x^2-118x+240=0$$

$$13x^2-78x-40x+240=0$$

$$\cancel{13x^2}\ 13x(x-6)-40(x-6)=0$$

$$(x-6)(13x-40)=0$$

$$x-6=0 \quad \text{or} \quad 13x-40=0$$

$$\boxed{x=6} \quad \text{or} \quad \boxed{x=40/13}$$

24) $\frac{(x+3)}{(x-2)} - \frac{(1-x)}{x} = \frac{17}{4}$, $x \neq 0, 2$

$\rightarrow$ Given equⁿ is, $\frac{(x+3)}{(x-2)} - \frac{(1-x)}{x} = \frac{17}{4}$

$$\frac{x(x+3) - (x-2)(1-x)}{x(x-2)} = \frac{17}{4}$$

$$\frac{x^2+3x - x + x^2 + 2 - 2x}{x^2-2x} = \frac{17}{4}$$

$$\frac{2x^2+2}{x^2-2x} = \frac{17}{4}$$

$$4(2x^2+2) = 17(x^2-2x)$$

$$\Rightarrow 8x^2+8 = 17x^2-34x$$

$$9x^2-34x-8 = 0 \quad \text{—— (1)}$$

$$9x^2-36x+2x-8 = 0$$

$$9x(x-4)+2(x-4) = 0$$

$$(9x+2)(x-4) = 0$$

$$(9x+2) = 0 \quad \text{or} \quad (x-4) = 0$$

$$\boxed{x = -2/9} \quad \text{or} \quad \boxed{x = 4}$$

This are the roots of quadratic equⁿ (1).

25) $\frac{(x-3)}{(x+3)} - \frac{(x+3)}{(x-3)} = \frac{48}{7}$, $x \neq 3$, $x \neq -3$

$\rightarrow$ Given equⁿ is, $\frac{(x-3)}{(x+3)} - \frac{(x+3)}{(x-3)} = \frac{48}{7}$

$$\frac{(x-3)^2 - (x+3)^2}{(x+3)(x-3)} = \frac{48}{7}$$

$$\frac{(x^2-6x+9) - (x^2+6x+9)}{(x^2-9)} = \frac{48}{7}$$

$$\frac{-12x}{x^2-9} = \frac{48}{7}$$

$$7(-12x) = 48(x^2-9)$$
$$-84x = 48x^2 - 432$$
$$48x^2 + 84x - 432 = 0$$
$$4x^2 + 7x - 36 = 0 \quad \text{——} ①$$
$$4x^2 + 16x - 9x - 36 = 0$$
$$4x(x+4) - 9(x-4) = 0$$
$$(4x-9)(x+4) = 0$$
$$4x - 9 = 0 \quad \text{or} \quad x + 4 = 0$$
$$\boxed{x = 9/4} \quad \text{or} \quad \boxed{x = -4}$$

This are the roots of quadratic equ^n ①.

26) $\frac{1}{(x-2)} + \frac{2}{(x-1)} = \frac{6}{x}$, $x \neq 0$

→ Given equ^n is, $\frac{1}{(x-2)} + \frac{2}{(x-1)} = \frac{6}{x}$

$$\frac{(x-1) + 2(x-2)}{(x-2)(x-1)} = \frac{6}{x}$$
$$\frac{(x-1) + 2x - 4}{(x^2 - 2x - x + 2)} = \frac{6}{x}$$
$$\frac{3x-5}{(x^2-3x+2)} = \frac{6}{x}$$
$$x(3x-5) = 6(x^2 - 3x + 2)$$
$$3x^2 - 5x = 6x^2 - 18x + 12$$
$$3x^2 - 13x + 12 = 0 \quad \text{——} ①$$
$$3x^2 - 9x - 4x + 12 = 0$$
$$3x(x-3) - 4(x-3) = 0$$
$$(x-3)(3x-4) = 0$$
$$\boxed{x = 3} \quad \text{or} \quad \boxed{x = 4/3}$$

This are the roots of quadratic equ^n ①.

27) $\frac{(x+1)}{(x-1)} - \frac{(x-1)}{(x+1)} = \frac{5}{6}$, $x \neq 1, -1$

$\longrightarrow$ The given equ^n is $\frac{(x+1)}{(x-1)} - \frac{(x-1)}{(x+1)} = \frac{5}{6}$

$$\frac{(x+1)^2 - (x-1)^2}{x^2-1} = \frac{5}{6}$$

$$\frac{4x}{x^2-1} = \frac{5}{6}$$

$$6(4x) = 5(x^2-1) = 24x$$

$$5x^2 - 5 - 24x = 0 \quad \text{——} ①$$

$$5x^2 - 25x + x - 5 = 0$$

$$5x(x-5) + 1(x-5) = 0$$

$$(5x+1)(x-5) = 0$$

$$\boxed{x = -1/5} \quad \text{or} \quad \boxed{x = 5}$$

This are the two roots of quadratic equ^n ①

28) $\frac{(x-1)}{(2x+1)} + \frac{(2x+1)}{(x-1)} = \frac{5}{2}$, $x \neq 1, -1/2$

$\longrightarrow$ The given equ^n is $\frac{(x-1)}{(2x+1)} + \frac{(2x+1)}{(x-1)} = \frac{5}{2}$

$$\frac{(x-1)^2 + (2x+1)^2}{(x-1)(2x+1)} = \frac{5}{2}$$

$$\frac{(x-1)^2 + (2x+1)^2}{2x^2 - 2x + x - 1} = \frac{5}{2}$$

$$\frac{x^2 - 2x + 1 + 4x^2 + 4x + 1}{2x^2 - x - 1} = \frac{5}{2}$$

$$\frac{5x^2 + 2x + 2}{2x^2 - x - 1} = \frac{5}{2}$$

$$2(5x^2 + 2x + 2) = 5(2x^2 - x - 1)$$

$$10x^2 + 4x + 4 = 10x^2 - 5x - 5$$

$$4x + 5x + 4 + 5 = 0$$

$9x+9=0$

$\boxed{x=-1}$

This is only one root of given eqn.

29) $\frac{4}{x}-3=\frac{5}{2x+3}$, $x \neq 0, \frac{-3}{2}$

$\longrightarrow$ Given equn is $\frac{4}{x}-3=\frac{5}{(2x+3)}$

$\Rightarrow 5x=(2x+3)(4-3x)$

$5x=8x-6x^2+12-9x$

$5x-8x+6x^2-12+9x=0$

$6x^2+6x-12=0$ —— ①

$x^2+x-2=0$

$x^2+2x-x-2=0$

$x(x+2)-1(x+2)=0$

$(x-1)(x+2)=0$

$x-1=0$ or $x+2=0$

$\boxed{x=1}$ or $\boxed{x=-2}$

This are the roots of quadratic equn ①.

30) $\frac{x-4}{x-5}+\frac{x-6}{x-7}=\frac{10}{3}$

$\rightarrow$ Given equn is $\frac{x-4}{x-5}+\frac{x-6}{x-7}=\frac{10}{3}$

$$\frac{(x-4)(x-7)+(x-5)(x-6)}{(x-5)(x-7)}=\frac{10}{3}$$

$$\frac{x^2-7x-4x+28+x^2-6x-5x+30}{x^2-7x-5x+35}=\frac{10}{3}$$

$$\frac{2x^2-22x+58}{x^2-12x+35}=\frac{10}{3}$$

$3(2x^2-22x+58)=10(x^2-12x+35)$

$$6x^2-66x+174=10x^2-120x+350$$
$$4x^2-54x+176=0 \text{ —— } ①$$
$$2x^2-27x+88=0$$
$$2x^2-16x-11x+88=0$$
$$(x-8)\ (2x-11)=0$$
$$x-8=0 \quad \text{or} \quad 2x-11=0$$
$$\boxed{x=8} \quad \text{or} \quad \boxed{x=11/2}$$

This are the two roots of equn ①

Exercise 8.4

Find the roots of the following quadratic equn (if they exist) by the method of completing the square.

1) $x^2-4\sqrt{2}x+6=0$

→ Given equation is $x^2-4\sqrt{2}x+6=0$ —— ①

$$x^2-2x(2\sqrt{2})+(2\sqrt{2})^2-(2\sqrt{2})^2+6=0$$
$$(x-2\sqrt{2})^2=(2\sqrt{2})^2-6$$
$$(x-2\sqrt{2})^2=(4\times2)-6=8-6$$
$$(x-2\sqrt{2})^2=2$$
$$(x-2\sqrt{2})=\pm\sqrt{2}$$
$$(x-2\sqrt{2})=\sqrt{2} \quad \text{or} \quad (x-2\sqrt{2})=-\sqrt{2}$$
$$\boxed{x=\sqrt{2}+2\sqrt{2}} \quad \text{or} \quad x=-\sqrt{2}+2\sqrt{2}$$
$$\boxed{x=3\sqrt{2}} \quad \text{or} \quad \boxed{x=\sqrt{2}}$$

This are the roots of equation ①.

2) $2x^2-7x+3=0$

→ Given equⁿ is $2x^2-7x+3=0$ —— ①

$$2\left(x^2-\frac{7x}{2}+\frac{3}{2}\right)=0$$

$$x^2-2\left(\frac{7}{2}\right)\left(\frac{1}{2}\right)x+\frac{3}{2}=0$$

$$x^2-2\left(\frac{7}{4}\right)x+\left(\frac{7}{4}\right)^2-\left(\frac{7}{4}\right)^2+\frac{3}{2}=0$$

$$x^2-2\left(\frac{7}{4}\right)x+\left(\frac{7}{4}\right)^2-\left(\frac{49}{16}\right)+\frac{3}{2}=0$$

$$\left(x-\frac{7}{4}\right)^2-\frac{49}{16}+\frac{3}{2}=0$$

$$\left(x-\frac{7}{4}\right)^2=\frac{49}{16}-\frac{3}{2}$$

$$\left(x-\frac{7}{4}\right)^2=\frac{49-26}{16}$$

$$\left(x-\frac{7}{4}\right)^2=\frac{25}{16}$$

$$\left(x-\frac{7}{4}\right)^2=\left(\frac{5}{4}\right)^2 \Rightarrow x-\frac{7}{4}=\pm\frac{5}{4}$$

$$\Rightarrow x=\frac{7}{4}+\frac{5}{4} \text{ or } x=-\frac{5}{4}+\frac{7}{4}$$

$$\boxed{x=12/4} \text{ or } \boxed{x=2/4=1/2}$$

This are the roots of quadratic equⁿ ①.

