(ii) L.H.S

$1 + \tan^2 30^\circ$

$= 1 + \left(\frac{1}{\sqrt{3}}\right)^2$

$= 1 + \frac{1}{3}$

$= \frac{3+1}{3}$

$= \frac{4}{3}$

R.H.S

$\sec^2 30^\circ$

$= \left(\frac{2}{\sqrt{3}}\right)^2$

$= \frac{4}{3}$

Now, L.H.S = R.H.S

Thus, $1 + \tan^2 30^\circ = \sec^2 30^\circ$ [verified].

(iii) we know that

$\cos 90^\circ = 0$ ---(i)

Now, $1 - 2\sin^2 45^\circ$

$= 1 - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2$

$= 1 - 2 \times \frac{1}{2}$

$= 1 - 1 = 0$

$\therefore 1 - 2\sin^2 45^\circ = 0$ ...(ii)

Now, $2\cos^2 45^\circ - 1$

$= 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1$

$= 2 \times \frac{1}{2} - 1$

$= 1 - 1 = 0$

$\therefore 2\cos^2 45^\circ - 1$ ...(iii)

Thus, from (i), (ii) and (iii), we get

Thus, $\cos 90^\circ = 1 - 2\sin^2 45^\circ = 2\cos^2 45^\circ - 1$.

(iv) L.H.S

$\sin 30^\circ \cdot \cos 60^\circ + \cos 30^\circ \cdot \sin 60^\circ$

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4}$

$= \frac{1+3}{4} = \frac{4}{4} = 1$

R.H.S

$\sin 90^\circ$

$= 1$

Now. L.H.S = R.H.S

Thus, $\sin 30^\circ \cdot \cos 60^\circ + \cos 30^\circ \cdot \sin 60^\circ = \sin 90^\circ$ [verified].

(2) Find the value of the following:

(i) $\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5 \sin 90^\circ}{2 \cos 0^\circ}$

(ii) $(\sin 90^\circ + \cos 60^\circ + \cos 45^\circ) \times (\sin 30^\circ + \cos 0^\circ - \cos 45^\circ)$

(iii) $\sin^2 30^\circ - 2 \cos^3 60^\circ + 3 \tan^4 45^\circ$

⇒ (i) $\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} - \frac{5 \sin 90^\circ}{2 \cos 0^\circ}$

$= \frac{1}{2} + \frac{2}{1} - \frac{5 \times 1}{2 \times 1}$

$= \frac{1}{2} + 2 - \frac{5}{2}$

$= \frac{1+4-5}{2}$

$= \frac{5-5}{2}$

$= 0.$

(ii) $(\sin 90^\circ + \cos 60^\circ + \cos 45^\circ) \times (\sin 30^\circ + \cos 0^\circ - \cos 45^\circ)$

$= \left(1 + \frac{1}{2} + \frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{2} + 1 - \frac{1}{\sqrt{2}}\right)$

$= \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right) \times \left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)$

$= \left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{9}{4} - \frac{1}{2}$

$= \frac{9-2}{4}$

$= \frac{7}{4}$

(iii) $\sin^2 30^\circ - 2\cos^3 60^\circ + 3\tan^4 45^\circ$

$= \cancel{\frac{1}{2}}\left(\frac{1}{2}\right)^2 - 2 \times \left(\frac{1}{2}\right)^3 + 3 \times (1)^4$

$= \frac{1}{4} - 2 \times \frac{1}{8} + 3 \times 1$

$= \frac{1}{4} - \frac{1}{4} + 3 \times 1$

$= 3$

(3) verify $\cos 3A = 4\cos^3 A - 3\cos A$, when $A = 30^\circ$.

$\Rightarrow$ Given that $A = 30^\circ$.

L.H.S

$\cos 3A$

$= \cos(3 \times 30^\circ)$

$= \cos 90^\circ$

$= 0$

R.H.S

$4\cos^3 A - 3\cos A$

$= 4\cos^3 30^\circ - 3\cos 30^\circ$

$= 4 \times \left(\frac{\sqrt{3}}{2}\right)^3 - 3 \times \frac{\sqrt{3}}{2}$

$= 4 \times \frac{3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0$