Chapter 21: Surface Area & Volume of a Sphere

Exercise 21.1

1.) Find the surface area of a sphere of radius:
i) 10.5 cm ii) 5.6 cm iii) 14 cm

→ i) Given that,
The radius of sphere (r) = 10.5 cm
The surface area of sphere = $4\pi r^2$
$= 4 \times \frac{22}{7} (10.5)^2$
$= 1386 \text{ cm}^2$
Thus, the surface area of sphere is found to be 1386 cm^2.

ii) Given that,
The radius of sphere (r) = 5.6 cm
The surface area of sphere = $4\pi r^2$
$= 4 \times \frac{22}{7} (5.6)^2$
$= 394.24 \text{ cm}^2$
Thus, the surface area of sphere is found to be 394.24 cm^2.

iii) Given that,
The radius of sphere (r) = 14 cm
The surface area of sphere = $4\pi r^2$
$= 4 \left(\frac{22}{7}\right) (14)^2$
$= 2464 \text{ cm}^2$
Thus, the surface area of sphere is found to be 2464 cm^2.

2.) Find the surface area of a sphere of diameter
i) 14cm ii) 21cm iii) 3.5cm

→ i) Given that,
diameter of a sphere = 14 cm
Then, Radius of a sphere = 7 cm
The surface area of sphere $= 4\pi r^2$
$= 4\left(\frac{22}{7}\right)(7)^2$
$= 616\ cm^2$
Thus, surface area of sphere is found to be $616\ cm^2$.

ii) Given that,
diameter of a sphere = 21 cm
Then, Radius of a sphere = 10.5 cm
Then, surface area of sphere $= 4\pi r^2$
$= 4\left(\frac{22}{7}\right)(10.5)^2$
$= 1386\ cm^2$
Thus, surface area of sphere is found to be $1386\ cm^2$.

iii) Given that,
diameter of a sphere = 3.5 cm
Then, Radius of a sphere = 1.75 cm
The surface area of sphere $= 4\pi r^2$
$= 4\left(\frac{22}{7}\right)(1.75)^2$
$= 38.5\ cm^2$
Thus, surface area of sphere is found to be $38.5\ cm^2$.

3.) Find the total surface area of a hemisphere of a solid hemisphere each of radius 10cm.

→ Given that,

Radius of solid hemisphere = 10cm

Then, surface area of hemisphere $= 2\pi r^2$

$= 2(3.14)(10)^2$

$= 628\ cm^2$

The total surface area of hemisphere $= 3\pi r^2$

$= 3(3.14)(10)^2$

$= 942\ cm^2$

Thus, the total surface area of solid hemisphere is found to be $942\ cm^2$.

4.) The surface area of a sphere is $5544\ cm^2$, find its diameter

→ Given that,

Surface area of a sphere $= 5544\ cm^2$

Then, if 'r' is the radius of a sphere then its surface area is given by,

S.A of sphere $= 4\pi r^2$

$5544 = 4(3.14)r^2$

$r^2 = 441$

$\boxed{r = 21\ cm}$

Thus, the radius of solid sphere is found to be 21cm.

Then, diameter of the sphere $= 2r = 42\ cm$.

5.) A hemispherical bowl made of brass has inner diameter 10.5cm. Find the cost of tin plating on it on the inside at the rate of Rs. 4 per $100\ cm^2$.

→

Given that, Inner diameter of hemispherical bowl = 10.5cm

Then, Radius of hemispherical bowl = r = 5.25cm

Surface area of hemispherical bowl = $2\pi r^2$

$= 2(3.14)(5.25)^2$

$= 173.25 cm^2$

Thus, Surface area of hemispherical bowl is found to be $173.25 cm^2$.

The cost of tin plating for $100 cm^2$ area is 4 Rs.

Then, the cost of tin plating for $173.25 cm^2 = 4 \times 173.25/100$

$= 6.93$

Thus, the cost of tin plating for $173.25 cm^2$ area required is Rs. 6.93

6) The dome of a building is in the form of a hemisphere. It's radius is 63 dm. Find the cost of painting it at the rate of Rs. 2 per square meter.