3) $3x^2+11x+10=0$

→ Given equⁿ is $3x^2+11x+10=0$

$$x^2+\frac{11}{3}x+\frac{10}{3}=0$$

$$x^2+2\times\frac{1}{2}\times\frac{11x}{3}+\frac{10}{3}=0$$

$$x^2+2\times\frac{11}{6}x+\left(\frac{11}{6}\right)^2-\left(\frac{11}{6}\right)^2+\frac{10}{3}=0$$

$$\left(x+\frac{11}{6}\right)^2=\left(\frac{11}{6}\right)^2-\frac{10}{3}$$

$$\left(x+\frac{11}{6}\right)^2=\frac{121}{36}-\frac{10}{3}$$

$$\left(x+\frac{11}{6}\right)^2=\frac{121-120}{36}$$

$$\left(x+\frac{11}{6}\right)^2=\frac{1}{36}$$

$$\left(x+\frac{11}{6}\right)^2=\left(\frac{1}{6}\right)^2$$

$$x+\frac{11}{6}=\pm\frac{1}{6}$$

$$\Rightarrow x+\frac{11}{6}=\frac{1}{6} \quad \text{or} \quad x+\frac{11}{6}=\frac{-1}{6}$$

$$x=\frac{1}{6}-\frac{11}{6} \quad \text{or} \quad x=\frac{-1}{6}-\frac{11}{6}$$

$$x=-10/6 \quad \text{or} \quad x=\frac{-12}{6}$$

$$\boxed{x=-5/3} \quad \text{or} \quad \boxed{x=-2}$$

This are the roots of equ^n ①

4) $2x^2+x-4=0$

→ Given equ^n is $2x^2+x-4=0$ —— ①

$$2\left(x^2+\frac{x}{2}-\frac{4}{2}\right)=0$$

$$x^2+2\times\frac{1}{2}\times\frac{1}{2}\,x-2=0$$

$$x^2+2\left(\frac{1}{4}\right)x+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2=0$$

$$\left(x+1/4\right)^2=\left(\frac{1}{4}\right)^2+2$$

$$\left(x+1/4\right)^2=\frac{1}{16}+2=\frac{33}{16}$$

$$\left(x+1/4\right)=\pm\sqrt{\frac{33}{16}}$$

$$\left(x+\frac{1}{4}\right)=\sqrt{\frac{33}{16}} \quad \text{or} \quad \left(x+1/4\right)=-\sqrt{\frac{33}{16}}$$

$$x=\frac{\sqrt{33}}{4}-\frac{1}{4} \quad \text{or} \quad x=-\frac{\sqrt{33}}{4}-\frac{1}{4}$$

$$\boxed{x=\frac{\sqrt{33}-1}{4}} \quad \text{or} \quad \boxed{x=-\frac{(\sqrt{33}+1)}{4}}$$

This are the two roots of equ^n ①

5) $2x^2+x+4=0$

$\longrightarrow$ Given eqn is $2x^2+x+4=0$ —— ①

$$x^2+x/2+2=0$$

$$x^2+2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)x+2=0$$

$$x^2+2\left(\frac{1}{4}\right)x+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2+2=0$$

$$x^2+2\left(\frac{1}{4}\right)x+\left(\frac{1}{4}\right)^2=\left(\frac{1}{4}\right)^2-2$$

$$\left(x+\frac{1}{4}\right)^2=\left(\frac{1-32}{16}\right)$$

$$\left(x+1/4\right)^2=\frac{-31}{16}$$

$$\left(x+1/4\right)=\pm\sqrt{\frac{-31}{16}}$$

$$\left(x+1/4\right)=\frac{\sqrt{-31}}{4} \quad \text{or} \quad \left(x+1/4\right)=\frac{-\sqrt{-31}}{4}$$

$$\boxed{x=\frac{\sqrt{-31}-1}{4}} \quad \text{or} \quad \boxed{x=\frac{-\sqrt{-31}-1}{4}}$$

This are the roots of quadratic eqn ①.

Exercise 8.5

1) Write the discriminant of the following quadratic equations.

i) $2x^2-5x+3=0$

→ Given eqn is $2x^2-5x+3=0$ —— ①

on compairing eqn ① with $ax^2+bx+c=0$

$a=2$, $b=-5$, $c=3$

Discriminant $= D = b^2-4ac$

$D = (-5)^2-4\times2\times3$

$D = 25-24$

$\boxed{D=1}$

Here, the discriminant of eqn ① is 1.

ii) $x^2+2x+4=0$

→ Given eqn is $x^2+2x+4=0$ —— ①

On compairing eqn ① with $ax^2+bx+c=0$,

we get $a=1$, $b=2$, $c=4$

Discriminant $= D = b^2-4ac$

$= (2)^2-4(1)(4)$

$= 4-16$

$\boxed{D=-12}$

iii) ~~x^2+2x~~ $(x-1)(2x-1)=0$

→ Given eqn is $(x-1)(2x-1)=0$

$2x^2-3x+1=0$ —— ①

on compairing eqn ① with $ax^2+bx+c=0$

$a=2$, $b=-3$, $c=1$

Discriminant $= D = b^2-4ac$

$D = (-3)^2-4(2)(1)$

$D = 9-8 = 1$ $\boxed{D=1}$

iv) $x^2 - 2x + k = 0, k \in R$

$\rightarrow$ Given equⁿ is $x^2 - 2x + k = 0$ —— ①

on compairing equⁿ ① with $ax^2 + bx + c = 0$

$a = 1, \ b = -2, \ c = k$

Discriminant $= b^2 - 4ac$

$D = (-2)^2 - 4(1)(k)$

$\boxed{D = 4 - 4k}$

is the discriminant of equⁿ ①.

v) $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

$\rightarrow$ Given equⁿ is $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$ —— ①

on compairing equⁿ ① with $ax^2 + bx + c = 0$

$\therefore a = \sqrt{3}, \ b = 2\sqrt{2}, \ c = -2\sqrt{3}$

Discriminant $= b^2 - 4ac$

$D = (2\sqrt{2})^2 - 4(\sqrt{3})(-2\sqrt{3})$

$= 8 + 24$

$\boxed{D = 32}$

is the required discriminant of equⁿ ①.

vi) $x^2 - x + 1 = 0$

$\rightarrow$ Given equⁿ is $x^2 - x + 1 = 0$ —— ①

on compairing equⁿ ① with $ax^2 + bx + c = 0$

$a = 1, \ b = -1, \ c = 1$

Discriminant $= D = b^2 - 4ac$

$D = (-1)^2 - 4(1)(1)$

$D = 1 - 4$

$\boxed{D = -3}$

is the required discriminant of equⁿ ①

Exercise 8.6

1) Determine the nature of the roots of the following quadratic equations.

i) $2x^2-3x+5=0$

→ Given equ^n is

$2x^2-3x+5=0$ —— ①

on compairing ① with

$ax^2+bx+c=0$

$a=2,\ b=-3,\ c=5$

$D=b^2-4ac$

$=(-3)^2-4(2)(5)$

$=9-40$

$\boxed{D=-31} < 0$

$\boxed{D<0}$

Hence, the equation has no real roots.

ii) $2x^2-6x+3=0$

→ Given equ^n is

$2x^2-6x+3=0$ —— ①

on compairing equ^n ① with

$ax^2+bx+c=0$

$a=2,\ b=-6,\ c=3$

$D=b^2-4ac$

$=(-6)^2-4(2)(3)$

$=36-24$

$D=12>0$

$\boxed{D>0}$

Hence, equ^n ① has real & distinct roots.

iii) $(3/5)x^2-(2/3)+1=0$

→ Given equ^n is

$(3/5)x^2-(2/3)+1=0$ —— ①

on compairing equ^n ① with

$ax^2+bx+c=0$

$a=3/5,\ b=-2/3,\ c=1$

$D=b^2-4ac$

$=\cancel{(3/5)^2-4(-2/}$

$=(-2/3)^2-4(3/5)(1)$

$=4/9-12/5$

$D=-\frac{88}{45}<0$

$\boxed{D<0}$

Hence, equ^n ① has no real roots.

iv) $3x^2-4\sqrt{3}x+4=0$

→ Given equ^n is

$3x^2-4\sqrt{3}x+4=0$ —— ①

on compairing equ^n ① with

$ax^2+bx+c=0$

$a=3,\ b=-4\sqrt{3},\ c=4$

$D=b^2-4ac$

$=(-4\sqrt{3})^2-4(3)(4)$

$=48-48$

$\boxed{D=0}$

Hence, equ^n ① has real & equal roots.

v) $3x^2 - 2\sqrt{6}x + 2 = 0$

→ Given equⁿ is $3x^2 - 2\sqrt{6}x + 2 = 0$ —— ①

on compairing equⁿ ① with $ax^2 + bx + c = 0$

$\therefore a = 3, \; b = -2\sqrt{6}, \; c = 2$

$D = b^2 - 4ac$

$= (-2\sqrt{6})^2 - 4(3)(2)$

$= 24 - 24$

$\boxed{D = 0}$

Hence, the equⁿ ① has real and ~~di~~ equal roots.

2. Find the values of k for which the roots are real & equal in each of the following equations.

i) $kx^2 + 4x + 1 = 0$

→ The given equⁿ is $kx^2 + 4x + 1 = 0$ —— ①

on comparing ① with $ax^2 + bx + c = 0$

$\therefore a = k, \; b = 4, \; c = 1$

$D = b^2 - 4ac = 0$ ∵ roots are real & equal

$4^2 - 4(k)(1) = 0$

$16 - 4k = 0$

$16 = 4k \Rightarrow \boxed{k = 4}$

ii) $kx^2 - 2\sqrt{5}x + 4 = 0$

→ The given equⁿ is $kx^2 - 2\sqrt{5}x + 4 = 0$ —— ①

on compairing ① with $ax^2 + bx + c = 0$

$\therefore a = k, \; b = -2\sqrt{5}, \; c = 4$

$D = b^2 - 4ac = 0$ (∵ roots are real & equal)

$(-2\sqrt{5})^2 - 4(k)(4) = 0$

$20 - 16k = 0$

$\boxed{k = 5/4}$

iii) $3x^2-5x+2k=0$

→ The given eqn is $3x^2-5x+2k=0$ —— ①

on compairing eqn ① with $ax^2+bx+c=0$

$\therefore a=3,\ b=-5,\ c=2k$

$D=b^2-4ac=0$ $\because$ roots are real & equal.

$(-5)^2-4(3)(2k)=0$

$25-24k=0$

$\boxed{k=25/24}$

iv) $4x^2+kx+9=0$

→ The given eqn is $4x^2+kx+9=0$ —— ①

on compairing eqn ① with $ax^2+bx+c=0$

$\therefore a=4,\ b=k,\ c=9$

$D=b^2-4ac=0$ $\because$ roots are real & equal.

$(k^2)-4(4)(9)=0$

$k^2-144=0$

$k^2=144 \Rightarrow \boxed{k=\pm 12}$

v) $2kx^2-40x+25=0$

→ Given eqn is $2kx^2-40x+25=0$ —— ①

on compairing eqn ① with $ax^2+bx+c=0$

$\therefore a=2k,\ b=-40,\ c=25$

$D=b^2-4ac=0$ $\because$ roots are real & equal.

