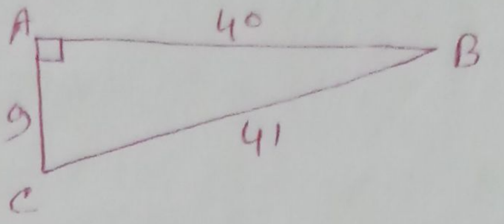
Exercise - 6.1

(1) From the given figure, find all the trigonometric ratios of angle B.



⇒ Given that $\triangle ABC$ is a right-angle triangle and $\angle A = 90^\circ$.

Now, BC = hypotenuse = 41

AC = opposite side = 9

AB = adjacent side = 40.

We know that,

$$\sin B = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AC}{BC} = \frac{9}{41}$$

$$\cos B = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{40}{41}$$

$$\tan B = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{AC}{AB} = \frac{9}{40}$$

$$\operatorname{cosec} B = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{BC}{AC} = \frac{41}{9}$$

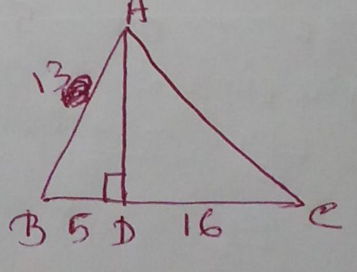
$$\sec B = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{BC}{AB} = \frac{41}{40}$$

$$\cot B = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{AB}{AC} = \frac{40}{9}.$$

(2) From the given figure, find the value of

(i) $\sin B$ (ii) $\sec B$ (iii) $\cot B$

(iv) $\cos C$ (v) $\tan C$ (vi) $\operatorname{cosec} C$



⇒ NOW, ΔABD is a right-angle triangle and $\angle D = 90°$

then, $AB^2 = AD^2 + BD^2$

$(13)^2 = AD^2 + (5)^2$

$169 = AD^2 + 25$

$(AD)^2 = 144$

$AD = 12$

NOW, ΔADC is a tright-angle triangle and $\angle D = 90°$

$(AD)^2 + (DC)^2 = (AC)^2$

$(12)^2 + (16)^2 = (AC)^2$

$(AC)^2 = 144 + 256 = 400$

$AC = 20$

~~(i) $\sin B = \frac{AD}{AB}$~~

ΔABD, AB = hypotenuse

AD = opposite side

BD = adjacent side.

(i) $\sin B = \frac{AD}{AB} = \frac{12}{13}$

(ii) $\sec B = \frac{AB}{BD} = \frac{13}{5}$

(iii) $\cot B = \frac{BD}{AD} = \frac{5}{12}$

ΔADC, AD = opposite side

DC = adjacent side

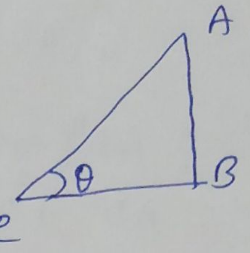
AC = hypotenuse.

(iv) $\cos C = \frac{DC}{AC} = \frac{16}{20} = \frac{4}{5}$

(v) $\tan C = \frac{AD}{DC} = \frac{12}{16} = \frac{3}{4}$

(vi) $\operatorname{cosec} C = \frac{AC}{AD} = \frac{20}{12} = \frac{5}{3}$

(3) If $2\cos\theta = \sqrt{3}$, then find all the trigonometric ratios of angle θ.



$\Rightarrow$ Now, $2\cos\theta = \sqrt{3}$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$$

Now, $\triangle ABC$, is a right-angle triangle.

then, $(AB)^2 + (BC)^2 = (AC)^2$

$$(AB)^2 + (\sqrt{3})^2 = (2)^2$$

$$(AB)^2 + 3 = 4$$

$$(AB)^2 = 4 - 3 = 1$$

$$AB = 1$$

Now,

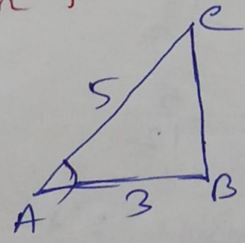
$$\sin\theta = \frac{AB}{AC} = \frac{1}{2}$$

$$\cos\theta = \frac{\sqrt{3}}{2}, \quad \tan\theta = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$$

$$\operatorname{cosec}\theta = \frac{AC}{AB} = \frac{2}{1} = 2$$

$$\sec\theta = \frac{AC}{BC} = \frac{2}{\sqrt{3}}, \quad \cot\theta = \frac{BC}{AB} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

(4) If $\cos A = \frac{3}{5}$, then find the value of $\frac{\sin A - \cos A}{2\tan A}$.