→ Given that, The radius of hemisphere = 63 dm = 6.3 m

Then, inner surface area of hemisphere = $2\pi r^2$

$2\pi r^2 = 2(3.14)(6.3)^2$

$= 249.48 cm^2$

Thus, inner surface area of hemisphere is found to be $249.48 cm^2$.

Cost of painting $1 m^2$ area is Rs. 2.

Then, total cost of painting $249.48 cm^2$ area is found to be $= 2 \times 249.48$

$=$ Rs. 498.96.

Exercise 21.2

1.) find the volume of a sphere whose radius is :
i) 2 cm ii) 3.5 cm iii) 10.5 cm

→ i) Given that, Radius of sphere $(r) = 2$ cm

The Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\left(\frac{22}{7}\right)(2)^3$

$= 33.52 \text{ cm}^3$

Thus, Volume of sphere is found to be 33.52 cm^3.

ii) Given that, Radius of sphere $(r) = 3.5$ cm

The Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\left(\frac{22}{7}\right)(3.5)^3$

$= 179.666 \text{ cm}^3$

Thus, the volume of sphere is found to be 179.666 cm^3.

iii) Given that, Radius of sphere $(r) = 10.5$ cm

The Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\left(\frac{22}{7}\right)(10.5)^3$

$= 4851 \text{ cm}^3$

Thus, the volume of sphere is found to be 4851 cm^3.

2.) Find the volume of a sphere whose diameter is
i) 14 cm ii) 3.5 dm iii) 2.1 m

→ i) Given that,
Diameter of a sphere = 14 cm
Then, Radius of a sphere = 7 cm
The Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}(3.14)(7)^3 = 1437.33 \text{ cm}^3$

ii) Given that,

Diameter of a sphere $= 3.5\,dm$

Then, Radius of a sphere $= 1.75\,dm$

Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}(3.14)(1.75)^3$

$= 22.46\,dm^3$

Thus, the volume of sphere is found to be $22.46\,dm^3$.

iii) Given that,

Diameter of a sphere $= 2.1\,m$

Then, Radius of a sphere $= 1.05\,m$

Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\left(\frac{22}{7}\right)(1.05)^3$

$= 4.851\,m^3$

Thus, the volume of sphere is found to be $4.851\,m^3$.

3.) A hemispherical tank has the inner radius of 2.8m. find its capacity in liters.

→ Given that,

Inner Radius of hemispherical tank $(r) = 2.8\,m$

Then, Capacity / Volume of hemispherical tank $\Big\} = \frac{2}{3}\pi r^3$

$= \frac{2}{3}\left(\frac{22}{7}\right)(2.8)^3$

$= 45.997\,m^3$

Thus, the capacity of hemispherical tank is found to be $45.997\,m^3$ or 45997 litres.

4.) A hemispherical bowl is made of steel 0.25 cm thick, The inside radius of the bowl is 5 cm. Find its volume of steel used in making the bowl.

→ Given that,

Inner Radius of hemispherical bowl $(r) = 5$ cm

Outer Radius of hemispherical bowl $= 5 + 0.25 = 5.25$ cm

Then, Volume of steel used in making the bowl $\}$ = Outer Volume − Inner Volume

$$= \frac{2}{3}\pi\left[(5.25)^3 - (5)^3\right]$$

$$= \frac{2}{3}\left(\frac{22}{7}\right)\left[(5.25)^3 - 125\right]$$

$$= 41.282 \text{ cm}^3$$

Thus, the volume of steel used in making the bowl is found to be 41.282 cm^3.

5.) How many bullets can be made out of a cube of lead, whose edge measures 22 cm, each bullet being 2 cm in diameter?

→ Hese, given that

Diameter of a bullet = 2cm

Then, Radius of a bullet $(r) = 1$ cm

Edge of the cube = 22cm

We know that, Volume of a cube $= (\text{side})^3 = (22)^3 = 10648 \text{ cm}^3$

Now, Volume of bullet which is spherical in shape $\}$ $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\left(\frac{22}{7}\right)(1)^3$$

$$= 88/21 \text{ cm}^3$$

Then, Number of bullets $= \frac{\text{Volume of cube}}{\text{Volume of bullet}} = \frac{10648}{88/21}$

$$= 2541$$

Thus, total bullets which are 2541 in number can be made from cube of lead.

6.) A shopkeeper has one laddoo of radius 5cm. With the same material, how many laddoos of radius 2.5cm can be made?