$(-40)^2-4(2k)(25)=0$

$1600-200k=0$

$\boxed{k=8}$

vi) $9x^2 - 24x + k = 0$

→ The given equation is $9x^2 - 24x + k = 0$ —— ①

on compairing equⁿ ① with $ax^2 + bx + c = 0$

$\therefore a = 9,\ b = -24,\ c = k$

$D = b^2 - 4ac = 0$ $\because$ roots are real & equal.

$(-24)^2 - 4(9)(k) = 0$

$576 - 36k = 0$

$\boxed{k = 16}$

vii) $4x^2 - 3kx + 1 = 0$

→ The given equⁿ is $4x^2 - 3kx + 1 = 0$ —— ①

on compairing equⁿ ① with $ax^2 + bx + c = 0$

$\therefore a = 4,\ b = -3k,\ c = 1$

$D = b^2 - 4ac = 0$ $\because$ roots are real & equal

$(-3k)^2 - 4(4)(1) = 0$

$9k^2 - 16 = 0$

$9k^2 = 16$

$\boxed{k = \pm 4/3}$

viii) $x^2 - 2(5+2k)x + 3(7+10k) = 0$

→ The given quadratic equⁿ is $x^2 - 2(5+2k)x + 3(7+10k) = 0$ —— ①

$\therefore a = 1,\ b = -2(5+2k),\ c = 3(7+10k)$

$D = b^2 - 4ac = 0$ $\because$ roots are real & equal.

$[-2(5+2k)]^2 - 4(1)(3)(7+10k) = 0$

$4(5+2k)^2 - 12(7+10k) = 0$

$25 + 4k^2 + 20k - 21 - 30k = 0$

$4k^2 - 10k + 4 = 0$

$2k^2 - 5k + 2 = 0$

$2k^2 - 4k - k + 2 = 0$

$2k(k-2) - 1(k-2) = 0$

$\Rightarrow (k-2)(2k-1) = 0$

$\boxed{k = 2}$

or $\boxed{k = 1/2}$

ix) $(3k+1)x^2+2(k+1)x+k=0$

$\rightarrow$ Given equn is $(3k+1)x^2+2(k+1)x+k=0$ —— ①

on compairing equn ① with $ax^2+bx+c=0$

$\therefore a=(3k+1),\ b=2(k+1),\ c=k$

$D=b^2-4ac=0$ $\because$ roots are real and equal

$2^2(k+1)^2-4(3k+1)k=0$

$4(k^2+2k+1)-(12k^2+4k)=0$

$4k^2+8k+4-12k^2-4k=0$

$-8k^2+4k+4=0$

$2k^2-k-1=0$

$2k^2-2k+k-1=0$

$2k(k-1)+1(k-1)=0$

$(k-1)(2k+1)=0$

$\boxed{k=1}$ and $\boxed{k=-1/2}$

x) $kx^2+kx+1=-4x^2-x$

$\rightarrow$ Given equn is $kx^2+kx+1=-4x^2-x$ —— ①

on compairing equn ① with $ax^2+bx+c=0$

$\therefore a=k,\ b=k,\ c=$

① $\Rightarrow$ $(k+4)x^2+(k+1)x+1=0$

$\therefore a=(k+4),\ b=(k+1),\ c=1$

$D=b^2-4ac=0$ $\because$ roots are real & equal

$(k+1)^2-4(k+4)(1)=0$

$k^2+2k+1-(4k+16)=0$

$k^2-2k-15=0$

$k^2-5k+3k-15=0$

$k(k-5)+3(k-5)=0$

$(k+3)(k-5)=0$

$\boxed{k=-3}$ and $\boxed{k=5}$

xi) $(k+1)x^2 + 2(k+3)x + (k+8) = 0$

→ Given eqⁿ is $(k+1)x^2 + 2(k+3)x + (k+8) = 0$ —— ①

on compairing equⁿ ① with $ax^2 + bx + c = 0$

$\therefore a = (k+1) \quad b = 2(k+3) \; ; \; c = k+8$

$D = b^2 - 4ac = 0$ $\because$ roots are real & equal.

$4(k+3)^2 - 4(k+1)(k+8) = 0$

$(k^2+6k+9) - (k^2+9k+8) = 0$

$-3k + 1 = 0$

$\boxed{k = 1/3}$

xii) $x^2 - 2kx + 7k - 12 = 0$

→ Given equⁿ is $x^2 - 2kx + (7k-12) = 0$ —— ①

on compairing equⁿ ① with $ax^2 + bx + c = 0$

$\therefore a = 1 \; , \; b = -2k \; , \; c = (7k-12)$

$D = b^2 - 4ac = 0$ $\because$ roots are real & equal.

$(-2k)^2 - 4(1)(7k+12) = 0$

$4k^2 - 4(7k-12) = 0$

$4k^2 - 4(7k-12) = 0$

$k^2 - 7k + 12 = 0$

$k^2 - 4k - 3k + 12 = 0$

$(k-4)(k-3) = 0$

$\boxed{k = 4}$ and $\boxed{k = 3}$

xiii) $(k+1)x^2-2(3k+1)x+(8k+1)=0$

$\longrightarrow$ Given equ^n is $(k+1)x^2-2(3k+1)x+(8k+1)=0$

$\therefore a=k+1,\quad b=-2(3k+1),\quad c=(8k+1)$

$D=b^2-4ac=0 \quad \because$ roots are real & equal.

$4(3k+1)^2-4(k+1)(8k+1)=0$

$4(3k+1)^2-4(k+1)(8k+1)=0$

$(3k+1)^2-(k+1)(8k+1)=0$

$9k^2+6k+1-(8k^2+9k+1)=0$

$9k^2+6k+1-8k^2-9k-1=0$

$k^2-3k=0$

$k(k-3)=0$

$\boxed{k=0}$ and $\boxed{k=3}$

xiv) $5x^2-4x+2+k(4x^2-2x+1)=0$

$\longrightarrow$ Given equ^n is $5x^2-4x+2+k(4x^2-2x+1)=0$

$x^2(5+4k)-x(4+2k)+(2-k)=0$ —— ①

on comparing equ^{n}① with $ax^2+bx+c=0$

$\therefore a=(4k+5),\quad b=-(2k+4),\quad c=(2-k)$

$D=b^2-4ac=0 \quad \because$ roots are real & equal.

$(2k+4)^2-4(4k+5)(2-k)=0$

$(2k+4)^2-4(4k+5)(2-k)=0$

$16+4k^2+16k-4(8k-4k^2+10-5k)=0$

$16+4k^2+16k-40+20k-32k+16k^2=0$

$20k^2+4k-24=0$

$5k^2+k-6=0$

$5k^2+6k-5k-6=0$

$5k(k-1) + 6(k-1) = 0$

$(k-1)(5k+6) = 0$

$\boxed{k=1}$ and $\boxed{k=-6/5}$

xv) $(4-k)x^2 + (2k+4)x + (8k+1) = 0$

→ Given equn is $(4-k)x^2 + (2k+4)x + (8k+1) = 0$ — ①

on compairing with equn ① $ax^2 + bx + c = 0$

$\therefore a = (4-k)$, $b = (2k+4)$, $c = (8k+1)$

$D = b^2 - 4ac = 0$ $\because$ roots are real & equal

$(2k+4)^2 - 4(4-k)(8k+1) = 0$

$4k^2 + 16k + 16 - 4(-8k^2 + 32k + 4 - k) = 0$

$4k^2 + 16k + 16 + 32k^2 - 124k - 16 = 0$

$36k^2 - 108k = 0$

$9k(k-3) = 0$

$\boxed{k=0}$ and $\boxed{k=3}$

xvi) $(2k+1)x^2 + 2(k+3)x + (k+5) = 0$

→ The given equn is $(2k+1)x^2 + 2(k+3)x + (k+5) = 0$ — ①

on compairing equn ① with $ax^2 + bx + c = 0$

$\therefore a = (2k+1)$, $b = 2(k+3)$, $c = (k+5)$

$D = b^2 - 4ac = 0$ $\because$ roots are real & equal.

$4(k+3)^2 - 4(2k+1)(k+5) = 0$

$4(k+3)^2 - 4(2k^2 + 11k + 5) = 0$

$(k+3)^2 - (2k^2 + 11k + 5) = 0$

$k^2 + 5k - 4 = 0$

$k^2 + 2(5/2)k + (5/2)^2 = 4 + (5/2)^2$

$(k + 5/2)^2 = 4 + 25/4 = \sqrt{41/4}$

$k + 5/2 = \pm \frac{\sqrt{41}}{2}$

$\boxed{k = (\sqrt{41} - 5)/2}$ or $\boxed{k = -(\sqrt{41} + 5)/2}$

xvii) $4x^2 - 2(k+1)x + (k+4) = 0$

→ Given equn is $4x^2 - 2(k+1)x + (k+4) = 0$ —— ①

on compairing equn ① with $ax^2 + bx + c = 0$

$\therefore a = 4, \quad b = -2(k+1), \quad c = (k+4)$

$D = b^2 - 4ac = 0$ ∵ roots are real & equal.

$4(k+1)^2 - 4(4)(k+4) = 0$

$4(k+1)^2 - 4(k+4) = 0$

$k^2 - 2k - 15 = 0$

$k^2 - 5k + 3k - 15 = 0$

$k(k-5) + 3(k-5) = 0$

$(k-5)(k+3) = 0$

$\boxed{k = 5}$ & $\boxed{k = -3}$

3. In the following, determine the set of values of k for which the given quadratic equn has real roots:

i) $2x^2 + 3x + k = 0$

→ Given equn is $2x^2 + 3x + k = 0$ —— ①

on comparing equn ① with $ax^2 + bx + c = 0$

$\therefore a = 2, \quad b = 3, \quad c = k$

$D = b^2 - 4ac \geq 0$ ∵ roots are real

$= 9 - 4(2)k \geq 0$

$\Rightarrow \boxed{k \leq 9/8}$

The value of k should be less than 9/8, for real roots.

ii) $2x^2+x+k=0$

$\rightarrow$ Given equⁿ is $2x^2+x+k=0$ —— ①

On compairing equⁿ ① with $ax^2+bx+c=0$

$\therefore a=2, \quad b=1, \quad c=k$

$D=b^2-4ac \geq 0$ $\quad \because$ roots are real

$(1)^2-4(2)k \geq 0$

$1-8k \geq 0$

$\cancel{(1-} \; 1 \geq 8k$

$8k \leq 1$

$\boxed{k \leq 1/8}$

The value of 'k' should be less than 1/8 to get real roots.

iii) $2x^2-5x-k=0$

$\rightarrow$ Given equⁿ is $2x^2-5x-k=0$ — ①

on compairing equⁿ ① with $ax^2+bx+c=0$

$\therefore a=2, \; b=-5, \; c=-k$

$D=b^2-4ac \geq 0$

$(-5)^2-4(2)(-k) \geq 0$

$(25+8k) \geq 0$

$\cancel{25 \geq -8k}$ $\quad 8k \geq -25$

$\cancel{8k \leq -25}$ $\quad \boxed{k \geq -25/8}$

$\cancel{k \leq -25/8}$

Hence, the value of k should be greater than (-25/8) to have real roots.

iv) $kx^2+6x+1=0$

$\rightarrow$ The given equⁿ is $kx^2+6x+1=0$ —— ①

on compairing equⁿ with $ax^2+bx+c=0$

$\therefore a=k, \; b=6, \; c=1$

$D=b^2-4ac \geq 0$ $\quad \because$ roots are real

$36-4k \geq 0$ $\quad 4k \leq 36$

$36 \geq 4k$ $\quad \boxed{k \leq 9}$

v) $3x^2+2x+k=0$

→ Given equⁿ is $3x^2+2x+k=0$ — ①

on compairing with $ax^2+bx+c=0$

$\therefore a=3,\ b=2,\ c=k$

$D=b^2-4ac \geq 0$ ∵ roots are real

$(2)^2-4(3)(k) \geq 0$

$4-12k \geq 0$

$4 \geq 12k$

$12k \leq 4$ The value of k should be less than 1/3 to get real roots.