$\Rightarrow$ $\cos A = \frac{3}{5}$

$$\cos A = \frac{AB}{AC} = \frac{3}{5}$$

$\triangle ABC$, then, $(BC)^2 = (AC)^2 - (AB)^2$

$$= (5)^2 - (3)^2$$

$$= 25 - 9$$

$$= 16 = (4)^2$$

$$BC = 4.$$

Now, $\sin A = \frac{BC}{AC} = \frac{4}{5}$

$\cos A = \frac{3}{5}$, $\tan A = \frac{BC}{AB} = \frac{4}{3}$.

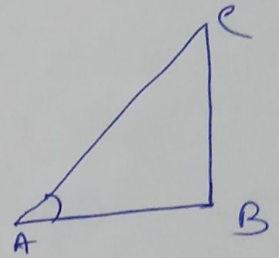
Now, $\frac{\sin A - \cos A}{2 \tan A}$

$= \frac{\frac{4}{5} - \frac{3}{5}}{2 \times \frac{4}{3}} = \frac{\frac{4-3}{5}}{\frac{8}{3}} = \frac{1/5}{8/3} = \frac{1 \times 3}{5 \times 8} = \frac{3}{40}$

Thus, $\frac{\sin A - \cos A}{2 \tan A} = \frac{3}{40}$.

5) If $\cos A = \frac{2x}{1+x^2}$, then find the values of $\sin A$ and $\tan A$ in terms of x.

⇒ Given that



$\cos A = \frac{2x}{1+x^2}$

$\cos A = \frac{AB}{AC} = \frac{2x}{1+x^2}$.

Now, $\triangle ABC$ is a right-angle triangle.

then, $BC^2 = AC^2 - AB^2$

$BC^2 = (1+x^2)^2 - (2x)^2$

$BC^2 = 1 + 2x^2 + x^4 - 4x^2$

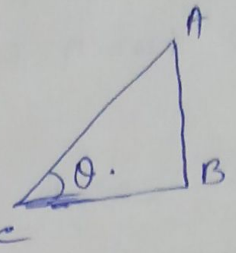
$BC^2 = 1 - 2x^2 + x^4 = \cancel{(1-2x^2)}\ (1-x^2)^2$

$BC = 1 - x^2$.

then, $\sin A = \frac{BC}{AC} = \frac{1-x^2}{1+x^2}$

$\tan A = \frac{BC}{AB} = \frac{1-x^2}{2x}$.

⑥ If $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$, then show that $b\sin\theta = a\cos\theta$.



$\Rightarrow$ $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$ ---(i)

$\sin\theta = \frac{AB}{AC} = \frac{a}{\sqrt{a^2+b^2}}$.

$\triangle ABC$ is a right angle triangle.

then, $(AB)^2 = \cancel{(AC)^2 - (BC)^2}$

$\cancel{= (\sqrt{a^2+b^2})^2 - (a)}$

$(BC)^2 = (AC)^2 - (AB)^2$

$= (\sqrt{a^2+b^2})^2 - (a)^2$

$= a^2 + b^2 - a^2$

$= b^2$

$BC = b$.

Now, $\cancel{\frac{\sin\theta}{\cos}}$ $\cos\theta = \frac{BC}{AC} = \frac{b}{\sqrt{a^2+b^2}}$ ---(ii)

From (i) and (ii), we get

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{a}{\sqrt{a^2+b^2}}}{\frac{b}{\sqrt{a^2+b^2}}}$$

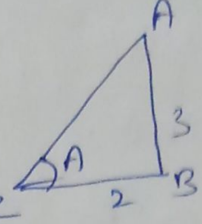
$$\frac{\sin\theta}{\cos\theta} = \frac{a \times \sqrt{a^2+b^2}}{b \times \sqrt{a^2+b^2}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{a}{b}$$

$b\sin\theta = a\cos\theta$.

[Proved]

(7) If $3\cot A = 2$, then find the value of $\frac{4\sin A - 3\cos A}{2\sin A + 3\cos A}$.