→ Here, given that

Radius of laddoo = 5cm

Then Volume of that laddoo $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\left(\frac{22}{7}\right)(5)^3$

$= 11000/21 \text{ cm}^3$

for second laddoo, Radius = 2.5cm

Then, Volume of Second laddoo $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\left(\frac{22}{7}\right)(2.5)^3$

$= 1375/21 \text{ cm}^3$

Thus, total no. of laddoos made $= \frac{11000/21}{1375/21} = 8$

Hence, 8 laddoos will be made from given laddoo.

8.) A sphere of radius 5cm is immersed in water filled in a cylinder, the level of water rises 5/3 cm. find the radius of the cylinder.

→ Given that,

Radius of sphere (r_1) = 5cm

Let 'r' is the radius of a cylinder.

Then, Volume of a sphere $= \frac{4}{3}\pi r_1^3$

$= \frac{4}{3}\left(\frac{22}{7}\right)(5)^3$

Here, water leves rises to 5/3 cm.

Then, Volume of water raised in Cylinder $= \pi r^2 h$

Now, Volume of water raised in cylinder after immersing sphere = Volume of sphere

$\Rightarrow \pi r^2 h = \frac{4}{3}\pi r^3$

$rh = \frac{4}{3}r$ $\qquad$ ~~$r = \frac{3}{4}h$~~

~~$r = \frac{4}{3h}$~~ $\qquad$ ~~$r = \frac{3}{4} \times \frac{3}{5}$~~

~~$r = \frac{4}{3} \times \frac{3}{5} = 4$~~

$\pi r^2 (5/3) = \frac{4}{3}(\pi)(5)^3$

$r^2\left(\frac{5}{3}\right) = \frac{4}{3}(125)$

$r^2 = 100$

$\boxed{r = 10\text{cm}}$

Thus, the radius of cylinder is found to be 10cm.

9.) If the radius of a sphere is doubled, what is the ratio of the volume of the first sphere to that of the second sphere?

→ Let us consider 'r' is the radius of first sphere then 2r is the radius of second sphere.

Now, $\frac{\text{Volume of first sphere}}{\text{Volume of second sphere}} = \frac{4/3\pi r^3}{4/3\pi (2r)^3}$

$\frac{V_1}{V_2} = \frac{1}{8}$

Thus, the ratio of volumes of spheres is found to be

$V_1 : V_2 = 1 : 8$

10.) A cone and a hemisphere have equal bases & equal volumes. Find the ratio of their heights.

→ Given that,

Volume of a cone = Volume of a hemisphere

Also, given that the base of a cone & hemisphere is same.
Let us consider 'r' is the radius of Base.

Then, Volume of a cone = Volume of a hemisphere

$$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$

$$r^2 h = 2r^3$$

$$\boxed{h = 2r}$$

$$\frac{h}{r} = \frac{2}{1}$$

Thus, the ratio of their heights is found to be 2:1
(Since, for hemisphere radius is the height only.)

11.) A vessel in the form of a hemispherical bowl is full of water. It contents are emptied in a right circular cylinder. The internal radii of the bowl & the cylinder are 3.5cm and 7cm respectively. Find the height to which water will rise in the cylinder.

→ Given that,
Inner radius of hemispherical bowl = 3.5cm (r_1)
Inner radius of cylinder = 7cm (r_2)

Then, from given condition

{Volume of water in the hemispherical bowl} = {Volume of water in the cylinder}

$$\frac{2}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\frac{2}{3} r_1^3 = r_2^2 h$$

$$h = \frac{2}{3}\frac{r_1^3}{r_2^2} = \frac{2}{3}\frac{(3.5)^3}{(7)^2}$$

$$\boxed{h = 7/12 \text{ cm}}$$

Thus, the height of the water which rises in the cylinder is found to be 7/12 cm.

12.) A cylinder whose height is two thirds of its diameter has the same volume as a sphere of radius 4cm. Calculate the radius of the base of the cylinder.

