3. Motion in a straight line

We are well familiar with basics of motion in std. IX. Running car, military parade, moving blades of fan, rotation of planets, blood flow in body are all different examples of motion.

'The change in position of object with respect to surrounding is known as motion.'

Basically there are three types of motion.
1) Rectilinear motion. 2) Rotational motion 3) Oscillatory motion.

In this lesson we are going to confine our studies upto rectilinear motion i.e 'the motion of body along a straight line'

* The branch of physics which deals with study of objects in motion is called Kinematics.

Following are the most essential parameters which helps us to understand details about motion.

1. Position :

A point where the object is located at given instant is termed as position of object. For explanation of position, a rectangular coordinate system with three mutually perpendicular axes (X, Y & Z) is taken. The point of intersection of these axes is termed as Origin or reference point. So if any object is located at point P in X, Y, Z plane then the position of that object can be find using coordinates of point P as P(x, y, z)

* if motion is restricted in one direction, only one axis is required (ie. X-axis)
* If motion is restricted in two directions, two $\perp^{ar}$ axes are required (ie. X & Y axes)
* for motion of object in three dimensions, three axes are required.

2. **Path length:**

The actual distance travelled by object during motion is called as path length.

Suppose a person walking on road and starts from point A, then to point B and finally to point C, as follows. (distances are in metre)

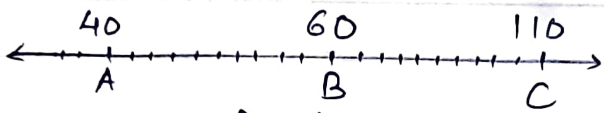


fig. 1)

Then the path length is

$AB + BC = 20 + 50$

$= 70$ m.

* It is scalar quantity, so it has magnitude only.
* It can not negative or zero.

3. **Displacement:**

The shortest distance between two positions of object is called as displacement.

It is vector quantity so it has magnitude and direction both.

Consider same case in above example where person starts from point A, move to B, C and returns to point B again. (fig. 1) the displacement is, $A = x_1 (40\text{ m})$, $B (= x_2 (60\text{ m})$

$\therefore \Delta x = \overline{x_2} - \overline{x_1} = 60 - 40 = 20$ m.

* Displacement can be negative, positive or zero.
* It is vector quantity
* Magnitude of displacement will be equal with path if the initial and final points in both cases are same and along same line.
* For another conditions, magnitude of displacement may or may not be equal to path length.
* Displacement is taken as zero if initial and final position of object is same. In this case path length will not be zero.

Eg in fig i) If object comes to point A again, then $\Delta x = 0$, and path length $= 70 + 70 = 140$ m.

* **Uniform motion :**
The object of object along straight line covers equal distances in equal intervals of time is known as uniform motion.
Eg. motion of planets around sun, military parade.

* **Non-uniform motion :**
The motion of object along straight line but do not covers equal distances in equal time intervals is known as non-uniform motion.

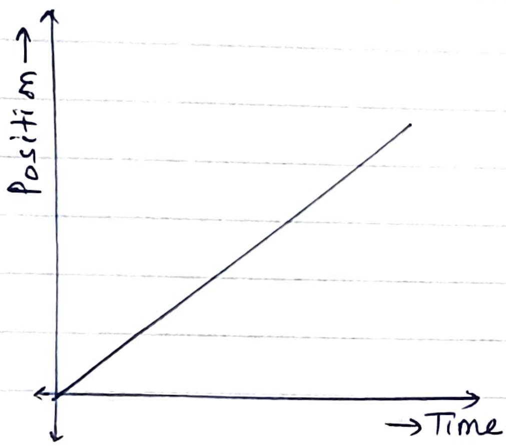


fig. Graph of uniform motion.

