Exercise - 5.2

(1) Find the distance between the following pairs of points.

(i) $(1,2)$ and $(4,3)$ (ii) $(3,4)$ and $(-7,2)$

(iii) (a,b) and (c,d) (iv) $(3,-9)$ and $(-2,3)$.

(i)

⇒ Now, $(x_1,y_1)=(1,2)$ and $(x_2,y_2)=(4,3)$

The distance between the points $(1,2)$ and $(4,3)$ is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(4-1)^2+(3-2)^2}$$

$$=\sqrt{9+1}$$

$$=\sqrt{10}$$

Thus, $(1,2)$ and $(4,3)$ points between distance is $\sqrt{10}$ units

(ii) Let $(x_1,y_1)=(3,4)$ and $(x_2,y_2)=(-7,2)$

The distance between the points $(3,4)$ and $(-7,2)$ is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(-7-3)^2+(2-4)^2}$$

$$=\sqrt{100+4}$$

$$=\sqrt{104}$$

$$=2\sqrt{26} \text{ units.}$$

Thus, the required distance is $2\sqrt{26}$ units.

(iii) Let $(x_1,y_1)=(a,b)$, $(x_2,y_2)=(c,b)$

The distance between the points (a,b) and (c,d) is

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{(c-a)^2+(b-b)^2}$$

$$=\sqrt{(c-a)^2+0}=\sqrt{(c-a)^2}$$

$$=(c-a) \text{ units.}$$

Thus, the required distance is $(c-a)$ units.

(iv) Let $(x_1, y_1) = (3, -9)$ and $(x_2, y_2) = (-2, 3)$.

The distance between the points $(3, -9)$ and $(-2, 3)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-2-3)^2 + (3+9)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13 \text{ units.}$$

Thus, the required distance is 13 units.

(2) Determine whether the given set of points in each case are collinear or not.

(i) $(7, -2), (5, 1), (3, 4)$ (ii) $(a, -2), (a, 3), (a, 0)$.

(i)
⇒ Let $A = (7, -2)$,
$B = (5, 1)$ and $C = (3, 4)$.

Now, $AB = \sqrt{(5-7)^2 + (1+2)^2}$
$= \sqrt{4+9}$
$= \sqrt{13}$

$BC = \sqrt{(3-5)^2 + (4-1)^2}$
$= \sqrt{4+9}$
$= \sqrt{13}$

Now, $AC = \sqrt{(3-7)^2 + (4+2)^2}$
$= \sqrt{16 + 36}$
$= \sqrt{52}$
$= \sqrt{4 \times 13}$
$= 2\sqrt{13}$.

Now, $AB + BC$
$= \sqrt{13} + \sqrt{13}$
$= 2\sqrt{13} = AC$

$\therefore AB + BC = AC$, Therefore the points are collinear.

(ii) Let $A = (a,-2)$, $B = (a,3)$ and $C = (a,0)$.

$AB = \sqrt{(a-a)^2 + (3+2)^2}$

$= \sqrt{0+25}$

$= 5$ units

$BC = \sqrt{(a-a)^2 + (0-3)^2}$

$= \sqrt{0+9}$

$= 3$ units.

$AC = \sqrt{(a-a)^2 + (0+2)^2}$

$= \sqrt{0+4}$

$= 2$ units.

Now, $BC + AC$

$= 3+2 = 5 = AB$

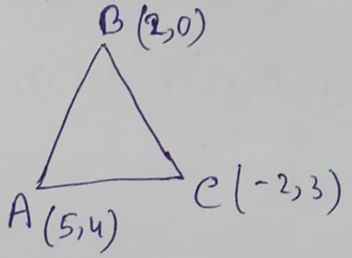
$\therefore AB = BC + AC$.

Therefore the points are collinear.

(3) Show that the following points taken in order form an isosceles triangle.

(i) $A(5,4)$, $B(2,0)$, $C(-2,3)$ (ii) $A(6,-4)$, $B(-2,-4)$, $C(2,10)$.

⇒ (i)



Now, $AB = \sqrt{(2-5)^2 + (0-4)^2}$

$= \sqrt{9+16}$

$= \sqrt{25} = 5$ units.

$AC = \sqrt{(-2-5)^2 + (3-4)^2}$

$= \sqrt{49+1}$

$= \sqrt{50}$ units.

$= 5\sqrt{2}$ units.

