Exercise - 7.1

(1) Using Heron's formula, find the area of a triangle whose sides are.

(i) 10 cm, 24 cm, 26 cm, (ii) 1.8 m, 8 m, 8.2 m.

⇒ (i) Let $a = 10\,cm$, $b = 24\,cm$, $c = 26\,cm$.

Now, $s = \frac{a+b+c}{2} = \frac{10+24+26}{2} = \frac{60}{2}\,cm = 30\,cm$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{30(30-10)(30-24)(30-26)}$

$= \sqrt{30 \times 20 \times 6 \times 4}$

$= \sqrt{5 \times 6 \times 5 \times 4 \times 6 \times 4}$

$= 5 \times 6 \times 4$

$= 120\,cm^2$

Thus, the Area of the triangle is $120\,cm^2$.

(ii) Let $a = 1.8\,m$, $b = 8\,m$, $c = 8.2\,m$

$s = \frac{a+b+c}{2} = \frac{1.8+8+8.2}{2} = \frac{18}{2} = 9\,m$

∴ Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{9(9-1.8)(9-8)(9-8.2)}$

$= \sqrt{9 \times 7.2 \times 1 \times .8}$

$= \sqrt{9 \times 9 \times .8 \times 1 \times .8}$

$= 9 \times .8 \times 1$

$= 7.2\,m^2$

Therefore, ~~the~~ Area of the triangle is $7.2\,m^2$.

(2) The sides of the triangular ground are 22 m, 120 m, and 122 m. Find the area and cost of levelling the ground at the rate of ₹ 20 per m².

⇒ Let $a = 22\,m$, $b = 120\,m$, $c = 122\,m$.

$$s = \frac{a+b+c}{2} = \frac{22+120+122}{2} = \frac{264}{2} = 132\,m$$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{132(132-22)(132-120)(132-122)}$$

$$= \sqrt{132 \times 110 \times 12 \times 10}$$

$$= \sqrt{12 \times 11 \times 11 \times 10 \times 12 \times 10}$$

$$= 12 \times 11 \times 10$$

$$= 1320\ m^2.$$

Given the cost of levelling the ground is ₹ 20 per m²

Then total cost $= 1320 \times 20$

$= 26400$

Thus, the the total cost ₹ 26400.

(3) The perimeter of a triangular plot is 600 m. If the sides are in the ratio 5:12:13, then find the area of the plot.

⇒ Given the triangulor plot sides ratio = 5:12:13.

∴ Let $a = 5x$, $b = 12x$, $c = 13x$.

then, $2s = 5x + 12x + 13x$

∴ $600 = 30x$

$x = 20$

∴ $a = 100$, $b = 240$, $c = 260$

$$s = \frac{a+b+c}{2} = \frac{30x}{2} = \frac{30 \times 20}{2} = 300\ m.$$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{300(300-100)(300-240)(300-260)}$

$= \sqrt{300 \times 200 \times 60 \times 40}$

$= \sqrt{\cancel{50 \times 60}\, 5 \times 60 \times 5 \times 40 \times 60 \times 40}$

$= 5 \times 60 \times 40 \ m^2$

$= 12000 m^2$

Therefore, the triangle are is $12000 \ m^2$.

(4) Find the area of an equilateral triangle whose perimeter is 180 cm.

⇒ Let the equilateral triangle side = a cm.

Now, perimeter = 180 (given)

$3a = 180$

$a = 60$ cm

∴ Area of the triangle $= \frac{\sqrt{3}}{4} \times a^2 \ cm^2$

$= \frac{\sqrt{3}}{\cancel{4}} \times \overset{15}{\cancel{60}} \times 60 \ cm^2$

$= 900 \times 1.7320$

$= 1558.84 \ cm^2$

Thus, the Area of the equilateral triangle is $1558.84 \ cm^2$

(5) An advertisement board is in the form of an isosceles triangle with perimeter 36 m and each of the equal sides are 13 m. Find the cost of painting it at ₹ 17.50 per square metre.

⇒ Let $a = b = 13m$ and $c = 36 - (13 \times 2)$ m

$= 36 - 26$

$= 10$ m

~~$2S = 36m$~~ $2S = 36 m$

$S = 18 m.$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-13)(18-13)(18-10)}$

$= \sqrt{18 \times 5 \times 5 \times 8}$

$= \sqrt{9 \times 2 \times 5 \times 5 \times 4 \times 2}$

$= 3 \times 2 \times 5 \times 2 \ m^2$

$= 60 \ m^2$

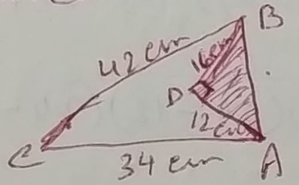
Given the cost of per square metre is ₹ 17.50

then total cost of painting $= 17.50 \times 60$

$= 1050$

Thus, the total cost of painting is ₹ 1050.

(6) Find the area of the unshaded region.