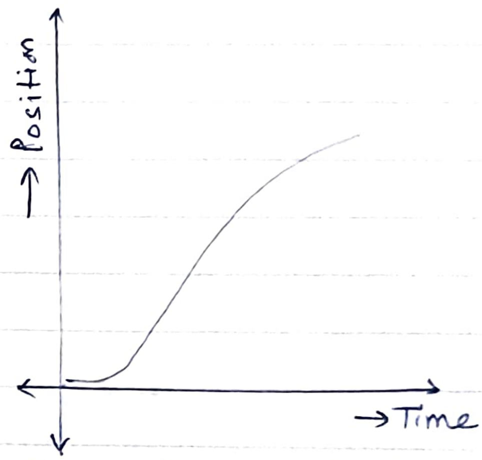


fig. Graph of non-uniform motion

* **Average velocity and average speed :-**

1. **Average velocity :**
Average rate of change of displacement OR change in position per unit time interval is called as average velocity ($\bar{v}$)

$$\therefore \text{average velocity} = \frac{\text{displacement}}{\text{time interval}}$$

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

* It is vector quantity, hence it has magnitude and direction.
* It can be zero, positive or negative

2. Average speed :

The total path length travelled by object per unit total time taken is called as average speed.

$\therefore$ Average speed $= \dfrac{\text{Total path length}}{\text{Total time}}$

* It is scalar quantity, hence has magnitude only.
* It can not be zero, negative.
* If motion of object is along straight line and in same direction, magnitude of displacement is equal to total path length.

SI unit of average velocity and average speed is same i.e.

metre per second (m/s or $m \cdot s^{-1}$)

* Instanteneous velocity and speed :

Speedometer of every vehicle gives us the information about the speeds of vehicle moving in different instants of time. This speed or velocity at the perticular instant of time is known as instanteneous velocity.

Instanteneous velocity is defined as the limit of average velocity in infinitesimal small interval of time. (Δt close to zero).

$$\therefore \quad v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$= \frac{dx}{dt}$$

Instanteneous speed is nothing but the magnitude of instanteneous velocity. Hence the magnitude of instanteneous velocity at the given instant is same as that of instanteneous speed at that instant.

* Acceleration :

We have oftenly seen that the velocity of moving car varies as per the conditions of traffic, road situation etc. The velocity can be changed (increased or decreased) with the help of accelerator in vehicles.

1. Average acceleration :

The rate of change of average velocity or change in velocity per unit time interval is called as average acceleration.

If V_1 = initial velocity of object at time t_1

V_2 = Final velocity of object at time t_2

∴ average acceleration = $\dfrac{\text{Change in velocity}}{\text{Time interval}}$

$$\bar{a} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta a}{\Delta t}$$

SI unit of acceleration is m/s^2 or $m \cdot s^{-2}$

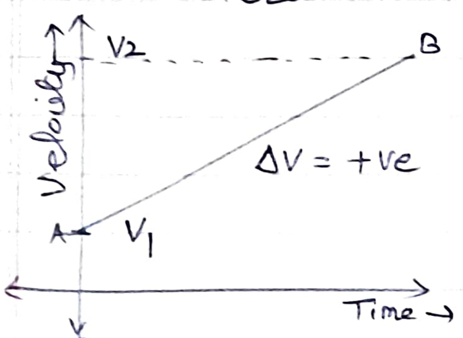
2. Instanteneous acceleration :

The limit of average acceleration is infinitesimal small interval of time (Δt) is called as instanteneous acceleration.

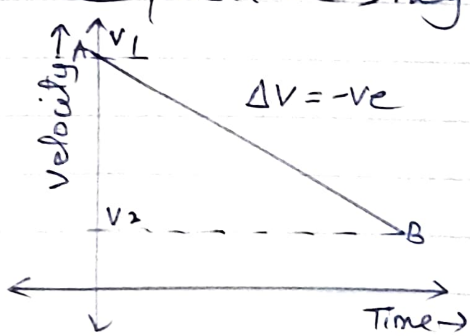
$$\therefore \text{Instanteneous acceleration} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$= \frac{dv}{dt}.$$