$BC = \sqrt{(-2-2)^2 + (3-0)^2}$

$= \sqrt{16+9} = \sqrt{25}$

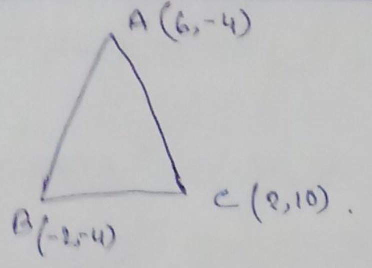
$= 5$ units.

$\therefore$ Now $AB = BC = 5$.

So, two sides are equal.

Thus, $\triangle ABC$ is an isosceles triangle.

(ii)



Now, $AB = \sqrt{(-2-6)^2 + (-4+4)^2}$ and $BC = \sqrt{(2+2)^2 + (10+4)^2}$

$= \sqrt{64 + 0}$ $\qquad = \sqrt{16 + 196}$

$= 8$ $\qquad = \sqrt{212}$

$\qquad = 2\sqrt{53}$

$AC = \sqrt{(2-6)^2 + (10+4)^2}$

$= \sqrt{16 + 196}$

$= \sqrt{212}$

$= 2\sqrt{53}$

Now, $AC = BC = 2\sqrt{53}$

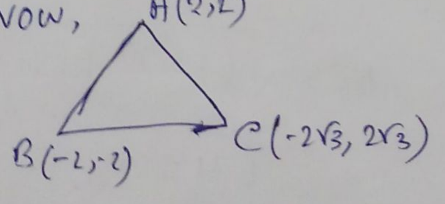
~~So~~ Two sides are equal.

Thus, ~~the~~ ΔABC is an isosceles triangle.

(4) Show that the following points taken in order form an equilateral triangle in each case.

(i) $A(2,2)$, $B(-2,-2)$, $C(-2\sqrt{3}, 2\sqrt{3})$ (ii) $A(\sqrt{3},2)$, $B(0,1)$, $C(0,3)$.

⇒ (i) Now,



Now,

ΔABC,

$AB = \sqrt{(-2-2)^2 + (-2-2)^2}$ $\qquad BC = \sqrt{(-2\sqrt{3}+2)^2 + (2\sqrt{3}+2)^2}$

$= \sqrt{16 + 16}$ $\qquad = \sqrt{12 - 8\sqrt{3} + 4 + 12 + 8\sqrt{3} + 4}$

$= \sqrt{32}$ $\qquad = \sqrt{16 + 16} = \sqrt{32}$

$AC = \sqrt{(-2\sqrt{3}-2)^2 + (2\sqrt{3}-2)^2}$

$AC = \sqrt{12 + 8\sqrt{3} + 4 + 12 - 8\sqrt{3} + 4}$

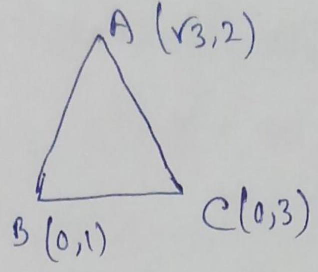
$= \sqrt{16+16} = \sqrt{32}$

Now, $\triangle ABC$,

$AB = BC = AC = \sqrt{32}$.

Thus, the $\triangle ABC$ is an equilateral triangle.

(ii)



Now, $\triangle ABC$,

$$AB = \sqrt{(0-\sqrt{3})^2 + (1-2)^2}$$
$$= \sqrt{3+1} = \sqrt{4}$$
$$= 2$$

$$BC = \sqrt{(0-0)^2 + (3-1)^2}$$
$$= \sqrt{0+4}$$
$$= 2$$

$$AC = \sqrt{(0-\sqrt{3})^2 + (3-2)^2}$$
$$= \sqrt{3+1}$$
$$= \sqrt{4} = 2$$

Now, $AB = BC = AC = 2$

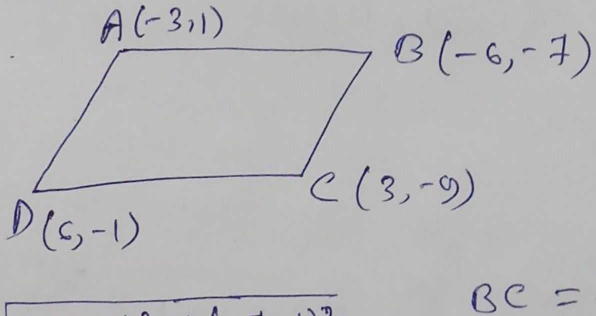
Therefore, $\triangle ABC$ is an equilateral triangle.