⇒ Given that △ ADB is a right-angled triangle and ∠D = 90°

then, $(AB)^2 = (AD)^2 + (BD)^2$

$(AB)^2 = (12)^2 + (16)^2 = 144 + 256$

$(AB)^2 = 400 = (20)^2$

$\therefore AB = 20$ cm.

then, Now, unshaded region ADBC

$\therefore$ area of ADBC = Area of ABC − Area of ADB.

△ ABC, $a_1 = AB = 20$ cm, $a_2 = AC = 34$ cm, $a_3 = BC = 42$ cm.

$S = \frac{20+34+42}{2} = \frac{96}{2} = 48$ cm.

Area of $\Delta ABC = \sqrt{s(s-a_1)(s-a_2)(s-a_3)}$

$= \sqrt{48(48-20)(48-34)(48-42)}$

$= \sqrt{48 \times 28 \times 14 \times 6}$

$= \sqrt{8 \times 6 \times 7 \times 4 \times 7 \times 2 \times 6}$

$= \sqrt{8 \times 6 \times 7 \times 8 \times 7 \times 6}$

$= 8 \times 6 \times 7 \text{ cm}^2$

$= 336 \text{ cm}^2.$

~~Area of~~
ΔADB, $b_1 = AB = 20$ cm, $b_2 = AD = 12$ cm, $b_3 = BD = 16$ cm.

$s = \frac{20+12+16}{2} = \frac{48}{2} = 24$ cm

Area of $\Delta ADB = \sqrt{s(s-b_1)(s-b_2)(s-b_3)}$

$= \sqrt{24(24-20)(24-12)(24-16)}$

$= \sqrt{24 \times 4 \times 12 \times 8}$

$= \sqrt{12 \times 8 \times 12 \times 8}$

$= 12 \times 8 \text{ cm}^2$

$= 96 \text{ cm}^2$

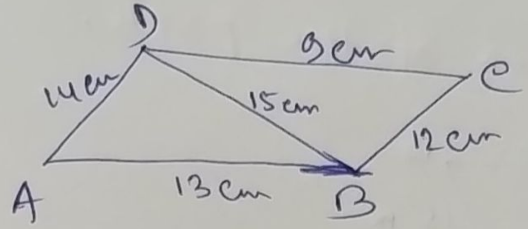
Therefore the unshaded region ADBC = Area of ΔABC - Area of ΔADB

$= (336 - 96) \text{ cm}^2$

$= 240 \text{ cm}^2$

(7) Find the area of a quadrilateral ABCD whose sides are AB = 13 cm, BC = 12 cm, CD = 9 cm, AD = 14 cm and diagonal BD = 15 cm.

⇒



Now, $\triangle ABD$, Let $a_1 = AB = 13$ cm, $a_2 = BD = 15$ cm, $a_3 = AD = 14$ cm.

$$\therefore s = \frac{a_1 + a_2 + a_3}{2} = \frac{13 + 15 + 14}{2} = \frac{42}{2} = 21 \text{ cm.}$$

$$\text{Area of } \triangle ABD = \sqrt{s(s-a_1)(s-a_2)(s-a_3)}$$

$$= \sqrt{21(21-13)(21-15)(21-14)}$$

$$= \sqrt{21 \times 8 \times 6 \times 7}$$

$$= \sqrt{7 \times 3 \times 4 \times 2 \times 3 \times 2 \times 7}$$

$$= 7 \times 3 \times 2 \times 2 \text{ cm}^2$$

$$= 84 \text{ cm}^2.$$

Now, $\triangle BCD$, Let $b_1 = BC = 12$ cm, $b_2 = CD = 9$ cm, $b_3 = BD = 15$ cm

$$s = \frac{b_1 + b_2 + b_3}{2} = \frac{12 + 9 + 15}{2} = \frac{36}{2} = 18 \text{ cm.}$$

$$\text{Area of } \triangle BCD = \sqrt{s(s-b_1)(s-b_2)(s-b_3)}$$

$$= \sqrt{18 \times (18-12)(18-9)(18-15)}$$

$$= \sqrt{9 \times 2 \times 6 \times 9 \times 3}$$

$$= 9 \times 2 \times 3 \text{ cm}^2$$

$$= 54 \text{ cm}^2$$

Thus, the area of quadrilateral ABCD

$= \text{Area of } \triangle ABD + \text{Area of } \triangle BCD$

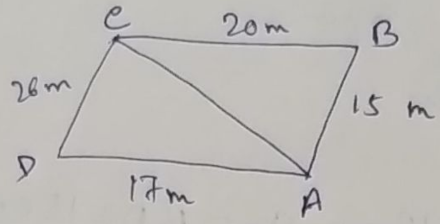
$= (84 + 54) \text{ cm}^2$

$= 138 \text{ cm}^2$

=

(8) A park is in the shape of a quadrilateral. The sides of the park are 15 m, 20 m, 26 m and 17 m and the angle between the first two sides is a right angle. Find the area of the park.

⇒



Now, given, $\angle ABC = 90^\circ$ then, $\triangle ABC$,

$$(AC)^2 = (AB)^2 + (BC)^2$$
$$= 225 + 400 = 625 = (25)^2$$

$\therefore AC = 25$ m.