$\boxed{k \leq 1/3}$

4. Find the values of k for which the following equations have real & equal roots.

i) $x^2-2(k+1)x+k^2=0$

→ Given equⁿ is $x^2-2(k+1)x+k^2=0$ — ①

on compairing equⁿ① with $ax^2+bx+c=0$

$\therefore a=1,\ b=-2(k+1),\ c=k^2$

$D=b^2-4ac=0$ ∵ roots are real & equal

$4(k+1)^2-4(1)(k^2)=0$

$4k^2+8k+4-4k^2=0$

$8k+4=0$

$k=-4/8$ The value of k must be $(-1/2)$ to get real & equal roots.

$\boxed{k=-1/2}$

ii) $k^2x^2 - 2(2k-1)x + 4 = 0$

→ Given eqn is $k^2x^2 - 2(2k-1)x + 4 = 0$ —— ①

on compairing ① with $ax^2 + bx + c = 0$

$\therefore a = k^2, \quad b = -2(2k-1), \quad c = 4$

$D = b^2 - 4ac = 0$ $\because$ roots are real & equal.

$4(2k-1)^2 - 4(k^2)4 = 0$

$4k^2 - 4k + 1 - 4k^2 = 0$

$-4k + 1 = 0$

$\boxed{k = 1/4}$

The value of k must be 1/4, to get real & equal roots.

iii) $(k+1)x^2 - 2(k-1)x + 1 = 0$

→ Given eqn is $(k+1)x^2 - 2(k-1)x + 1 = 0$ —— ①

on compairing ① with $ax^2 + bx + c = 0$

$\therefore a = (k+1), \quad b = -2(k-1), \quad c = 1$

$D = b^2 - 4ac = 0$

$4(k-1)^2 - 4(k+1)(1) = 0$

$4k^2 - 2k + 1 - k - 1 = 0$

$k^2 - 3k = 0$

$k(k-3) = 0$

$\boxed{k = 0}$ or $\boxed{k = 3}$

The value of k must be 0 or 3, to get real & equal roots.

5. find the values of k for which the following equations have real roots.

i) $2x^2+kx+3=0$

$\rightarrow$ Given equn is $2x^2+kx+3=0$ — ①

on compairing equn ① with $ax^2+bx+c=0$

$\therefore a=2, \quad b=k, \quad c=3$

$D=b^2-4ac \geq 0 \quad \because$ roots are real

$k^2-2(4)(3) \geq 0$

$k^2-24 \geq 0$

$k^2 \geq 24$

$\boxed{k \geq 2\sqrt{6}}$ and $\boxed{k \leq -2\sqrt{6}}$

ii) $kx(x-2)+6=0$

$\rightarrow$ Given equn is $kx(x-2)+6=0$ — ①

$kx^2-2kx+6=0$

on compairing above equn with $ax^2+bx+c=0$

$\therefore a=k, b=-2k, \quad c=6$

$D=b^2-4ac \geq 0 \quad \because$ roots are real

$(-2k)^2-4(k)(6) \geq 0$

$4k^2-24k \geq 0$

$4k(k-6) \geq 0$

$k \geq 0$ and $k \geq 6$

$\boxed{k \geq 6}$

The k should be greater than 6 for real roots.

iii) $x^2 - 4kx + k = 0$

$\rightarrow$ Given equn is $x^2 - 4kx + k = 0$ —— ①

on compairing equn ① with $ax^2 + bx + c = 0$

$\therefore a = 1, \; b = -4k, \; c = k$

$D = b^2 - 4ac \geq 0$ $\quad$ $\because$ roots are real

$(-4k)^2 - 4(1)(k) \geq 0$

$4k(4k - 1) \geq 0$

$k \geq 0$ and $k \geq 1/4$

$\Rightarrow \boxed{k \geq 1/4}$

The value of k should be greater than 1/4, for real roots.

iv) $kx(x - 2\sqrt{5}) + 10 = 0$

$\rightarrow$ Given equn is $kx(x - 2\sqrt{5}) + 10 = 0$

$kx^2 - 2\sqrt{5}kx + 10 = 0$ —— ①

on compairing ① with equn $ax^2 + bx + c = 0$

$\therefore a = k, \; b = -2\sqrt{5}k, \; c = 10$

$D = b^2 - 4ac \geq 0$

$(-2\sqrt{5}k)^2 - 4(k)(10) \geq 0$

$20k^2 - 40k \geq 0$

$20k(k - 2) \geq 0$

$k \geq 0$ and $k \geq 2$

$\boxed{k \geq 2}$

The value of k should be greater than or equal to 2 to get real roots.

v) $kx(x-3)+9=0$

$\rightarrow$ Given equ^n is $kx(x-3)+9=0$

$$kx^2-3kx+9=0 \quad \text{——} \; (1)$$

on compairing equ^n (1) with $ax^2+bx+c=0$

$$\therefore a=k, \; b=-3k, \; c=9$$

$D=b^2-4ac \geq 0$ $\quad \because$ roots are real

$(-3k)^2-4(k)(9) \geq 0$

$9k^2-36k \geq 0$

$9k(k-4) \geq 0$

$k \geq 0$ & $k \geq 4$

$\Rightarrow \boxed{k \geq 4}$

The value of k must be greater than or equal to 4, to get real roots.

vi) $4x^2+kx+3=0$

$\rightarrow$ The given equ^n is $4x^2+kx+3=0$ —— (1)

compairing equ^n (1) with $ax^2+bx+c=0$

$$\therefore a=4, \; b=k, \; c=3$$

$D=b^2-4ac \geq 0$

$k^2-4(4)(3) \geq 0$

$k^2-48 \geq 0$

$k^2 \geq 48$

$\boxed{k \geq 4\sqrt{3}}$ & $\boxed{k \leq -4\sqrt{3}}$

6. find the values of k for which the given quadratic equation has real & distinct roots.

i) $kx^2+2x+1=0$

$\rightarrow$ Given equn is $kx^2+2x+1=0$ —— ①

on compairing equn ① with $ax^2+bx+c=0$

$\therefore a=k, \quad b=2, \quad c=1$

$D=b^2-4ac>0$

$4-4k>0$

$4k<4$ The value of k must be less than

$\boxed{k<1}$ 1, to get real & distinct roots.

ii) $kx^2+6x+1=0$

$\rightarrow$ Given equn is $kx^2+6x+1=0$ —— ①

on compairing equn ① with $ax^2+bx+c=0$

$\therefore a=k, \quad b=6, \quad c=1$

$D=b^2-4ac>0$

$6^2-4(1)k>0$

$36-4k>0$

$4k<36$ The value of k is less than 9

$\boxed{k<9}$ then roots are real & distinct.

7. for what value of k, $(4-k)x^2+(2k+4)x+(8k+1)=0$, is a perfect square.

$\rightarrow$ Given equn is $(4-k)x^2+(2k+4)x+(8k+1)=0$ —— ①

on compairing equn ① with ①

$\therefore a=(4-k), \quad b=(2k+4), \quad c=(8k+1)$

$=(2k+4)^2-4(4-k)(8k+1)$

$=4k^2+16+4k-4(32+4-8k^2-k)$

$=4(k^2+4+k-32-4+8k^2+k)$

$=4(9k^2-27k)$

$D = 0$ $\quad$ ∵ equation is perfect square

$4(9k^2 - 27k) = 0$

$(9k^2 - 27k) = 0$

$3k(k-3) = 0$

$3k = 0$ or $k - 3 = 0$

$\boxed{k=0}$ $\quad$ $\boxed{k=3}$

The value of k must be 0 or 3, to get perfect square.

8. Find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots.

→ Given equⁿ is $x^2 + kx + 4 = 0$ — ①

on comparing equⁿ ① with $ax^2 + bx + c = 0$

∴ $a = 1$, $b = k$, $c = 4$

$D = b^2 - 4ac \geq 0$ $\quad$ ∵ roots are real

$k^2 - 4(1)(4) \geq 0$

$\Rightarrow k^2 - 16 \geq 0$

$\boxed{k \geq 4}$ and $\boxed{k \leq -4}$

Since, we consider the least positive value.

$\boxed{k = 4}$

The least value of k is 4, to get real roots.

9. Find the values of k for which the quadratic equation $(3k+1)x^2 + 2(k+1)x + 1 = 0$ has equal roots. Also, find the roots.

→ Given equⁿ is $(3k+1)x^2 + 2(k+1)x + 1 = 0$ — ①

on comparing equⁿ ① with $ax^2 + bx + c = 0$.

∴ $a = (3k+1)$, $b = 2(k+1)$, $c = 1$

For roots to be real & equal:

$$D = b^2 - 4ac = 0$$
$$2^2(k+1)^2 - 4(3k+1)(1) = 0$$
$$(k+1)^2 - (3k+1) = 0$$
$$k^2 + 2k + 1 - 3k - 1 = 0$$
$$k^2 - k = 0$$
$$k(k-1) = 0$$
$$\boxed{k=0} \text{ or } k-1=0 \Rightarrow \boxed{k=1}$$

Now, $\boxed{k=0} \Rightarrow [3(0)+1]x^2 + 2(0+1)x + 1 = 0$
$$x^2 + 2x + 1 = 0$$
$$(x+1)^2 = 0$$
$\boxed{x=-1}$ is one of the root of quadratic equⁿ ①

Now, $\boxed{k=1} \Rightarrow [3(1)+1]x^2 + 2(1+1)x + 1 = 0$
$$4x^2 + 4x + 1 = 0$$
$$(2x+1)^2 = 0$$
$$2x = -1 \Rightarrow \boxed{x = -1/2}$$
$x = -1/2$ is also the root of quadratic equⁿ①.

10. Find the values of p for which the quadratic equation $(2p+1)x^2 - (7p+2)x + (7p-3) = 0$ has equal roots. Also, find the roots.