(5) Show that the following points taken in order form the vertices of a parallelogram.

(i) $A(-3,1)$, $B(-6,-7)$, $C(3,-9)$, $D(6,-1)$

(ii) $A(-7,-3)$, $B(5,10)$, $C(15,8)$, $D(3,-5)$

⇒ (i)



Now,

$$AB = \sqrt{(-6+3)^2 + (-7-1)^2}$$
$$= \sqrt{9+64}$$
$$= \sqrt{73}$$

$$BC = \sqrt{(3+6)^2 + (-9+7)^2}$$
$$= \sqrt{81+4}$$
$$= \sqrt{85}$$

$$CD = \sqrt{(6-3)^2 + (-1+9)^2}$$
$$= \sqrt{9+64}$$
$$= \sqrt{73}.$$

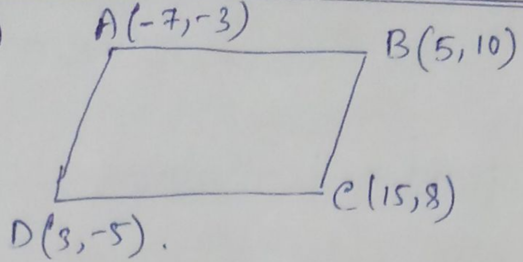
$$DA = \sqrt{(-3-6)^2 + (1+1)^2}$$
$$= \sqrt{81+4}$$
$$= \sqrt{85}$$

Now, $AB = CD = \sqrt{73}$ and $BC = DA = \sqrt{85}$

∴ Opposite sides are equal.

Thus, ABCD is a parallelogram.

(ii)



Now, $AB = \sqrt{(5+7)^2 + (10+3)^2}$ $\quad BC = \sqrt{(15-5)^2 + (8-10)^2}$

$= \sqrt{144 + 169}$ $\quad = \sqrt{100 + 4}$

$= \sqrt{313}$ $\quad = \sqrt{104}$.

$CD = \sqrt{(3-15)^2 + (-5-8)^2}$ $\quad DA = \sqrt{(-7-3)^2 + (-3+5)^2}$

$= \sqrt{144 + 169}$ $\quad = \sqrt{100 + 4}$

$= \sqrt{313}$ $\quad = \sqrt{104}$

Now, $AB = CD = \sqrt{313}$ and $BC = DA = \sqrt{104}$.

So, opposite sides are equal.

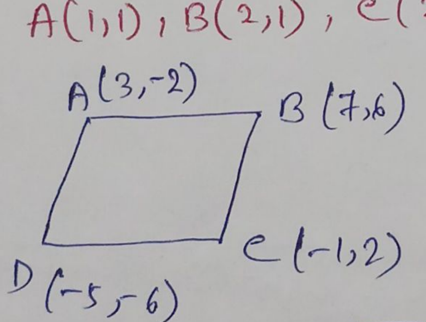
Thus, ABCD is a parallelogram.

(6) Verify that the following points taken in order form the vertices of a rhombus.

(i) A(3, -2), B(7, 6), C(-1, 2), D(-5, -6)

(ii) A(1,1), B(2,1), C(2,2), D(1,2).

⇒



$AB = \sqrt{(7-3)^2 + (6+2)^2}$ $\quad BC = \sqrt{(-1-7)^2 + (2-6)^2}$

$= \sqrt{16 + 64}$ $\quad = \sqrt{64 + 16}$

$= \sqrt{100}$ $\quad = \sqrt{100}$

$= 10$ $\quad = 10$

$CD = \sqrt{(-5+1)^2+(-6-2)^2}$ $\quad$ $DA = \sqrt{(3+5)^2+(-2+6)^2}$

$= \sqrt{16+64}$ $\quad$ $= \sqrt{64+36}$

$= \sqrt{100}$ $\quad$ $= \sqrt{100}$

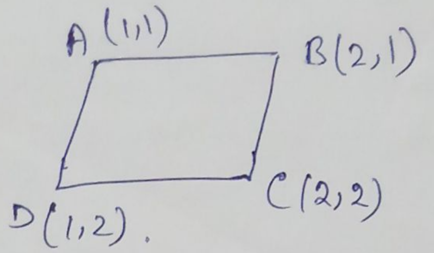
$= 10$ $\quad$ $= 10.$

Now, $AB = BC = CD = DA = 10.$

So, all sides are equal.

