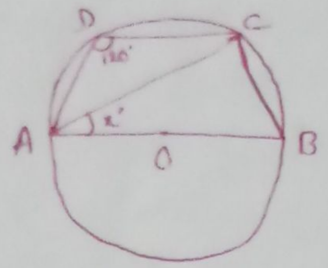
Exercise - 4.4

(1) Find the value of x in the given figure.



$\Rightarrow$ Now, ABCD is a Parallelogram

$\therefore \angle ADC + \angle DAB = 180^\circ$

$120^\circ + \angle DAB = 180^\circ$

$\angle DAB = 180 - 120^\circ$

$\angle DAB = 60^\circ$

Now, $\angle DAC = \angle BAC$ [$\because$ AC bisector of $\angle A$]

$\therefore \angle DAC + \angle BAC = \angle DAB$

$\angle BAC + \angle BAC = 60^\circ$

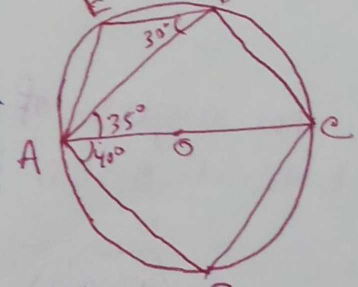
$2\angle BAC = 60^\circ$

$2x^\circ = 60^\circ$

$x^\circ = 30^\circ$

Thus, the angles of $x = 30^\circ$

(2) In the given figure, AC is the diameter of the circle with centre O. If $\angle ADE = 30^\circ$; $\angle DAC = 35^\circ$ and $\angle CAB = 40^\circ$ find (i) $\angle ACD$ (ii) $\angle ACB$, (iii) $\angle DAE$



$\Rightarrow$ (i) we know that

$\angle ADC = 90^\circ$ [$\because$ Angle is a semi-circle]

$\therefore \triangle ADC$,

$\angle DAC + \angle ACD + \angle ADC = 180^\circ$

$35^\circ + \angle ACD + 90^\circ = 180^\circ$

$\angle ACD = 180^\circ - 125^\circ$

$\angle ACD = 55^\circ$

(ii) $\angle ABC$ is a semi-circle.

then, $\angle ABC = 90^\circ$

Now, $\triangle ABC$,

$\angle ABC + \angle BCA + \angle CAB = 180^\circ$

$90^\circ + \angle BCA + 40^\circ = 180^\circ$

$\angle ACB = 180^\circ - 130^\circ$

$\angle ACB = 50^\circ$

(iii) ACDE is a quadrilateral.

then, ~~opposite angles~~ sum of opposit side angle is 180°.

$\therefore \angle ACD + \angle AED = 180^\circ$

$55^\circ + \angle AED = 180^\circ$ [by (i)]

$\angle AED = 180^\circ - 55^\circ$

$\angle AED = 125^\circ$

Now, $\triangle ADE$,

$\angle AED + \angle EDA + \angle EAD = 180^\circ$

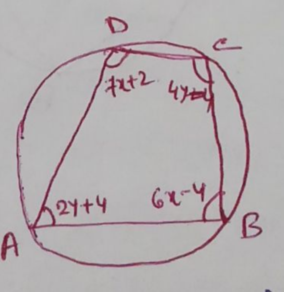
$125^\circ + 30^\circ + \angle DAE = 180^\circ$

$\angle DAE = 180 - 155^\circ$

$\angle DAE = 25^\circ$

$\therefore \angle DAE = 25^\circ$.

(3) Find all the angles of the given cyclic quadrilateral ABCD is the figure.



⇒ Given that ABCD is a cyclic quadrilateral.

thus, sum of the opposite angles of a cyclic quadrilateral is 180°.