Now, $\triangle ABC$, $a_1 = AB = 15$ m, $a_2 = BC = 20$m, $a_3 = AC = 25$ m.

$$S = \frac{a_1 + a_2 + a_3}{2} = \frac{15 + 20 + 25}{2} = \frac{60}{2} = 30 \text{ m}$$

Area of $\triangle ABC = \sqrt{S(S-a_1)(S-a_2)(S-a_3)}$
$$= \sqrt{30(30-15)(30-20)(30-25)}$$
$$= \sqrt{5 \times 3 \times 2 \times 5 \times 3 \times 5 \times 2 \times 5}$$
$$= 5 \times 5 \times 3 \times 2 \text{ m}^2$$
$$= 150 \text{ m}^2.$$

Now, $\triangle ADC$, $b_1 = AD = 17$m, $b_2 = DC = 26$m, $AC = b_3 = 25$ m.

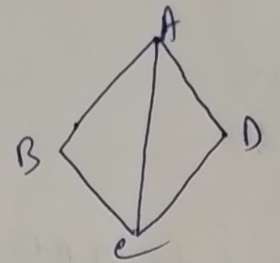
$$S = \frac{b_1 + b_2 + b_3}{2} = \frac{17 + 26 + 25}{2} = \frac{68}{2} = 34 \text{ m}.$$

Area of $\triangle ADC$, $= \sqrt{S(S-b_1)(S-b_2)(S-b_3)}$
$$= \sqrt{34(34-17)(34-26)(34-25)}$$
$$= \sqrt{17 \times 2 \times 17 \times 4 \times 2 \times 3 \times 3}$$
$$= 17 \times 2 \times 2 \times 3 \text{ m}^2 = 204 \text{ m}^2.$$

Thus, the area of the park = Area of $\triangle ABC$ + Area of $\triangle ACD$
$$= (150 + 204) \text{ m}^2$$
$$= 354 \text{ m}^2.$$

(9) A land is in the shape of rhombus. The perimeter of the land is 160 m and one of the diagonal is 48 m. Find the area of the land.

⇒



We know that rhombus all sides are equal.

then, perimeter $= 4a$ [Let one side $= a$]

$\therefore 4a = 160$

$a = 40 m.$

$AC =$ diagonal $= 48 m.$

ΔABC, ~~$AB = a_1 = 40 m$~~,

$AB = BC = a_1 = a_2 = 40 m$, $AC = a_3 = 48 m.$

$s = \frac{a_1+a_2+a_3}{2} = \frac{40+40+48}{2} = \frac{128}{2} = 64 m.$

Area of $\Delta ABC = \sqrt{s(s-a_1)(s-a_2)(s-a_3)}$

$= \sqrt{64 \times (64-40)(64-40)(64-48)}$

$= \sqrt{8 \times 8 \times 24 \times 24 \times 4 \times 4}$

$= 8 \times 24 \times 4 = 768 m^2$

ΔACD, $AD = DC = b_1 = b_2 = 40 m$, $AC = b_3 = 48 m.$

$s = \frac{b_1+b_2+b_3}{2} = \frac{40+40+48}{2} = 64 m.$

Area of $\Delta ADC = \sqrt{s(s-b_1)(s-b_2)(s-b_3)}$

$= \sqrt{64 \times (64-40)(64-40)(64-48)}$

$= \sqrt{8 \times 8 \times 24 \times 24 \times 4 \times 4} = 768 m^2.$

Therefore the area of the land

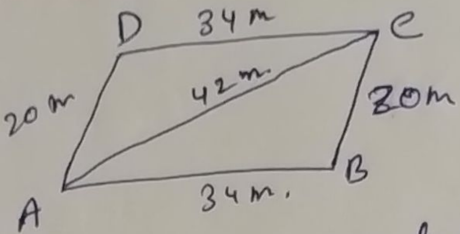
$=$ Area of $\Delta ABC +$ Area of ΔADC

$= (768 + 768) m^2$

$= 1536 m^2$

(10) The adjacent sides of a parallelogram measures 34 m, 20 m and the measure of one of the diagonal is 42 m. Find the area of parallelogram.

⇒



Now, $\triangle ADC$ ~~is similar~~ equal to $\triangle ABC$.

$\therefore a = 34,\ b = 20\,m,\ c = 42\,m.$

$$s = \frac{a+b+c}{2} = \frac{34+20+42}{2} = \frac{96}{2} = 48\,m.$$

Area of $\triangle ADC$ or $\triangle ABC$,

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{48(48-34)(48-20)(48-42)}$$

$$= \sqrt{8 \times 6 \times 7 \times 2 \times 7 \times 4 \times 6}$$

$$= \sqrt{8 \times 6 \times 7 \times 8 \times 7 \times 6}$$

$$= 8 \times 6 \times 7$$

$$= 336\ m^2$$

Thus, the area of parallelogram

$$= 2 \times \text{area of } \triangle ABC \text{ or } \triangle ADC$$

$$= 2 \times 336\ m^2$$

$$= 672\ m^2.$$