$\Rightarrow$ Given that,

$$3\cot A = 2$$
$$\cot A = \frac{2}{3}$$
$$\cot A = \frac{BC}{AB} = \frac{2}{3}$$

ΔABC is a right-angle triangle.

$$(AC)^2 = (AB)^2 + (BC)^2$$
$$(AC)^2 = (3)^2 + (2)^2 = 9 + 4 = 13$$
$$AC = \sqrt{13}$$

Now, $\sin A = \frac{AB}{AC} = \frac{3}{\sqrt{13}}$, $\cos A = \frac{BC}{AC} = \frac{2}{\sqrt{13}}$.

$$\therefore \frac{4\sin A - 3\cos A}{2\sin A + 3\cos A}$$

$$= \frac{4 \times \frac{3}{\sqrt{13}} - 3 \times \frac{2}{\sqrt{13}}}{2 \times \frac{3}{\sqrt{13}} + 3 \times \frac{2}{\sqrt{13}}}$$

$$= \frac{\frac{12 - 6}{\sqrt{13}}}{\frac{6 + 6}{\sqrt{13}}}$$

$$= \frac{6/\sqrt{13}}{12/\sqrt{13}}$$

$$= \frac{6 \times \sqrt{13}}{12 \times \sqrt{13}}$$

$$= \frac{1}{2}$$

Therefore, $\frac{4\sin A - 3\cos A}{2\sin A + 3\cos A} = \frac{1}{2}$

(8) If $\cos\theta : \sin\theta = 1:2$, then find the value of

$$\frac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta}$$

⇒ Given that, $\cos\theta : \sin\theta = 1:2$

$$\frac{\cos\theta}{\sin\theta} = \frac{1}{2}$$

$$\frac{\cos\theta}{1} = \frac{\sin\theta}{2} = k \text{ (let)}$$

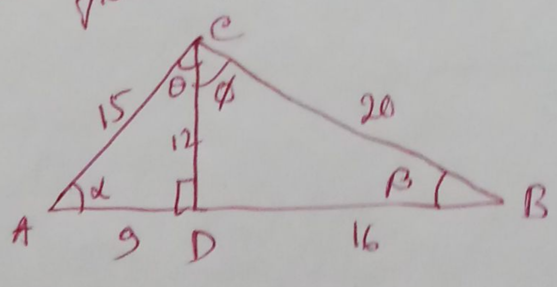
$\cos\theta = k$ and $\sin\theta = 2k$.

Now, $\dfrac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta}$

$$= \frac{8k - 2\times 2k}{4k + 2\times 2k} = \frac{8k - 4k}{4k + 4k} = \frac{4k}{8k} = \frac{1}{2}$$

Therefore, $\dfrac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta} = \dfrac{1}{2}$.

(9) From the given figure, prove that $\theta + \phi = 90°$. Also prove that there are two other right angled triangles. Find $\sin\alpha$, $\cos\beta$ and $\tan\phi$.



⇒ Given that

$AC = 15$, $AB = AD + DB = 9 + 16 = 25$

and $BC = 20$.

Now, $(AC)^2 + (BC)^2 = (15)^2 + (20)^2$
$= 225 + 400$
$= 625$
$= (25)^2 = (AB)^2$.

~~Now~~ then, $\triangle ABC$, is a ~~right~~ angle triangle.

$\angle C = 90°$

~~then~~ Now, $\theta + \phi = 90°$ $[\because \angle C = \theta + \phi]$

where θ and ϕ are two angle in triangle ACD and DCD respectively. [Proved].

Now, $\triangle ADC$,

$$\sin\alpha = \frac{DC}{AC} = \frac{12}{15} = \frac{4}{5}$$

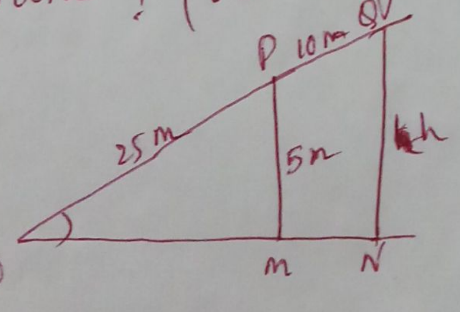
$\triangle DBC$,

$$\cos\beta = \frac{BD}{BC} = \frac{16}{20} = \frac{4}{5}$$

~~$\triangle ABC$~~ $\triangle CBD$,

$$\tan\phi = \frac{BD}{CD} = \frac{16}{12} = \frac{4}{3}$$

(10) A boy standing at a point O finds his kite flying at a point P with distance OP = 25 m. It is at a height of 5 m from the ground. When the thread is extended by 10 m from P, it reaches a point Q. What will be the height QN of the kite from the ground? (use trigonometric ratios)



$\Rightarrow$ Let $\angle MOP = \theta$. ~~then~~

then $\triangle OMP$,

$$\sin\theta = \frac{MP}{PO} = \frac{5}{25} = \frac{1}{5}$$

$$\therefore \sin\theta = \frac{1}{5} \quad \text{---(i)}$$