→ Here, Given that

Radius of sphere (r_1) = 4cm

Volume of Sphere (V_1) = Volume of Cylinder (V_2)

Again, Height of cylinder (h) = $\frac{2}{3}$ (Diameter of sphere)

$$h = \frac{2}{3}(2r_1) = \frac{4r_1}{3}$$

Then, $V_1 = V_2$

$$\frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

∵ Let 'r_2' is the radius of base of a cylinder.

$$\frac{4}{3}r_1^3 = r_2^2 h$$

$$\frac{4}{3}(4)^3 = r_2^2\left(\frac{4}{3}r_1\right)$$

$$4^2 = r_2^2$$

$$\boxed{r_2 = 4\text{cm}}$$

Thus, the radius of base of a cylinder is found to be 4cm.

13.) A vessel in the form of a hemispherical bowl is full of water. The contents are emptied into a cylinder. The internal radii of the bowl and cylinder are respectively 6cm and 4cm. Find the height of water in the cylinder.

→ Here, Given that

Inner radius of hemispherical bowl (r_1) = 6cm

Inner radius of cylinder (r_2) = 4cm

Let 'h' is the height of a cylinder.

From given condition,

Volume of hemispherical bowl = Volume of Cylinder

$$\frac{2}{3}\pi r_1^3 = \pi r_2^2 h$$

$\frac{2}{3}\pi(6)^3 = \pi(9)^2 h$

$\boxed{h = 9\text{cm}}$

Thus, the height of water in the cylinder is found to be 9cm.

14) A cylindrical tube of radius 16cm contains water to a depth of 30cm. A spherical iron ball is dropped into the tube & thus level of water is raised by 9cm. What is the radius of the ball?

→ Let us consider, 'r' is the radius of iron ball.

'R' be the radius of cylinder.

Given that, Radius of cylinder (R) = 16 cm.

Height of cylinder (h) = 9cm

from given condition, we can write

Volume of iron ball = Volume of water raised in the tub

$\frac{4}{3}\pi r^3 = \pi R^2 h$

$\frac{4}{3} r^3 = (16)^2 (9)$

$r^3 = 1728$

$\boxed{r = 12\text{cm}}$

Thus, the radius of iron ball is found to be 12cm.

Exercise VSAQs

1.) Find the surface area of a sphere of radius 14cm.

→ Given that, Radius of sphere (r) = 14cm

Then, Surface area of a sphere $= 4\pi r^2$

$= 4\left(\frac{22}{7}\right)(14)^2$

$= 2464 cm^2$

Thus, surface area of a sphere is found to be $2464 cm^2$

2.) Find the total surface area of a hemisphere of radius 10cm.

→ Given that, Radius of hemisphere (r) = 10cm

Then, Total surface area of a hemisphere $= 3\pi r^2$

$= 3\left(\frac{22}{7}\right)(10)^2$

$= 942 cm^2$

Thus, total surface area of a hemisphere is found to be $942 cm^2$.

3.) Find the radius of a sphere whose surface area is $154 cm^2$.

→ Here given that,

Surface area of a sphere $= 154 cm^2$.

Let 'r' be the radius of a sphere.

Then, $4\pi r^2 = 154$

$r^2 = \frac{154}{4\pi} = \frac{154}{4 \times (22/7)}$

$r^2 = 49/4$

$r = 7/2 = 3.5 cm$

Thus, radius of a sphere is found to be 3.5cm.

4) The hollow sphere, in which the circus motor cyclist performs his stunts, has a diameter of 7m. Find the area available to the motor cyclist for riding.

→ Here, Given that

Diameter of hollow sphere = 7m

Then, Radius of hollow sphere = 3.5m

Thus, we can say

Area available to the motor cyclist for riding } = { Surface area of the sphere

$= 4\pi r^2$

$= 4\left(\frac{22}{7}\right)(3.5)^2$

$= 154 m^2$

Thus, the area available to the motorcyclist for riding is found to be $154 cm^2$.

5) Find the volume of a sphere whose surface area is $154 cm^2$.

→ Given that, Surface area of a sphere = $154 cm^2$

Let 'r' be the radius of a sphere.

Then, Surface area of sphere = $4\pi r^2$

$154 = 4\left(\frac{22}{7}\right) r^2$

$r^2 = 49/4 \Rightarrow r = 7/2 = 3.5 cm$

Then, Volume of a sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\left(\frac{22}{7}\right)(3.5)^3$

$= 179.66 cm^3$

Thus, the volume of a sphere is found to be $179.66 cm^3$.