2. Units & Measurement

Physics deals with calculative analysis and studies, for this purpose some standards are required. termed as units.
Physical quantities are of two types :-

1) Fundamental quantities :-
The quantities which do not depends upon any other quantity for its measurement is called as Fundamental quantities.
Eg. mass, length, time, temperature etc.

2) Derived quantities :
The quantities which depends upon two or more than two fundamental quantities are derived quantities.
Eg. speed, density, work, pressure etc.

Units : The reference standard required for the measurement are called as units.

- Units of fundamental quantities are known as Fundamental units.
  eg. kilogram, meter, second etc.
- Units of derived quantities are known as derived units.
  Eg. meter/second, newton.meter etc.

Unit should possess following properties :-

- It should be reproducible. It should be invariable.
- It should be universally accepted

Systems of unit

1) CGS system – Centrimetre, Gram, Second.
2) FPS system – Foot, Pound, Second.
3) MKS system – Metre, Kilogram, Second.

At present 'Systeme Internationaled' system is accepted internationally for measurement from 1971. It is termed as SI units.
Following are some important fundamental / base quantities and their units with the defination.

1) Length : Unit is meter. It's symbol is 'm'.
One metre = path travelled by light in vaccum in 1/299792458 sec

2) Mass : Unit is kilogram. Symbol is 'kg'
One kilogram = mass of prototype of platinum-iridium alloy cylinder.

3) Time : Unit is second. Symbol is 's'
One second = time for 9,192,631,770 vibrations of ground state of cesium-133 atom

4) Current : Unit is ampere. symbol is 'A'
One ampere = One coulomb charge flow per second through conductor.

5) Temperature : Unit is kelvin. symbol is 'K'
One kelvin = $(1/273)^{th}$ fraction of temperature of triple point of water.

6) Amount of substance : Unit is mole. Symbol is 'mol'
One mole = number of atoms present in 0.012 kg of carbon-12

7) Luminous intensity : Unit is candela. Symbol 'cd'
One candela = if monochromatic soure emits radiation of frequency 54×10^{13} Hz.

Supplimentary units for measuring angle :-

1) Radian :
Unit of plane angle. Symbol. 'rad' 1 rad = 57.3^0

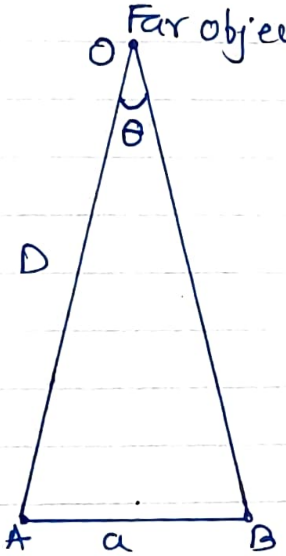
2) steradian :
Unit of solid angle. Symbol 'star'

Measurement of length :-

For measurement of lengths beyond the range of instruments, following methods are used.

1> Measurement of large distances :

Distances of planets, stars from earth or any other selestial object can be measured using parallax method.



Consider any object at long (infinite) distance from point A (shown in fig.) Using parallax method, observe the object from point A (like how one can observe a tree from long distance) Mark this distance of object from point A as 'D'. Now observe the object by changing position from A to B at very small distance 'a'

Let $\angle AOB = \theta$ (parallax angle)

As compared to 'D' and 'a' we can say that

$$D >> a \therefore \frac{a}{D} << 1$$

Let's imagine that distance AB is arc of large circle, then, $AO = \text{radius} = BO$.

$\therefore$ By formula for length of arc,

$$b = \text{radius} \times \text{angle}$$

$$b = D \times \theta$$

$$\therefore D = \frac{b}{\theta}$$

2) Measurement of small distances :

Small distances of order 10^{-10} m, 10^{-12} m, 10^{-15} m are measured with some special techniques which involve use of electron microscope and chemical methods.

Measurement of mass:

Mass is the most important fundamental quantity which represents amount of matter present in substance. It SI unit is kilogram (kg) and CGS unit is gram (g).

Kilogram or gram are inconvenient while measuring mass of atoms, molecules or even small things. So 'unified atomic mass unit (u)' is introduced.

where 1 unified atomic mass unit = 1 u

= $(1/12)^{th}$ mass of C-12 atom

Measurement of time:

Time is fundamental quantity, can be measured in unit 'second'. Cesium clock or atomic clock which measures time for vibrations of cesium atom in ground state.

Where, 1 s = 9,192,631,770 vibrations of cesium atom in that time.

* Following are some important ranges used for length and mass.

1) Length:

- 1 fermi = 1F = 10^{-15} m.
- 1 angstrom = 1 Å = 10^{-10} m.
- 1 light year = 1 ly = 9.46×10^{15} m
- 1 parsec = 3.08×10^{16} m

2) Mass:

- Mass of electron = 10^{-30} kg
- Mass of proton = 10^{-27} kg
- Rain drop = 10^{-6} kg
- Earth = 10^{25} kg
- Milky way galaxy = 10^{41} kg

* Accuracy, Precision and Errors in measurement:

Every measurement contains some uncertainity; due to which errors are introduced in measurement.

'The difference between measured value and theoretical value is called as error.'

- Accuracy of measurement defines the closeness of measurement to the true value of measured quantity.
- Precision in measurement gives an idea that till what limit/end the quantity can be measured.