Thus, ABCD is a rhombus.

(ii)



Now, $AB = \sqrt{(2-1)^2+(1-1)^2}$ $\quad$ $BC = \sqrt{(2-2)+(2-1)^2}$

$= \sqrt{1+0}$ $\quad$ $= \sqrt{0+1}$

$= 1$ $\quad$ $= 1$

$CD = \sqrt{(1-2)^2+(2-2)^2}$ $\quad$ $DA = \sqrt{(1-1)+(1-2)^2}$

$= \sqrt{1+0}$ $\quad$ $= \sqrt{0+1}$

$= 1$ $\quad$ $= 1$

Now, $AB = BC = CD = DA = 1.$

So, all sides are equal.

Thus, ABCD is a rhombus.

(7) $A(-1,1)$, $B(1,3)$ and $C(3,a)$ are points and if $AB = BC$, then find 'a'.

$\Rightarrow$ Now, let $A(x_1,y_1) = (-1,1)$, $B(x_2,y_2) = (1,3)$ and $C(x_3,y_3) = (3,a)$.

Now, $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(1+1)^2+(3-1)^2}$

$= \sqrt{4+4} = \sqrt{8}$

$$BC = \sqrt{(x_3-x_2)^2 + (y_3-y_2)^2}$$

$$= \sqrt{(3-1)^2 + (a-3)^2}$$

$$= \sqrt{4 + (a-3)^2}$$

Now, $AB = BC$,

$$\sqrt{8} = \sqrt{4+(a-3)^2}$$

$$4 + (a-3)^2 = 8$$ ~~both sides~~ [square by both sides]

$$4 + a^2 - 6a + 9 = 8$$

$$a^2 - 6a + 13 = 8$$

$$a^2 - 6a + 13 - 8 = 0$$

$$a^2 - 6a + 5 = 0$$

$$a^2 - 5a - a + 5 = 0$$

$$a(a-5) - 1(a-5) = 0$$

$$(a-5)(a-1) = 0.$$

$a-5=0$	$a-1=0$
$a=5$	$a=1$

Thus, the value of a is 5 or 1.

(8) The abscissa of a point A is equal to its ordinate, and its distance from the point B(1,3) is 10 units, what are the coordinates of A?

→ Let the abscissa of a point A is a

then, point A ordinate is a.

Now, point A is (a,a).

Now, $AB = 10$

$$\sqrt{(1-a)^2 + (3-a)^2} = 10$$

[square by both sides]

$(1-a)^2 + (3-a)^2 = 100$

$1 - 2a + a^2 + 9 - 6a + a^2 = 100$

$2a^2 - 8a + 10 = 100$

$a^2 - 4a + 5 = 50$

$a^2 - 4a + 5 - 50 = 0$

$a^2 - 4a - 45 = 0$

~~$a^2 - 12a + 4a - 48 =$~~

$a^2 - 9a + 5a - 45 = 0$

$a(a-9) + 5(a-9) = 0$

$(a-9)(a+5) = 0$

$a - 9 = 0$ | $a + 5 = 0$

$a = 9$ | $a = -5$

Now, if $a = 9$ then point of A is $(9, 9)$

if $a = -5$ then point of A is $(-5, -5)$.

9) The point (x, y) is equidistant from the points $(3, 4)$ and $(-5, 6)$. Find a relation between x and y.

$\Rightarrow$ Let $A = (3, 4)$, $B = (x, y)$ and $C = (-5, 6)$.

then, $AB = BC$

$\sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(-5-x)^2 + (6-y)^2}$

$(x-3)^2 + (y-4)^2 = (-5-x)^2 + (6-y)^2$

$x^2 - 6x + 9 + y^2 - 8y + 16 = 25 + 10x + x^2 + 36 - 12y + y^2$

$-6x - 8y + 25 = 25 + 36 - 12y + 10x$

$-6x - 8y = 36 - 12y + 10x$

~~$-8y + 12y = 36 + 6x$~~

$12y - 8y = 10x + 6x + 36$

$4y = 16x + 36$.

$\therefore y = 4x + 9$.

This is the relation between x and y.

(10) Let $A(2,3)$ and $B(2,-4)$ be two points. If P lies on the x-axis, such that $AP = \frac{3}{7}AB$, find the coordinates of P.