→ The given equⁿ $(2p+1)x^2 - (7p+2)x + (7p-3) = 0$ — ①
on comparing equⁿ ① with $ax^2 + bx + c = 0$
$$\therefore a = (2p+1), \quad b = -(7p+2), \quad c = (7p-3)$$
For the roots to be real & equal roots:
$$D = b^2 - 4ac = 0$$
$$[-(7p+2)]^2 - 4(2p+1)(7p-3) = 0$$

$(7p+2)^2 - 4(14p^2+p-3) = 0$

$49p^2 + 28p + 4 - 56p^2 - 4p + 12 = 0$

$-7p^2 + 24p + 16 = 0$

$-7p^2 + 28p - 4p + 16 = 0$

$-7p(p-4) - 4(p-4) = 0$

$(p-4)(-7p-4) = 0$

$p - 4 = 0 \Rightarrow \boxed{p=4}$

$-7p - 4 = 0 \Rightarrow \boxed{p=-4/7}$

when $p=4$: $[2(4)+1]x^2 - [7(4)+2]x + [7(4)-3] = 0$

$9x^2 - 30x + 25 = 0$

$(3x-5)^2 = 0$

$\boxed{x = 5/3}$ is one of the root of given quadratic equ^n.

when $p=-4/7$:

$[2(-4/7)+1]x^2 - [7(-4/7)+2]x + [7(-4/7)-3] = 0$

$x^2 - 14x + 49 = 0$

$(x-7)^2 = 0$

$x - 7 = 0$

$\boxed{x=7}$ is the root of the given equ^n 1.

11) If -5 is a root of the quadratic equ^n $2x^2 + px - 15 = 0$ and the quadratic equ^n $p(x^2+x) + k = 0$ has equal roots, find the value of k.

→ Given quadratic equ^n is $2x^2 + px - 15 = 0$ — 1

Given that, equ^n 1 ha $x = -5$ is one of the root.

$\Rightarrow 2(-5)^2 + p(-5) - 15 = 0$

$50 - 5p - 15 = 0$

$35 = 5p$

$\boxed{p = 7}$, put in equn (*) $p(x^2+x) + k = 0$

$\Rightarrow$ (*) $7(x^2+x) + k = 0$

$7x^2 + 7x + k = 0$ —— ②

Now, given that equn ② has two equal roots.

Here, $a = 7$, $b = 7$, $c = k$

$D = b^2 - 4ac = 0$

$(7)^2 - 4(7)(k) = 0$

$49 - 28k = 0$

$k = 49/28 \Rightarrow \boxed{k = 7/4}$

12) If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find the value of k.

$\longrightarrow$ Given quadratic equn $3x^2 + px - 8 = 0$ —— ① has $x = 2$ is one of the root.

$\Rightarrow 3(2)^2 + p(2) - 8 = 0$

$3(4) + p(2) - 8 = 0$

$12 + p(2) - 8 = 0$

$2p = -4$

$\boxed{p = -2}$ put the value of p in equn

$4x^2 - 2px + k = 0$

$\Rightarrow 4x^2 - 2(-2)x + k = 0$

$4x^2 + 4x + k = 0$ —— ②

Now, equn ② has equal roots.

Here, $a = 4$, $b = 4$, $c = k$

$D = b^2 - 4ac = (4)^2 - 4(4)k = 0$

$16 - 16k = 0$

$1 - k = 0$

$\boxed{k = 1}$

Exercise 8.7

1. Find two consecutive numbers whose squares have the sum of 85.

→ Let us consider the two consecutive numbers as x and $(x+1)$ respectively.

Given that, the sum of squares of number is 85.

$$\Rightarrow x^2+(x+1)^2=85$$
$$x^2+x^2+2x+1=85$$
$$2x^2+2x+1-85=0$$
$$2x^2+2x-84=0$$
$$x^2+x-42=0$$
$$\Rightarrow x^2+7x-6x-42=0$$
$$x(x+7)-6(x+7)=0$$
$$(x-6)(x+7)=0$$
$$(x-6)=0 \quad \text{or} \quad x+7=0$$
$$\boxed{x=6} \quad \text{or} \quad \boxed{x=-7}$$

The required two consecutive numbers may b $(6 \,\&\, 7)$ or $(-7 \,\&\, -6)$.

2) Divide 29 into two parts so that the sum of the squares of the parts is 425.

→ Let us consider one part is 'x' and hence other part must be $(29-x)$.

Given that, the sum of the squares of x and $(29-x)$ is 425.

$$\Rightarrow x^2+(29-x)^2=425$$
$$x^2+x^2+841+(-58x)=425$$
$$2x^2-58x+841-425=0$$
$$2x^2-58x+416=0$$
$$x^2-29x+208=0$$

$$\Rightarrow x^2 - 13x - 16x + 208 = 0$$

$$x(x-13) - 16(x-13) = 0$$

$$(x-13)(x-16) = 0$$

$$\Rightarrow x - 13 = 0 \quad \text{or} \quad x - 16 = 0$$

$$\boxed{x = 13} \quad \text{or} \quad \boxed{x = 16}$$

In this way, the two parts whose sum of the squares is 425 are 13 and 16 respectively.

3) Two squares have sides x cm and $(x+4)$ cm. The sum of their areas is 656 cm². Find the sides of the squares.

$\longrightarrow$ Given that, two squares have sides x cm and $(x+4)$ cm. respectively.

And the sum of their areas is 656 cm².

But, we already know that

Area of the square = $(\text{side})^2$

Hence, areas of the squares are x^2 and $(x+4)^2$.

from given condition,

$$\Rightarrow x^2 + (x+4)^2 = 656$$

$$x^2 + x^2 + 8x + 16 = 656$$

$$2x^2 + 8x - 640 = 0$$

$$x^2 + 4x - 320 = 0$$

$$\Rightarrow x^2 + 20x - 16x - 320 = 0$$

$$x(x+20) - 16(x+20) = 0$$

$$(x+20)(x-16) = 0$$

$$x + 20 = 0 \quad \text{or} \quad x - 16 = 0$$

$$\boxed{x = -20} \quad \text{or} \quad \boxed{x = 16}$$

But, the side of the square is never negative.

Hence, the side of square is 16 cm.

Thus, $x + 4 = 16 + 4 = 20$ cm

And hence, the other square is having side 20cm.

4) The sum of two numbers is 48 and their product is 432. Find the numbers.

→ • Given that, the sum of two numbers is 48.

• Let us consider, 'x' be the one number & hence another number will be $(48-x)$.

• Also, given that the product of these two numbers is 432.

$$\Rightarrow \cancel{48x-} \; x(48-x) = 432$$

$$48x - x^2 = 432$$

$$x^2 - 48x + 432 = 0$$

$$x^2 - 36x - 12x + 432 = 0$$

$$x(x-36) - 12(x-36) = 0$$

$$(x-36)(x-12) = 0$$

$$x-36 = 0 \quad \text{or} \quad x-12 = 0$$

$$\boxed{x = 36} \quad \text{or} \quad \boxed{x = 12}$$

Thus, the required two numbers are 12 & 36 respectively.

5) If an integer is added to its square, the sum is 90. Find the integer with the help of quadratic equation.

→ Let us consider, the integer is x.

Then, the square of it is x^2.

Also, the sum of these two numbers is 90.

$$\Rightarrow x + x^2 = 90$$

$$x^2 + x - 90 = 0$$

$$x^2 + 10x - 9x - 90 = 0$$

$$x(x+10) - 9(x+10) = 0$$

$$(x-9)(x+10) = 0$$

$$x-9 = 0 \quad \text{or} \quad x+10 = 0$$

$$\boxed{x = 9} \quad \text{or} \quad \boxed{x = -10}$$

Thus, the required integers are -10 & 9 respectively.

6) Find the whole number which when decreased by 20 is equal to 69 times the reciprocal of the number.

→ Let us consider the number as 'x'.

As number 'x' is decreased by 20 ⇒ $(x-20)$

And hence, reciprocal of the whole number is $1/x$.

from first given condition,

$$(x-20) = 69 \times (1/x)$$

$$x(x-20) = 69$$

$$\Rightarrow x^2 - 20x - 69 = 0$$

$$x^2 - 23x + 3x - 69 = 0$$

$$x(x-23) + 3(x-23) = 0$$

$$(x-23)(x+3) = 0$$

$$\boxed{x = 23} \quad \text{or} \quad \boxed{x = -3}$$

we already know that, the whole number is always positive.

Hence, the required number should be $x = 23$ only.

7) Find the two consecutive natural numbers whose product is 20.

→ Let us consider, the two consecutive natural numbers be x and $(x+1)$ respectively.

But, given that their product is 20.

$$\Rightarrow \quad x(x+1) = 20$$

$$x^2 + x - 20 = 0$$

$$x^2 + 5x - 4x - 20 = 0$$

$$x(x+5) - 4(x+5) = 0$$

$$(x+5)(x-4) = 0$$

$$x+5 = 0 \quad \text{or} \quad x-4 = 0$$

$$\boxed{x = -5} \quad \text{or} \quad \boxed{x = 4}$$

But, natural number is always positive.

Thus, $x = 4$ & $(x+1) = 5$ are the two consecutive natural numbers required.

8) The sum of the squares of two consecutive odd positive integers is 394. find them.

→ Let us consider, the consecutive odd positive integers be $(2x-1)$ and $(2x+1)$ respectively.

From given condition,

$$(2x-1)^2+(2x+1)^2=394$$

$$4x^2+1-4x+4x^2+1+4x=394$$

$$\Rightarrow 8x^2+2=394$$

$$8x^2=392$$

$$x^2=49 \Rightarrow x=\pm 7$$

$$\boxed{x=7} \text{ and } \boxed{x=-7}$$

But, here we want only consecutive odd positive integers.

So $\boxed{x=7}$ we can consider.

Thus, $2x-1=14-1=13$

$2x+1=14+1=15$

Hence, the two consecutive odd positive numbers required are 13 and 15 respectively.

9) The sum of two numbers is 8 and 15 times the sum of the reciprocal is also 8. Find the numbers.

→ Let us consider one number be 'x'.

Then other number will be $(8-x)$.

From given condition,

$$15\left(\frac{1}{x}+\frac{1}{(8-x)}\right)=8$$

$$15\,\frac{(8-x+x)}{x(8-x)}=8$$

$$15\left(\frac{8}{8x-x^2}\right)=8$$

$$120=8(8x-x^2)$$

$120 = 8(8x - x^2)$

$120 = 64x - 8x^2$

$8x^2 - 64x + 120 = 0$

$8(x^2 - 8x + 15) = 0$

$x^2 - 8x + 15 = 0$

$\Rightarrow x^2 - 5x - 3x + 15 = 0$

$x(x-5) - 3(x-5) = 0$

$(x-5)(x-3) = 0$

$\boxed{x = 5}$ or $\boxed{x = 3}$

Hence, the required two numbers are 5 & 3 respectively.

10) The sum of a number and its positive square root is 6/25. Find the numbers.

$\rightarrow$ Let us consider the number be 'x'.