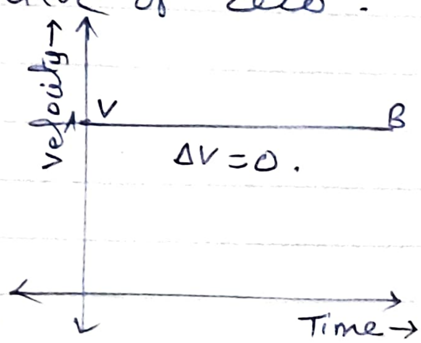
* Acceleration can be positive, negative or zero.



i. Positive acceleration



ii. Negative acceleration

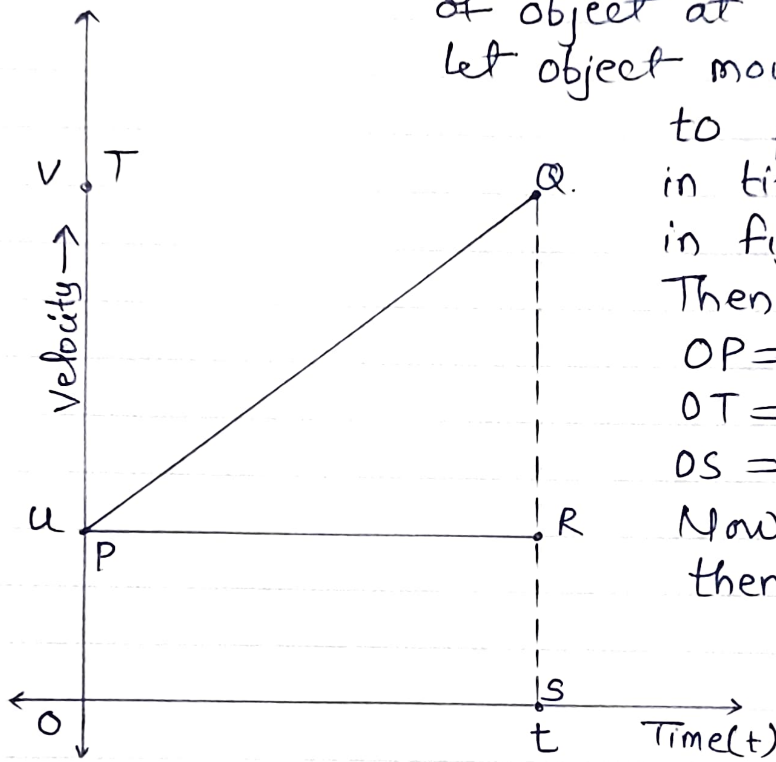


iii. zero acceleration

* Kinematical equations for uniformly accelerated motion :

1. By Graphical method :

Consider an object is performing uniformly accelerated motion. Let 'u' be the initial velocity of object at point (P) in time(0). Let object moves from point P to point Q in time 'Q' in time t, as shown in fig.



Then from graph,

OP = u = initial velocity

OT = v = final velocity

OS = t = time taken.

Now draw QS ⊥ OS, then OP = RS and PT = QR.

From graph, $PT = OT - OP$

$= v - u$. (i.e. Change in speed)

By defination,

$$\text{acceleration} = \frac{PT}{OS} = \frac{v-u}{t}$$

$$\therefore a = \frac{v-u}{t}$$

$$\therefore at = v - u \quad \text{--- ①}$$

or $\boxed{v = u + at}$

This is 1st equation of motion.

ΔPRQ of graph represents the 1st equation of motion.

To find 2nd equation of motion, consider the distance travelled = area of graph between O & S

= area of triangle PRQ + area of rectangle OPRS

$\therefore S = \frac{1}{2}(PR \times QR) + (OP \times PR)$

$= \frac{1}{2}[t \times (v-u)] + (u \times t)$

$= \frac{1}{2}(t \times at) + ut$ (from eqn 1)

$\therefore \boxed{S = ut + \frac{1}{2}at^2}$

This is 2nd equation of motion.