$\Rightarrow$ Let the P point coordinates is $(a,0)$.
[P lies on the x-axis].

Now, $A = (2,3)$, $B = (2,-4)$, $P = (a,0)$.

Now, ~~AB~~ $AP = \frac{3}{7}AB$.

$$\sqrt{(a-2)^2 + (0-3)^2} = \frac{3}{7}\sqrt{(2-2) + (-4-3)^2}$$

$$(a-2)^2 + 9 = \frac{3}{7}\{0 + 49\}$$

$$a^2 - 4a + 4 + 9 = \frac{3}{7} \times 7$$

$$a^2 - 4a + 13 = 3$$

$$a^2 - 4a - 8 = 0 \quad a^2 - 4a + 13 - 3 = 0$$

$$\therefore a = \frac{4 \pm \sqrt{16 + 32}}{2}$$

$$a = \frac{4 \pm \sqrt{48}}{2}$$

$$a = 2 \pm \sqrt{12}$$

$$AP = \frac{3}{7}AB$$

$$\sqrt{(a-2)^2 + (0-3)^2} = \frac{3}{7}\sqrt{(2-2)^2 + (-4-3)^2}$$

$$\sqrt{(a-2)^2+9} = \frac{3}{7}\sqrt{0+49}$$

$$\sqrt{(a-2)^2+9} = \frac{3}{7} \times 7$$

$$\sqrt{(a-2)^2+9} = 3$$

$(a-2)^2+9 = 9$ [~~both~~ square by both sides]

$$(a-2)^2 = 0$$

$$a-2 = 0$$

$$a = 2$$

Thus, the coordinates of P is $(2, 0)$.

11) Show that the point $(11, 2)$ is the centre of the circle passing through the point $(1, 2)$, $(3, -4)$, and $(5, -6)$.

⇒ Let $O = (11, 2)$ and $A = (1, 2)$

$B = (3, -4)$

$C = (5, -6)$.

Now, if the points $(11, 2)$ centre circle passing throught the ~~pts~~ A, B, C points.

then, $OA = OB = OC$.

Now, $OA = \sqrt{(1-11)^2 + (2-2)^2} = \sqrt{(10)^2+0} = \sqrt{100} = 10$

$OB = \sqrt{(3-11)^2 + (-4-2)^2} = \sqrt{64+36} = \sqrt{100} = 10$

$OC = \sqrt{(5-11)^2 + (-6-2)^2} = \sqrt{36+64} = \sqrt{100} = 10$.

Now, $OA = OB = OC = 10$.

Thus, the all point passing ~~the~~ through the circle.

(12) The radius of a circle with centre at origin is 30 units. write the coordinates of the points where the circle intersects the axes. find the distance between any two such points.

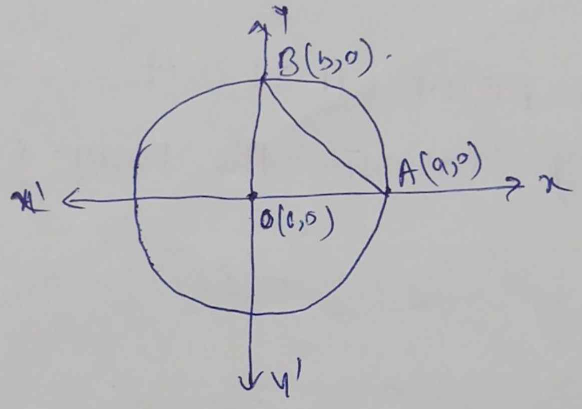
⇒ Given that the radius of circle = 30 units.

the centre of the circle = (0, 0).

Let the circle intersects the x-axis is (a, 0)

and the circle intersects the y-axis is (0, b).

~~NOW,~~



NOW, $OA = \sqrt{(a-0)^2+(0-0)^2} = \sqrt{a^2} = a$

$a = 30.$

Similarly, $b = 30.$

Thus, the point (30, 0) and (0, 30) are the intersects the axes.

Now, ~~the~~ This point distance

$= \sqrt{(0-30)^2+(30-0)^2}$

$= \sqrt{(30)^2+(30)^2}$

$= \sqrt{900+900}$

$= \sqrt{1800}$

$= 30\sqrt{2}$

Thus, the distance between ~~an~~ two point is $30\sqrt{2}$ units.