Exercise - 5.5

1. Find the centroid of the triangle whose vertices are.

(i) $(2,-4)$, $(-3,-7)$ and $(7,2)$

(ii) $(-5,-5)$, $(1,-4)$ and $(-4,-2)$.

⇒ (i) Let $(x_1, y_1) = (2,-4)$

$(x_2, y_2) = (-3,-7)$

$(x_3, y_3) = (7,2)$.

Let G be the centroid of the triangle.

then, $G(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$G(x, y) = \left(\frac{2 - 3 + 7}{3}, \frac{-4 - 7 + 2}{3}\right)$

$G(x, y) = \left(\frac{6}{3}, \frac{-9}{3}\right)$

$G(x, y) = (2, -3)$

Thus, $(2,-3)$ be the centroid of the triangle.

(ii) Let $(x_1, y_1) = (-5,-5)$, $(x_2, y_2) = (1,-4)$, $(x_3, y_3) = (-4,-2)$

Let G be the centroid of the triangle.

thus $G(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$G(x, y) = \left(\frac{-5 + 1 - 4}{3}, \frac{-5 - 4 - 2}{3}\right)$

$G(x, y) = \left(\frac{-8}{3}, \frac{-11}{3}\right)$

Thus, $\left(-\frac{8}{3}, \frac{-11}{3}\right)$ be the centroid of the triangle.

(2) If the centroid of a triangle is at $(4,-2)$ and two of its vertices are $(3,-2)$ and $(5,2)$ then find the third vertex of the triangle.

$\Rightarrow$ Let third vertex is (x_3, y_3).

Now, Let $(x_1, y_1) = (3,-2)$ and $(x_2, y_2) = (5,2)$.

~~then~~, given that this triangle centroid is $(4,-2)$.

then,
$$(4,-2) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$\left(\frac{3+5+x_3}{3}, \frac{-2+2+y_3}{3}\right) = (4,-2)$$

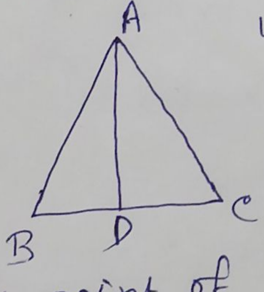
$$8 + x_3 = 12$$
$$x_3 = 12 - 8$$
$$x_3 = 4$$

$$-\cancel{2}\, y_3 = -6$$

Thus, the third vertex is $(4,-6)$.

(3) Find the length of median through A of a triangle whose vertices are $A(-1,3)$, $B(1,-1)$ and $C(5,1)$.

$\Rightarrow$



Let $A(x_1, y_1) = (-1, 3)$
$B(x_2, y_2) = (1, -1)$
$C(x_3, y_3) = (5, 1)$

D is the mid-point of BC.

then,
$$D(x,y) = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$
$$D(x,y) = \left(\frac{1+5}{2}, \frac{-1+1}{2}\right)$$
$$D(x,y) = (3,0)$$

Now, AD length is the median of the triangle.

then, length of $AD = \sqrt{(3+1)^2 + (0-3)^2}$
$$= \sqrt{16+9}$$
$$= \sqrt{25} = 5 \text{ units}$$

Therefore 5 units is the length of the triangle.

(4) The vertices of a triangle are (1,2), (h,-3) and (-4,k). If the centroid of the triangle is at the point (5,-1) then find the value of $\sqrt{(h+k)^2+(h+3k)^2}$.

⇒ Let $(x_1,y_1)=(1,2)$, $(x_2,y_2)=(h,-3)$ and $(x_3,y_3)=(-4,k)$.

Given that this triangle centroid is (5,-1).

then,
$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)=(5,-1)$$

$$\left(\frac{1+h-4}{3}, \frac{2-3+k}{3}\right)=(5,-1)$$

$$\left(\frac{h-3}{3}, \frac{-1+k}{3}\right)=(5,-1)$$

$$\frac{h-3}{3}=5 \qquad \frac{-1+k}{3}=-1$$

$$h-3=15 \qquad -1+k=-3$$

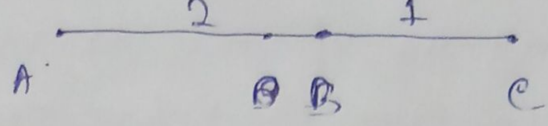
$$h=18 \qquad k=-2$$

Now,
$$\sqrt{(h+k)^2+(h+3k)^2}$$
$$=\sqrt{(18-2)^2+(18-6)^2}$$
$$=\sqrt{256+144}$$
$$=\sqrt{400}$$
$$=20 \; \cancel{\text{units}}$$

Thus the required value ~~of~~ is 20.

(5) Orthocentre and centroid of a triangle are A(-3,5) and B(3,3) respectively. If C is the circumcentre and AC is the diameter of this circle, then find the radius of the circle.

⇒ We know that any triangle circumcentre, centriod and orthocentre are collinear and centriod divided 1:2 ratio ~~orthocentre an~~ circumcentre and orthocentre.



Let $m:n = 2:1$

orthocentre $= A(x_1, y_1) = (-3, 5)$

centroid $= B(x_2, y_2) = (3, 3)$

Let circumcentre $= C(x_3, y_3) = (a, b)$.

then, $\left(\frac{mx_3 + nx_1}{m+n}, \frac{my_3 + ny_1}{m+n}\right) = (3, 3)$

$\left(\frac{2a-3}{3}, \frac{2b+5}{3}\right) = (3, 3)$

$2a - 3 = 9$	$2b + 5 = 9$
$2a = 12$	$2b = 4$
$a = 6$	$b = 2$

Now, C point is $(6, 2)$.

then, AC length is $= \sqrt{(6+3)^2 + (2-5)^2}$

$= \sqrt{81 + 9}$

$= \sqrt{90}$

$= 3\sqrt{5 \times 2}$

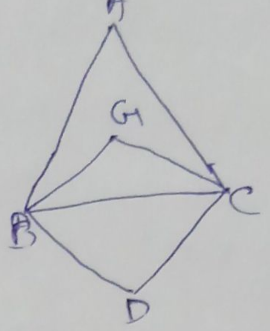
Given that AC = diameter.

$\therefore$ radius $= \frac{AC}{2} = \frac{3\sqrt{5 \times 2}}{2} = 3\sqrt{\frac{5}{2}}$ units.

Therefore the radius of the circle is $3\sqrt{\frac{5}{2}}$ units.

⑥ ABC is a triangle whose vertices are A(3,4), B(-2,-1) and C(5,3). If G is the centroid and BDCA is a parallelogram then find the coordinates of the vertex D.

⇒



ABC triangle centroid (G) is $= \left(\frac{3-2+5}{3}, \frac{4-1+3}{3}\right)$

$= \left(\frac{6}{3}, \frac{6}{3}\right)$

$= (2, 2)$

Now, ~~BDC~~ Let $D = (x, y)$.

Given that BDCA is a parallelogram.

then, mid-point of BC = mid-point of DG.

$\left(\frac{-2+5}{2}, \frac{-1+3}{2}\right) = \left(\frac{2+x}{2}, \frac{2+y}{2}\right)$

$\left(\frac{2+x}{2}, \frac{2+y}{2}\right) = \left(\frac{3}{2}, \frac{2}{2}\right)$

$2+x = 3$ | $2+y = 2$

$x = \underline{1}$ | $y = 0$.

Therefore, the ~~c~~ coordinates of the vertex D is $(1, 0)$.