Errors in measurement can be classified as,

1> Systematic error:

These errors are occured due to instruments, imperfections in handling, wrong methods of readings. Hence the following are sources of systematic error.

a> Instrumental error:

Errors in measurement which introduced due to faulty instruments, defective calibration are termed as instrumental error.

b> Imperfection in experimental techniques:

c> Personal errors:

Lac of attention of person handling experiment, Faulty observations recording, parallax error are some aspects which introduces errors in measurement.

2> Random Errors:

Errors which can occurs due to atmospheric conditions like temperature, humidity etc are random errors.

* Errors can be minimized by using proper instruments, careful setting and observations.

Absolute error, relative error and percentage error:

Consider that $a_1, a_2, \dots a_n$ are measurements recorded in any experiment.

a) Mean value (a_m)

Arithmetic mean of all measured values is called as mean value.

$$\therefore a_m = \frac{a_1 + a_2 + \dots + a_n}{n}$$

b) Absolute error & Mean absolute error:

Magnitude of difference between arithemetic mean and each measured value is called absolute error.

$$\Delta a_1 = |a_m - a_1|, \quad \Delta a_2 = |a_m - a_2|, \quad \Delta a_3 = |a_m - a_3|,$$

$$\vdots$$

$$\Delta a_n = |a_m - a_n|$$

Mean absolute error is arithmetic mean of all absolute error.

$$\therefore \Delta a_{mean} = \frac{\Delta a_1 + \Delta a_2 + \dots + \Delta a_n}{n}$$

c) Relative error:

Ratio of mean absolute error to mean value is called relative error or fractional error.

$$\therefore \text{Relative error} = \frac{\Delta a_{mean}}{a_{mean}}$$

d) Percentage error:

Percentage errors can be obtained by representing relative errors in percent.

$$\therefore \text{Percentage errors} = \text{Relative errors} \times 100$$

$$= \frac{\Delta a_m}{a_m} \times 100 \%$$

From above calculations one can find the total errors introduced in measurement.

Combination of errors :

1) Error of sum and difference :-

If ΔA and ΔB are errors in measurements of quantities A & B, then error in measurement of A±B is ΔZ, given as

$$\Delta Z = \Delta A + \Delta B$$

2) Error of a product and qustient

If ΔA and ΔB are measurements of errors of quantities A and B, then errors in measurement of Z=AB and Z=A/B is,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

3) Errors in measurement of higher order quantities

If $Z = A^n$ and ΔZ = ΔA are errors, then

$$\frac{\Delta Z}{Z} = n \frac{\Delta A}{A}$$

Significant figures

Results of measurement contains digits that has significance (of reliable values) as well as some digits which does not make large impact in given calculation ie. they are uncertain.

The reliable or certain digits plus first uncertain digits in measurement are called as significant digits or significant figures.

eg. consider that the diameter of spherical bob is measured as 1.97 cm. Here digits 1 & 9 are certain, whereas digit 7 is uncertain, hence measured value has 3 significant digits.

* Significant digits indicates the precision in the measurement which depends on the least count of instrument used in measurement.

Rules for determining significant figures :-

1) All the non-zero digits are significant
2) All zeros between non-zero digits are significant.
3) If number is less than 1, zeros on right of decimal point (left of first non-zero digit) are not significant. Eg. In 0.000735, underlined zero's aren't significant.
4) Values of measurement written in different system of units will have same significant as that in SI system.

   For eg. 325 cm = 3.25 m has 3 significant digit.

   145 m = 14500 cm, has 3 significant digits

   (underlined zero's aren't significant)
5) Zero's trailing a number are not significant

   Eg. 5 cm, 5.00 cm, 5.000 cm has 1 significant digit only.

Rounding off the uncertain digits :-

In measured value, the most uncertain digits are rounded off to nearest value.

For eg. i) if any calculation has value 4.532, then last digit 2 is most uncertain & less than 5, hence neglected.

ii) if any calculation has value 4.536, then last digit is rounded off as it is greater than 5, so new value is 4.54

Dimensions of physical quantities :-

Power raised for fundamental quantities so as to represent the given quantity is known as dimensional analysis.

* Representation of quantity in terms of mass, length & time ie. [M, L, T] is called <u>dimensional</u> <u>formula</u>.

Eg. 1) Consider area of rectangle as,

Area = length x breadth = length x length.

$= [M^0 L^2 T^0]$

$\therefore [M^0 L^2 T^0]$ are dimensional eqn of 'area'

2) density = $\frac{mass}{volume} = \frac{mass}{(length)^3} = \left[\frac{M}{L^3}\right]$

$\therefore$ density $= [M^1 L^{-3} T^0]$

3) speed = $\frac{distance}{Time} = \frac{length}{time} = \left[\frac{L}{T}\right]$

$\therefore$ speed $= [M^0 L^1 T^{-1}]$

Applications of dimensional analysis :-

1) To check correctness of given physical equation.
2) To find the conservation factor between units of same quantity in different system.
3) To deduce the physical equation (formula) for given quantity.

Dimensions of some important quantities.

1) Acceleration $= [M^0 L^1 T^{-2}]$
2) Force $= [M^1 L^1 T^{-2}]$
3) Work $= [M^1 L^2 T^{-2}]$
4) Pressure $= [M^1 L^{-1} T^{-2}]$
5) Energy $= [M^1 L^2 T^{-2}]$
6) Momentum $= [M^1 L^1 T^{-1}]$
7) Surface tension $= [M^1 L^0 T^{-2}]$
8) Impulse $= [M^1 L^1 T^{-2}]$
9) stress $= [M^1 L^{-1} T^{-2}]$