Now, given,

$\angle A = 2y + 4$, $\angle B = 6x - 4$

$\angle C = 4y - 4$, $\angle D = 7x + 2$.

then,

$\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

$2y + \cancel{4} + 4y - \cancel{4} = 180°$

$6y = 180°$

$y = 30°$

$6x - 4 + 7x + 2 = 180°$

$13x - 2 = 180°$

$13x = 182°$

$x = \frac{182}{13}$

$x = 14°$

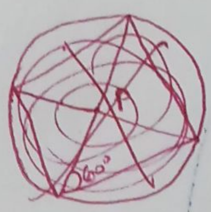
Now, $\angle A = 2y + 4 = 2 \times 30 + 4 = 64°$

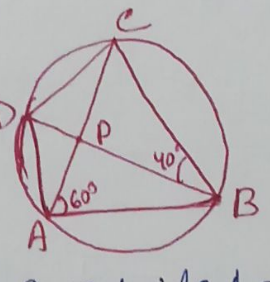
$\angle B = 6x - 4 = 6 \times 14 - 4 = 80°$

$\angle C = 4y - 4 = 4 \times 30 - 4 = 116°$

$\angle D = 7x + 2 = 7 \times 14° + 2 = 100°$

(4) In the given figure, ABCD is a cyclic quadrilateral where diagonals intersect at P such that $\angle DBC = 40°$ and $\angle BAC = 60°$ find (i) $\angle CAD$ (ii) $\angle BCD$





We know that cyclic quadrilateral angles in the same segment are equal.

then,

$\angle BDC = \angle BAC = 60°$ and $\angle CAD = \angle CBD = 40°$

(i) $\angle CAD = \angle CBD = 40°$

(ii) $\triangle BDC$, $\angle DBC + \angle BCD + \angle CDB = 180°$

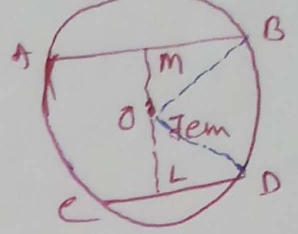
$40° + \angle BCD + 60° = 180°$

$\angle BCD = 180° - 100°$

$\angle BCD = 80°$.

$\therefore \angle BCD = 80°$.

(5) In the given figure, AB and CD are the parallel chords of a circle with centre O, such that AB = 8 cm and CD = 6 cm. If OM ⊥ AB and OL ⊥ CD distance between LM is 7 cm, find the radius of the circle?



⇒ Let the radius = r cm.

Now, OB = OD = r cm.

~~Now, △~~ BM = 4 cm and LD = 3 cm.

Let OM = x and LO = (7 − x) cm

△OBM,

$$(OB)^2 = (OM)^2 + (BM)^2$$

$$(OM)^2 = \cancel{(BM)^2} \; (OB)^2 - (BM)^2$$

$$(OM)^2 = r^2 - (4)^2$$

$$\cancel{(OM) = \sqrt{r^2 - 16} \cdots (i)}$$

$$(x)^2 = (r)^2 - 16 \Rightarrow (r)^2 = (x)^2 + 16 \cdots (i)$$

△ODL,

$$(OL)^2 = (OD)^2 - (LD)^2$$

$$(OL)^2 = r^2 - (3)^2$$

$$\cancel{(OL) = \sqrt{r^2 - 9} \cdots (ii)}$$

$$(7 - x)^2 = (x)^2 + 16 - 9 \quad [\text{by equation (i)}]$$

$$49 - 14x + \cancel{x^2} = \cancel{x^2} + 7$$

$$14x = 49 - 7$$

$$14x = 42$$

$$x = 3 \text{ cm.}$$

∴ from (i), putting x = 3 cm,

$$(r)^2 = (3)^2 + 16$$

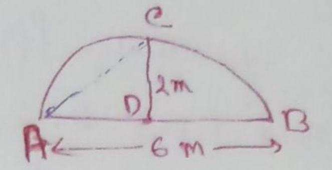
$$(r)^2 = 9 + 16 = 25$$

$$r^2 = (5)^2$$

$$r = 5.$$

Thus, the radius, 5 cm.

(6) The arch of a bridge has dimensions as shown, where the arch measure 2m at its highest point and its width is 6 cm. What is the radius of the circle that contains the arch?



⇒ $AB = AD + DB$ and $AD = DB$

$\therefore AD = DB = 3m.$

and $DC = 2m.$

$\triangle ADC,$

$(AD)^2 + (DC)^2 = (AC)^2$

$(3)^2 + (2)^2 = (AC)^2$

$(AC)^2 = 9 + 4$

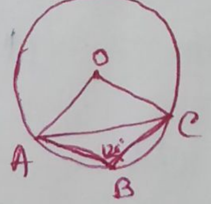
$(AC)^2 = 13$

$AC = \sqrt{13}$

$AC = 3 \cdot 6\ m$

Thus, the required length is 3·6 m.