From given condition,

$$x + \sqrt{x} = \frac{6}{25} \quad \text{---} \quad ①$$

put $x = y^2$ in ①

$\Rightarrow y^2 + y = 6/25$

$25y^2 + 25y - 6 = 0$

$25y^2 + 30y - 5y - 6 = 0$

$5y(5y+6) - 1(5y+6) = 0$

$(5y+6)(5y-1) = 0$

$5y + 6 = 0$ & $5y - 1 = 0$

$\boxed{y = -6/5}$ & $\boxed{y = 1/5}$

But, given that the positive square root is needed only.

Thus, $x = (1/5)^2 = 1/25$

Hence, the required number is 1/25.

11) The sum of a number & its square is 63/4, find the numbers

→ Let us consider the number be 'x'.

Hence, the square of the number is x^2.

from given condition,

$$x + x^2 = 63/4$$

$$4x + 4x^2 = 63$$

$$4x^2 + 4x - 63 = 0$$

$$4x^2 + 18x - 14x - 63 = 0$$

$$2x(2x+9) - 7(2x-9) = 0$$

$$(2x+9)(2x-7) = 0$$

$$\boxed{x = -9/2} \text{ or } \boxed{x = 7/2}$$

Hence, the required numbers be $-9/2$ & $7/2$.

12) There are three consecutive integers such that the square of the first increased by the product of the other two gives 154. What are the integers?

→ Let us consider, the three consecutive integers be x, $(x+1)$ & $(x+2)$ respectively.

from given condition,

$$x^2 + (x+1)(x+2) = 154$$

$$x^2 + x^2 + 3x + 2 = 154$$

$$2x^2 + 3x - 152 = 0$$

$$2x^2 + 19x - 16x - 152 = 0$$

$$x(2x+19) - 8(2x+19) = 0$$

$$(2x+19)(x-8) = 0$$

$$\boxed{x = -19/2} \text{ or } \boxed{x = 8}$$

Hence, the required numbers are 8, 9 & 10 respectively.

13) The product of two successive integral multiples of 5 is 300. Determine the multiples.

→ from given condition,

The product of two successive integral multiples of 5 is 300.

Let us consider that two numbers be $5x$ and $(5x+1)$

So that x and $(x+1)$ are two consecutive multiples.

$$\Rightarrow 5x[5(x+1)] = 300$$

$$25x(x+1) = 300$$

$$x^2 + x = 12$$

$$x^2 + x - 12 = 0$$

$$x^2 + 4x - 3x - 12 = 0$$

$$x(x+4) - 3(x+4) = 0$$

$$(x+4)(x-3) = 0$$

$$\boxed{x=-4} \quad \text{or} \quad \boxed{x=3}$$

When, $x=-4$

$5x = -20$ & $5(x+1) = -15$

When $x=3$

$5x = 15$ and $5(x+1) = 20$

Hence, the two successive integral multiples may be 15, 20 or -15, -20 respectively.

14) The sum of the squares of two numbers is 233 and one of the number is 3 less than twice the other number. Find the numbers.

→ Let us consider the number be 'x'.

Then another number will be $(2x-3)$.

From given condition,

$$x^2+(2x-3)^2=233$$

$$x^2+4x^2+9-12x=233$$

$$5x^2-12x-224=0$$

$$5x^2-40x+28x-224=0$$

$$5x(x-8)+28(x-8)=0$$

$$(5x+28)(x-8)=0$$

$$\boxed{x=8} \quad \text{or} \quad \boxed{x=-28/5}$$

But $(5x+28)\neq 0$

$$\Rightarrow \boxed{x=8}$$

Hence, the other number will be $2x-3=2(8)-3$

$=16-3$

$=13.$

Thus, the two required numbers are 8 & 13 respectively.

15) Find the consecutive even integers whose squares have the sum 340.

→ Let us assume that, the three consecutive even numbers be $2x$ and $(2x+2)$ respectively.

From given condition,

$$(2x)^2+(2x+2)^2=340$$

$$4x^2+4x^2+8x+4=340$$

$$8x^2+8x-336=0$$

$$8(x^2+x-42)=0$$

$x^2 + x - 42 = 0$

$x^2 + 7x - 6x - 42 = 0$

$(x+7)(x-6) = 0$

$\boxed{x = -7}$ or $\boxed{x = 6}$

When $x = -7$

The required numbers are $2x = -14$ & $(2x+2) = -12$.

When $x = 6$

The required numbers are $2x = 12$ and $(2x+2) = 14$.

Thus, the consecutive integers which are even may be $-12, -14$ or $12, 14$ respectively.

16) The difference of two numbers is 4. If the difference of their reciprocal is 4/21, find the numbers.

→ Let us consider the two numbers be x and $(x-4)$ respectively.

From given condition,

$$\frac{1}{(x-4)} - \frac{1}{x} = \frac{4}{21}$$

$$\frac{(x - x + 4)}{x(x-4)} = \frac{4}{21}$$

$84 = 4x(x-4)$

$x^2 - 4x - 21 = 0$

$x^2 - 7x + 3x - 21 = 0$

$(x-7)(x+3) = 0$

$\boxed{x = 7}$ or $\boxed{x = -3}$

Hence, the required numbers may be 7 & -3 respectively.

17) Find the two natural numbers which differ by 3 & whose squares have the sum 117.

→ Let us consider the numbers be x & $(x-3)$.

from given condition,

$$x^2 + (x-3)^2 = 117$$

$$x^2 + x^2 + 9 - 6x - 117 = 0$$

$$2x^2 - 6x - 108 = 0$$

$$x^2 - 3x - 54 = 0$$

$$x^2 - 9x + 6x - 54 = 0$$

$$x(x-9) + 6(x-9) = 0$$

$$(x-9)(x+6) = 0$$

$$\boxed{x = 9} \text{ or } \boxed{x = -6}$$

But, natural number is always positive.

Hence, when $x = 9 \Rightarrow x - 3 = 9 - 3 = 6$,

Thus, the required numbers are 6 & 9 respectively.

18) The sum of the squares of three consecutive natural numbers is 149. Find the numbers.

→ Let us consider, the three natural consecutive numbers be x, $(x+1)$ & $(x+2)$ respectively.

from given condition,

$$x^2 + (x+1)^2 + (x+2)^2 = 149$$

$$x^2 + x^2 + x^2 + 1 + 2x + 4 + 4x = 149$$

$$3x^2 + 6x - 144 = 0$$

$$x^2 + 2x - 48 = 0$$

$$x^2 + 8x - 6x - 48 = 0$$

$$x(x+8) - 6(x+8) = 0$$

$$(x+8)(x-6)=0$$

$$\boxed{x=-8} \text{ or } \boxed{x=6}$$

We are considering positive value of 'x' only.

When $\boxed{x=6}$ $\Rightarrow$ $x+1=7$, $x+2=8$.

Thus, the three conseative natural numbers are 6, 7 & 8 respectively.

19) The sum of two numbers is 16. The sum of their reciprocals is 1/3. Find the numbers.

→ Let us consider, the one of the natural number is 'x'.

Then another natural number is ($\cancel{x}$ $16-x$).

from given condition,

$$\frac{1}{x}+\frac{1}{(16-x)}=\frac{1}{3}$$

$$\frac{(16-x+x)}{x(16-x)}=\frac{1}{3}$$

$$\frac{16}{x(16-x)}=\frac{1}{3}$$

$$\Rightarrow 16x-x^2=48$$

$$-16x+x^2+48=0$$

$$x^2-12x-4x+48=0$$

$$x(x-12)-4(x-12)=0$$

$$(x-12)(x-4)=0$$

$$\boxed{x=12} \text{ or } \boxed{x=4}$$

Thus, the required two numbers are 4 & 12 respectively.

20) Determine the two consecutive multiples of 3 whose product is 270

→ Let us assume that, the two consecutive multiples of 3 may be $3x$ and $(3x+3)$.

from given condition,

$$3x(3x+3) = 270$$

$$x(3x+3) = 90$$

$$3x^2+3x-90=0$$

$$x^2+x-30=0$$

$$x^2+6x-5x-30=0$$

$$x(x+6)-5(x+6)=0$$

$$(x+6)(x-5)=0$$

$$\boxed{x=-6} \text{ or } \boxed{x=5}$$

We are considering here only positive value of x.

When $x=5$, $3x=15$ and $(3x+3)=18$.

Thus, the two consecutive multiples of 3 are 15 and 18 respectively.

21) The sum of a number & its reciprocal is 17/4. find the number.

→ Let us consider the number be 'x'.

from given condition,

$$x + 1/x = 17/4$$

$$\frac{x^2+1}{x} = \frac{17}{4}$$

$$4(x^2+1) = 17x$$

$$4x^2+4-17x=0$$

$$4x^2+4-16x-x=0$$

$$4x(x-4)-1(x-4)=0$$

$$(4x-1)(x-4)=0$$

$$\boxed{x=1/4} \text{ or } \boxed{x=4}$$

Here, the required number is $x=4$.

Exercise 8.8

1) The speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream.

→ Let us consider the speed of the stream be x km/hr.

The speed of boat in still water is given as 8 km/hr.

Thus, speed of downstream $= (8+x)$ km/hr

speed of upstream $= (8-x)$ km/hr

we know that, $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

Now, the time required to boat to go 15 km upstream

$= \dfrac{15}{(8-x)}$ hr.

And the time required to boat to return 22 km downstream $\Big\} = \dfrac{22}{(8+x)}$ hr

And the boat returns to the same point in 5 hrs.

Thus, $$\frac{15}{(8-x)} + \frac{22}{(8+x)} = 5$$

$$\frac{15(8+x)+22(8-x)}{(8-x)(8+x)} = 5$$

$$\frac{120+15x+176-22x}{64-x^2} = 5$$

$$\frac{296-7x}{64-x^2} = 5$$

$$5x^2-7x+296-320=0$$
$$5x^2-7x-24=0$$
$$5x^2-15x+8x-24=0$$
$$5x(x-3)+8(x-3)=0$$
$$(x-3)(5x+8)=0$$
$$\boxed{x=3} \quad \text{or} \quad \boxed{x=-8/5}$$

But, the speed becomes never negative.
Hence, the speed of the stream is 3 km/hr.

2) A train, traveling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/hr more. find the original speed of the train?

→ Let us consider that, the original speed of the train is x km/hr.

When the speed is increased by 5 then,
speed of train = $(x+5)$ km/hr

we know that,
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Time required by the train to cover 360 km $\Big\}$ = $\frac{360}{x}$ hr.

Time required by the train when speed is increased by 5 to cover 360 km $\Big\}$ = $\frac{360}{(x+5)}$ hr.

But, given that, the difference in the times is 48 mins. $\Rightarrow \frac{48}{60}$ hour

$$\Rightarrow \frac{360}{x} - \frac{360}{(x+5)} = \frac{48}{60}$$

$$\frac{360(x+5)-360x}{x(x+5)} = \frac{4}{5}$$

$$\frac{360x+1800-360x}{x^2+5x} = \frac{4}{5}$$

$$1800(5) = 4(x^2+5x)$$

$$9000 = 4x^2 + 20x$$

$$4x^2+20x-9000=0$$

$$x^2+5x-2250=0$$

$$x^2+50x-45x-2250=0$$

$$x(x+50)-45(x+50)=0$$

$$(x+50)(x-45)=0$$

$$\therefore \boxed{x=-50} \text{ or } \boxed{x=45}$$

But, speed is never negative.
Hence, the original speed of the train is 45 km/hr.