Now consider □OPQS as trapezium

$\therefore$ distance travelled = area of trapezium OPQS

$= \frac{1}{2}(OP + QS) \times OS$

$= \frac{1}{2}(u+v) \times t$

From 1st equation, $v - u = at$

$\therefore t = \frac{v-u}{a}$

$\therefore S = \frac{1}{2}(u+v) \times \left(\frac{v-u}{a}\right)$

$\therefore 2aS = (v+u)(v-u)$

$\therefore 2aS = v^2 - u^2$

$\therefore \boxed{v^2 = u^2 + 2aS}$

This is the 3rd equation of motion.

* If S & S_0 are positions at $t=0$. then eqns will be,

$v = u + at$

$S - S_0 = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2a(S - S_0)$

classmate

2. Calculus method :

Consider the motion of object along straight line.

Let u = initial velocity (at t = 0)
v = final velocity (at t = t)
a = acceleration
x = displacement.

Then using calculus we can write

$$a = \frac{dv}{dt} \quad \text{and} \quad v = \frac{dx}{dt}$$

Now, consider motion of object with constant accelⁿ,

$$a = \frac{dv}{dt}$$

$\therefore a \cdot dt = dv$ (seperating variables)

Now integrating both sides from t = 0 to t = 1 & u to v,

$$\int_0^t a \cdot dt = \int_u^v dv$$

$$a \int_0^t dt = \int_u^v dv$$

$$\therefore a[t]_0^t = [v]_u^v$$

$$\therefore a[t-0] = [v-u]$$

$$\therefore at = v - u$$

$$\therefore \boxed{v = u + at}$$

Also, $v = \frac{dx}{dt} \Rightarrow dx = v\,dt$

On integrating, we get

$$\int_0^x dx = \int_0^t v\,dt$$

$$[x]_0^x = \int_0^t (u+at)\,dt \quad \text{(from 1st eqⁿ)}$$

$$x = \int_0^t u\,dt + \int_0^t at \cdot dt$$

$$\therefore x = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

$$= u[t-0] + a\left[\frac{t^2}{2} - 0\right]$$

$$\therefore \boxed{x = ut + \frac{1}{2}at^2}$$

Now using Chain Rule,

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$a = v \cdot \frac{dv}{dx}$$

$$\therefore a \cdot dx = v \cdot dv.$$

On integrating both sides

$$\int_0^x a \cdot dx = \int_u^v v\,dv$$

$$\therefore ax = \left[\frac{v^2}{2}\right]_u^v$$

$$\therefore ax = \left[\frac{v^2}{2} - \frac{u^2}{2}\right]$$

$$\therefore ax = \frac{v^2 u^2}{2}$$

$$\therefore v^2 - u^2 = 2ax$$

$$\therefore \boxed{v^2 = u^2 + 2ax}$$

Calculus method involving differentiation and integration is very helpful in solving problems of mechanical physics.

Relative velocity :

Frame of reference plays important role in understanding mechanics and kinematics. A car moving with high speed that appears to be in motion for person on road (outside car) but two persons in car feels that they are at rest with respect to each other.

Consider two cars A and B are travelling with speeds V_1 & V_2 and coveres distances x_1 & x_2 at the instant (t), then their distances can be given as

$$x_1 = V_1 t \quad \text{--- (I)}, \qquad x_2 = V_2 t \quad \text{--- (II)}$$

Then the displacement of car A w.r.t. B is

$$x_{BA} = x_2 - x_1$$

$$= (V_2 t - V_1 t)$$

$$\therefore x_{BA} = (V_2 - V_1) t$$

But velocity is displacement/time, then Velocity of car B w.r.t. A is,

$$V_{BA} = \frac{x_{BA}}{t}$$

$$= \frac{(V_2 - V_1) t}{t} = V_2 - V_1$$

If we want to find, velocity of object/car A w.r.t. B, then,

$$V_{AB} = V_1 - V_2$$

ie if $V_1 = 40$ m/s, $V_2 = 60$ m/s

then, $V_{AB} = (40 - 60) = -20$ m/s &

$V_{BA} = (60 - 40) = +20$ m/s

This mean for car B, speed of car A is slower by 20 m/s.