(7) In figure, $\angle ABC = 120°$, where A, B, and C are points on the circle with centre O. Find $\angle OAC$?



⇒

We know that reflex $\angle AOC = 2\angle ABC$

reflex $\angle AOC = 2 \times 120°$

$360° - \angle AOC = 240°$

$\angle AOC = 360° - 240° = 120°$

Now, $OA = OC =$ radius.

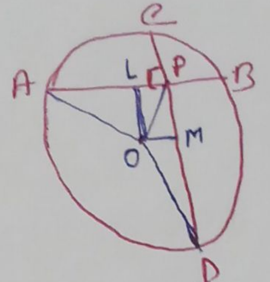
then, $\angle OAC = \angle OCA$.

$\therefore \angle OAC + \angle OCA = 180° - 120° = 60°$ [$\because \triangle AOC$]

$\therefore 2\angle OAC = 60°$

$\therefore \angle OAC = 30°.$

(8) A School wants to conduct tree plantation programme. For this a teacher allotted a circle of radius 6m ground to nineth standard students for planting sapplings. Four students plant trees at the point A, B, C, and D as shown in figure. Here AB = 8cm, CD = 10m and AB ⊥ CD. If another student places a flower pot at the point P, the intersection of AB and CD, then find the distance from the centre to P.



⇒ $OA = OD = 6\,m$

$AB = 8cm.$

$\therefore AL = LB = 4\,cm.$

$\Delta AOL,$

$(AO)^2 = (AL)^2 + (OL)^2$

$(6)^2 = (4)^2 + (OL)^2$

$(OL)^2 = 36 - 16$

$(OL)^2 = 20$

$OL = \sqrt{20}$

$CD = 10cm.$

$CM = MD = 5cm.$

$\Delta ODM,$

$(OD)^2 = (OM)^2 + (MD)^2$

$(6)^2 = (OM)^2 + (5)^2$

$(OM)^2 = 36 - 25$

$(OM)^2 = 11$

$OM = \sqrt{11}$

Now, OMLP is a rectangle.

$\therefore LP = OM = \sqrt{11}$ and $OL = PM = \sqrt{20}.$

$\therefore \Delta LOP,$

$(OP)^2 = (OL)^2 + (LP)^2$

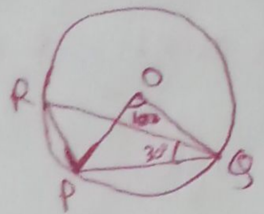
$(OP)^2 = (\sqrt{20})^2 + (\sqrt{11})^2$

$(OP)^2 = 20 + 11 = 31$

$OP = \sqrt{31} = 5.56 = 5.6\,m.$

$\therefore$ Thus, the required distance is 5.6 m.

(9) In the given figure, $\angle POQ = 100^\circ$ and $\angle PQR = 30^\circ$, then find $\angle RPO$.



$\Rightarrow$ we know that

$$\angle PRQ = \frac{1}{2}\angle POQ$$

$$\angle PRQ = \frac{1}{2} \times 100 = 50^\circ.$$

Now, $OP = OQ$ = radius.

then, $\angle OPQ = \angle OQP$.

Now, $\triangle POQ$,

$$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$$

$$100^\circ + 2\angle OPQ = 180^\circ$$

$$2\angle OPQ = 180^\circ - 100^\circ = 80^\circ$$

$$\angle OPQ = 40^\circ$$

Now, $\angle RPQ = \angle RPO + \angle OPQ$

$$= \angle RPO + 40^\circ$$

Now, $\triangle PRQ$,

$$\angle PRQ + \angle RPQ + \angle PQR = 180^\circ$$

$$50^\circ + \angle RPQ + 30^\circ = 180^\circ$$

$$\angle RPQ + 80^\circ = 180^\circ$$

$$\angle RPO + 40^\circ + 80^\circ = 180^\circ$$

$$\angle RPO + 120^\circ = 180^\circ$$

$$\angle RPO = 180^\circ - 120^\circ$$

$$\angle RPO = 60^\circ$$