3) A fast train takes one hour less than a slow train for a journey of 200km. If the speed of the slow train is 10 km/hr less than that of the fast train, find the speed of the two trains.

→ Let us consider, the speed of the fast train is $= x$ km/hr
Then the speed of the slow train is $= (x-10)$ km/hr
we know that, $\text{speed} = \frac{\text{distance}}{\text{time}}$

So, time required to fast train to cover 200km $\Big\} = \frac{200}{x}$ hr

Time required to slow train to cover 200 km $\Big\} = \frac{200}{(x-10)}$ hr

But, the difference in time given is 1 hr.

$$\Rightarrow \quad \frac{200}{x} - \frac{200}{(x-10)} = 1$$

$$\frac{[200(x-10) - 200x]}{x(x-10)} = 1$$

$$\frac{200x - 2000 - 200x}{x^2 - 10x} = 1$$

$$\therefore x^2 - 10x = -2000$$

$$x^2 - 10x + 2000 = 0$$

$$x^2 - 50x + 40x + 2000 = 0$$

$$x(x-50) + 40(x-50) = 0$$

$$(x-50)(x+40) = 0$$

$$\boxed{x = 50} \quad \text{or} \quad \boxed{x = -40}$$

But, speed is never negative.
Hence, the speed of fast train is 50 km/hr.
And the speed of slow train = (50−10) = 40 km/hr.

4) A passenger train takes one hour less for a journey of 150km if its speed is increased 5 km/hr from its usual speed. find the speed of the train.

→ Let us consider that, the usual speed of train is x km/hr.

The increased speed of train = $(x+5)$ km/hr

we know that, $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

Time required by the train with usual speed to cover 150 km $\Big\}$ = $\dfrac{150}{x}$ hr

$$\left.\begin{array}{l}\text{Time required by the train}\\ \text{with increased speed to cover}\\ \text{150 km}\end{array}\right\} = \frac{150}{(x+5)} \text{ hr.}$$

But, the difference in time is given 1 hr.

$$\frac{150}{x} - \frac{150}{(x+5)} = 1$$

$$\frac{150(x+5) - 150x}{x(x+5)} = 1$$

$$\frac{150x + 750 - 150x}{x^2 + 5x} = 1$$

$$750 = x^2 + 5x$$

$$x^2 + 5x - 750 = 0$$

$$x^2 - 25x + 30x - 750 = 0$$

$$x(x-25) + 30(x-25) = 0$$

$$(x-25)(x+30) = 0$$

$$\boxed{x = 25} \text{ or } \boxed{x = -30}$$

But, speed becomes never negative.
Hence, the usual speed of the train is found to be 25 km/hr.

5) The time taken by a person to cover 150 km was 2.5 hrs more than the time taken in the return journey. If he returned at the speed of 10 km/hr more than the speed of going, what was the speed per hour in each directions?

→ Let us consider, x km/hr be the ongoing speed of a person.

Then, While returning the speed of the person is $= (x+10)$ km/hr.

we know that, $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

Now, Time taken by the person in going direction to cover 150km $\}= 150/x$ hr

Time taken by the person in returning to cover 150 km $\} = \frac{150}{(x+10)}$ hr

But, given that difference in time is 2.5 hr $= \frac{5}{2}$ hours

$$\Rightarrow \quad \frac{150}{x} - \frac{150}{(x+10)} = \frac{5}{2}$$

$$\frac{150(x+10)-150x}{x(x+10)} = \frac{5}{2}$$

$$\frac{150x+1500-150x}{x^2+10x} = \frac{5}{2}$$

$$\frac{1500}{x^2+10x} = \frac{5}{2}$$

$$3000 = 5x^2+50x$$

$$5x^2+50x-3000=0$$

$$x^2+10x-600=0$$

$$x^2-20x+30x-600=0$$

$$x(x-20)+30(x-20)=0$$

$$(x-20)(x+30)=0$$

$$\boxed{x=20} \text{ or } \boxed{x=-30}$$

But, speed is never negative.

$x=20 \Rightarrow (x+10)=(20+10)=30$

Thus, the ongoing speed of the person is 20 km/hr.

And the returning speed of the person is 30 km/hr.

6) A plane left 40 minutes late due to bad weather and in order to reach the destination, 1600 km away in time, it had to increase its speed by 400 km/hr from its usual speed. Find the usual speed of the plane.

→ Let us consider, the speed of the plane is x km/hr.

The increased speed of the plane is $(x+4000)$ km/hr

we know that, $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

$$\left.\begin{array}{l}\text{Time required by the plane} \\ \text{with usual speed to cover} \\ \text{1600 km}\end{array}\right\} = \frac{1600}{x} \text{ hr}$$

$$\left.\begin{array}{l}\text{Time required by the plane} \\ \text{with increased speed to cover} \\ \text{1600 km}\end{array}\right\} = \frac{1600}{(x+400)} \text{ hr}$$

But, the difference in the time given is 40min. $= \frac{40}{60}$ hr

$$\Rightarrow \frac{1600}{x} - \frac{1600}{(x+400)} = \frac{40}{60}$$

$$\frac{1600(x+400) - 1600x}{x(x+400)} = \frac{2}{3}$$

$$\frac{1600x + 640000 - 1600x}{x^2+400x} = \frac{2}{3}$$

$$1920000 = 2x^2 + 800x$$

$$2x^2 + 800x - 1920000 = 0$$

$$x^2 + 400x - 960000 = 0$$

$$x^2 - 800x + 1200x - 960000 = 0$$

$$x(x-800) + 120(x-800) = 0$$

$$(x-800)(x+120) = 0$$

$\boxed{x=800}$ or $\boxed{x=-1200}$

But, speed is never negative.

Hence, the usual speed of the train is 800 km/hr.

7) An aeroplane takes 1 hour less for a journey of 1200 km if its speed is increased by 100 km/hr from its usual speed of the plane. Find its usual speed.

→ Let us consider, the usual speed of plane be 'x' km/hr.

Then, the increased speed of the plane is $= (x+100)$ km/hr.

we know that, $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

Now, time required by the plane with usual speed to cover 1200km $\Big\} = \dfrac{1200}{x}$ hr

Time required by the plane with increased speed to cover 1200km $\Big\} = \dfrac{1200}{(x+100)}$ hr

But, given that the time difference is 1 hr.

$$\frac{1200}{x} - \frac{1200}{(x+100)} = 1$$

$$\frac{1200(x+100) - 1200x}{x(x+100)} = 1$$

$$\frac{1200x + 120000 - 1200x}{x^2 + 100x} = 1$$

$$12000 = x^2 + 100x$$

$$x^2 + 100x - 120000 = 0$$

$$x^2 - 300x + 400x - 120000 = 0$$

$$x(x-300) + 400(x-300) = 0$$

$\boxed{x = 300}$ or $\boxed{x = -400}$

But speed is never negative.

Thus, the usual speed of the plane is 300 km/hr.

Exercise 8.9

1) Ashu is x years old while his mother Mrs. Veena is x^2 years old. Five years hence Mrs. Veena will be three times old as Ashu. Find their present ages.

→ Let us consider,

The Ashu's present age is 'x' years and his mother Mrs. Veena is 'x^2' years old.

After 5 years,

Ashu's age $= (x+5)$ years

Mrs. Veena's age $= (x^2+5)$ years

$$\Rightarrow \quad x^2+5 = 3(x+5)$$

$$x^2+5 = 3x+15 \qquad x^2+5-3x-15$$

$$x^2-5x+2x+10=0$$

$$x(x-5)+2(x-5)=0$$

$$(x+2)(x-5)=0$$

$$\boxed{x=-2} \quad \text{or} \quad \boxed{x=5}$$

Thus, Ashu's present age is 5 years.

Ashu's mother Mrs. Veena's present age is 25 years

2) The sum of the ages of a man and his son is 45 years. Five years ago, the product of their ages was four times the man's age at the time. Find their present age.

→ Let us consider, the present age of the man is 'x' years.

Then present age of his son $= (45 - x)$ years.

Five years ago,

A man's age was $= (x - 5)$ years

His son's age was $= (45 - x - 5)$ years

from given condition,

$$(x-5)(40-x) = 4(x-5)$$

$$40x - x^2 + 5x - 200 = 4x - 20$$

$$-x^2 + 45x - 200 - 4x + 20 = 0$$

$$-x^2 + 45x - 200 - 4x + 20 = 0$$

$$-x^2 + 41x - 180 = 0$$

$$x^2 - 36x - 5x + 180 = 0$$

$$x(x-36) - 5(x-36) = 0$$

$$(x-36)(x-5) = 0$$

$$\boxed{x = 36} \quad \text{or} \quad \boxed{x = 5}$$

But, father's age can never 5 years.

Hence, father's present age is 36 years.

And his son's age $= 45 - x = 45 - 36 = 9$ years.

3) The product of Shikha's age five years ago and her age 8 years later is 30, her age at both times being given in years find her present age

→ Let us consider that, present age of Shikha is 'x' years.

8 years later her age = $(x+8)$ years

five years ago her age = $(x-5)$ years

From given condition,

$$(x-5)(x+8)=30$$

$$x^2+8x-5x-40=30$$

$$x^2+3x-40-30=0$$

$$x^2+3x-70=0$$

$$x(x-7)+10(x-7)=0$$

$$(x-7)(x+10)=0$$

$$\boxed{x=7} \text{ or } \boxed{x=-10}$$

But, age is never negative.

Thus, the present age of Shikha is 7 years.

4) The product of Ramu's age (in years) five years ago & his age (in years) nine years later is 15. Find Ramu's present age.

→ Let us consider the Ramu's present age is 'x' years.

Then, 9 years later his age = $(x+9)$ years

five years ago his age = $(x-5)$ years

$$\Rightarrow (x-5)(x+9)=15$$

$$x^2+9x-5x-45=15$$

$$x^2+4x-60=0$$

$$x^2-6x+10x-60=0$$

$$x(x-6)+10(x-6)=0$$

$$(x+10)(x-6)=0$$

$$\boxed{x=-10} \text{ or } \boxed{x=6}$$

But age is never negative.

Thus, the present age of Ramu is 6 years.

Exercise 8.10

1) The hypotenuse of a right triangle is 25 cm. The difference betn the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

→

Let us consider, the length of one side of triangle is 'x cm'.
Then, other side will be $(x+5)$ cm in length.

Given, hypotenuse = 25 cm

Then, By Pythagoras Theorem,

$$x^2 + (x+5)^2 = 25^2$$

$$x^2 + x^2 + 10x + 25 = 625$$

$$2x^2 + 10x + 25 - 625 = 0$$

$$2x^2 + 10x - 600 = 0$$

$$x^2 + 5x - 300 = 0$$

$$x^2 - 15x + 20x - 300 = 0$$

$$\cancel{x^2}\ x(x-15) + 20(x-15) = 0$$

$$(x-15)(x+20) = 0$$

$$\boxed{x = 15} \text{ or } \boxed{x = -20}$$

Length can never be negative.

Thus, $x = 15 \Rightarrow x+5 = 15+5 = 20$

Hence, length of one side of right triangle is 15 cm and the length of the other side is 20 cm.

2) The diagonal of a rectangular field is 60 m more than the shorter side. If the longer side is 30 m more than the shorter side, find the ~~le~~ sides of field.

→ Let us consider, the rectangle is having smaller side in length 'x m'

Then, the larger side of rectangle is having length $(x+30)$ m and diagonal is $(x+60)$ m.

Then, By Pythagoras Theorem,

$$x^2+(x+30)^2=(x+60)^2$$

$$x^2+x^2+60x+900=x^2+120x+3600$$

$$2x^2+60x+900-x^2-120x-3600=0$$

$$x^2-60x-2700=0$$

$$x^2-90x+30x-2700=0$$

$$x(x-90)+30(x-90)=0$$

$$(x-90)(x+30)=0$$

$$\boxed{x=90} \text{ or } \boxed{x=-30}$$

But, length can never be negative.

$$\Rightarrow x+30=90+30=120$$

Thus, the smaller side of rectangle is having length 90m and the larger side is in length of 120m.

Exercise 8.11

1) The perimeter of the rectangular field is 82m and its area is $400 m^2$. Find the breadth of the rectangle.

→

Given that,

perimeter = 82m

Area = $400 m^2$

Let us, consider ~~length~~ breadth is x m.

we know that,

Perimeter of rectangle = 2 (length + breadth)

$$82 = 2(\text{length} + x)$$

$$41 = (\text{length} + x)$$

$$\Rightarrow \text{Length} = (41 - x)\ m$$

And also, Area = length × breadth

$$400 = (41 - x)(x)$$

$$400 = 41x - x^2$$

$$x^2 - 41x + 400 = 0$$

$$x^2 - 25x - 16x + 400 = 0$$

$$x(x-25) - 16(x-25) = 0$$

$$(x-16)(x-25) = 0$$

$$x - 16 = 0 \quad \text{or} \quad x - 25 = 0$$

$$\boxed{x = 16} \quad \text{or} \quad \boxed{x = 25}$$

Thus, breadth of rectangular field may be 16m or 25m respectively.

2) The length of the hall is 5m more than its breadth. If the area of the floor of the hall is $84 m^2$, what are the length & breadth of the hall?

→ Let us consider, the breadth of the rectangle is 'x' m.

Then, length of the hall should be $= (x+5)$ m

Area of the hall $= 84 m^2$

Area of the rectangular hall = length × breadth

$$84 = x(x+5)$$

$$x^2 + 5x - 84 = 0$$

$$x^2 + 12x - 7x - 84 = 0$$

$$x(x+12) - 7(x+12) = 0$$

$$(x+12)(x-7) = 0$$

$$\boxed{x = -12} \text{ or } \boxed{x = 7}$$

Thus, $\boxed{x = 7}$ is the breadth of rectangular hall.

$\Rightarrow x + 5 = 12$ m

Thus, length is 7m and breadth is 12m in length of the rectangular hall.

3) Two squares have sides x and $(x+4)$ cm. The sum of their area is $656 cm^2$. Find the sides of the square.

→ Let us consider, S_1 & S_2 are the two squares.

Let x cm is the side of square S_1.

Then $(x+4)$ cm is the side of square S_2.

Thus, Area of square $S_1 = x^2 cm^2$

Area of square $S_2 = (x+4)^2 cm^2$

from given condition,

Area of S_1 + Area of S_2 = 656 cm^2

$x^2 + (x+4)^2 = 656$

$x^2 + x^2 + 16 + 8x - 656 = 0$

$2x^2 + 16 + 8x - 656 = 0$

$x^2 + 4x - 320 = 0$

$x^2 + 20x - 16x - 320 = 0$

$x(x+20) - 16(x+20) = 0$

$(x+20)(x-16) = 0$

$x+20 = 0$ or $x-16 = 0$

$\boxed{x=-20}$ or $\boxed{x=16}$

But, the x can never be negative.

Thus $x = 16$ cm is the side of square S_1.

Then $x+4 = 16+4 = 20$ cm is the side of square S_2.

4) The area of a right-angled triangle is 165 cm^2. Determine its base & altitude if the latter exceeds the former by 7m.

→ Let us consider, the altitude of right angled triangle is x m.

~~Altitude = $(x-7)$ m~~

Base = $(x-7)$ m

we have,

Area of triangle = ½ (base) (height)

$165 = \frac{1}{2}(x-7)x$

$x(x-7) = 330$

$x^2 - 7x - 330 = 0$

$$x^2-22x+15x-330=0$$
$$x(x-22)+15(x-22)=0$$
$$(x-22)(x+15)=0$$
$$x-22=0 \quad \text{or} \quad x+15=0$$
$$\boxed{x=22} \quad \text{or} \quad \boxed{x=-15}$$

Thus, $x-7=22-7=15$

Hence, the base & altitude of right angled triangle are 15cm & 22cm respectively.

Exercise 8.12

1) A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

→ Let us consider that, B takes 'x' days to complete a work.

In one day B's work is $= 1/x$

Then, A takes $(x-10)$ days to complete same work.

A's one day work $= \frac{1}{(x-10)}$

And (A's & B's together) one day work $= 1/12$

from given condition,

$$\frac{1}{x}+\frac{1}{(x-10)}=\frac{1}{12}$$

$$\frac{x-10+x}{x(x-10)}=\frac{1}{12}$$

$$12(2x-10)=x(x-10)$$
$$24x-120=x^2-10x$$
$$x^2-10x-24x+120=0$$
$$x^2-34x+120=0$$

$$x^2 - 30x - 4x + 120 = 0$$
$$x(x-30) - 4(x-30) = 0$$
$$(x-4)(x-30) = 0$$
$$x-4=0 \text{ or } x-30=0$$
$$\boxed{x=4} \text{ or } \boxed{x=30}$$

But, the value of 'x' should not be less than 10.

Thus, $x = 30$

Hence, the B can finish his work in 30 days.

2) If two pipes function simultaneously, a reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hrs faster than the other. How many hours will the second pipe take to fill the reservoir.

→ Let us consider that,

Through faster pipe the reservoir tak 'x' hours to fill completely.

Thus, in 1 hour the portion of reservoir filled is $\} = 1/x$

And the slower pipe takes $(x+10)$ hrs to fill the reservoir completely.

Then, the portion of reservoir filled by slower pipe in 1 hour $\} = \frac{1}{(x+10)}$

And the portion of the reservoir filled by both pipes together in 1 hour $\} = \frac{1}{12}$

From given condition,

$$\frac{1}{x} + \frac{1}{(x+10)} = \frac{1}{12}$$

$$12(2x+10) = x(x+10)$$

$$x^2 - 14x - 120 = 0$$

$$x^2 - 20x + 6x - 120 = 0$$

$$x(x-20) + 6(x-20) = 0$$

$$(x-20)(x+6) = 0$$

$$x - 20 = 0 \quad \text{or} \quad x + 6 = 0$$

$$\boxed{x = 20} \quad \text{or} \quad \boxed{x = -6}$$

But, time cannot be negative.

Thus, $x = 20$ hrs and $x + 10 = 10 + 20 = 30$ hrs.

Thus, time required to fill the reservoir by slower pipe is 30 hours.

Exercise 8.13

1) A piece of cloth costs rs. 35. If the piece were 4m longer & each meter costs rs. 1 less, the cost would remain unchanged. How long is the piece?

→ Let us consider that, the length of cloth is 'a'm.
Given, piece of cloth costs Rs. 35 & if the piece were 4m longer & each meter costs Rs. 1 less, the cost remains unchanged.

⇒ cost of 1m of cloth $= 35/a$

$$(a+4) \times \left(\frac{35}{a} - 1\right) = 35$$

$$(a+4)(35-a) = 35a$$

$$35a + 140 - a^2 - 4a = 35a$$

$$a^2 + 4a - 140 = 0$$

$$a^2 + 14a - 10a - 140 = 0$$

$$a(a+14) - 10(a+14) = 0$$

$$(a-10)(a+14) = 0$$

$$\boxed{a=10} \text{ or } \boxed{a=-14}$$

But, length is never negative.
Thus, $a = 10$
Hence, the piece of cloth was 10 m in length.

2) Some students planned a picnic. The budget for food was Rs. 480. But eight of these failed to go and thus the cost of food for each member increased by Rs. 10. How many students attended the picnic?

→

Let us consider, 'x' be the numbers of students planned for picnic.

Budget for the food was Rs. 480.

Then, cost of food for each member $= \frac{480}{x}$

from given condition,

$$(x-8) \times \left(\frac{480}{x} + 10\right) = 480$$

$$(x-8)(480 + 10x) = 480x$$

$$480x + 10x^2 - 3840 - 80x = 480x$$

$$x^2 - 8x - 384 = 0$$

$$x^2 - 24x + 16x - 384 = 0$$

$$x(x-24) + 16(x-24) = 0$$

$$(x+16)(x-24) = 0$$

$$\boxed{x = -16} \text{ or } \boxed{x = 24}$$

$x = -16$ cannot be considered.

Thus, $x = 24$ are the students planned for picnic.

And the number of students who attended the picnic $= (24-8) = 16$.

3) A dealer sells an article for Rs. 24 and gains as much percent as the cost price of the article. Find the cost price of the article.

→ Let us consider, the cost price is Rs. x.

The gain in percent $= x\%$.

$$\boxed{\text{Gain}\% = \frac{SP - CP}{CP} \times 100}$$

$$x = \frac{24 - x}{x} \times 100$$

$$x^2 = (24-x) \times 100$$

$$x^2 + 100x - 2400 = 0$$

$$x^2 + 120x - 20x - 2400 = 0$$

$$(x+120)(x-20) = 0$$

$$\boxed{x=20} \quad \text{or} \quad \boxed{x=-120}$$

Thus, the cost price of the article is Rs. 20.

4) Out of a group of swans, 7/2 times the square root of the total number are playing on the share of a pond. The two remaining ones are swimming in water. Find the total number of swans.

→ Let us consider, the number of swans in a pond be 'a'.

from given condition,

$$\frac{7}{2}\sqrt{a} + 2 = a$$

$$7\sqrt{a} = 2a - 4$$

$$49a = 4a^2 + 16 - 16a$$

$$4a^2 - 65a + 16 = 0$$

$$4a^2 - 64a - a + 16 = 0$$

$$4a(a-16) - (a-16) = 0$$

$$(4a-1)(a-16) = 0$$

$$\boxed{a = 1/4} \quad \text{or} \quad \boxed{a=16}$$

The no. of swans only in the form of natural number.

Hence, the total no. of swans are 16.