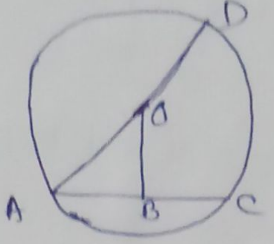
Exercise - 4.3

1. The diameter of the circle is 52 cm and the length of one of its chord is 20 cm. Find the distance of the chord from the centre.

⇒



Given, AD = 52 cm.

∴ AO = ~~29~~ 26 cm.

and AC = 20 cm

∴ AB = 10 cm.

OB = the distance of the chord from the centre.

now, △ABO,

$(AB)^2 + (OB)^2 = (AO)^2$

$(10)^2 + (OB)^2 = (26)^2$

$(OB)^2 = 676 - 100$

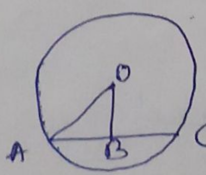
$(OB)^2 = 576$

OB = ~~18~~ 24

Thus, the distance of the chord from the centre is 24 cm.

2. The chord of length 30 cm is drawn at the distance of 8 cm from the centree of the circle. Find the radius of the circle.

⇒



Given, AC = 30 cm

AB = 15 cm

OB = 8 cm.

now, △AOB, $(AB)^2 + (OB)^2 = (AO)^2$

$(AO)^2 = (15)^2 + (8)^2$

$(AO)^2 = 225 + 64$

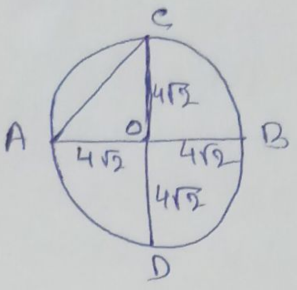
$(AO)^2 = 289$

AO = 17

Therefore, the circle radius 17 cm.

(3) Find the length of the chord AC where AB and CD are two diameters perpendicular to each other of a circle with radius $4\sqrt{2}$ cm and also find $\angle OAC$ and $\angle OCA$.

⇒



NOW, $AO = 4\sqrt{2}$ and $OC = 4\sqrt{2}$

then, ΔAOC, $\angle O = 90^\circ$

$$(OC)^2 + (OA)^2 = (AC)^2$$

$$(AC)^2 = (4\sqrt{2})^2 + 4\sqrt{2}$$

$$(AC)^2 = 32 + 32$$

$$(AC)^2 = 64$$

$$AC = 8$$

NOW, $OC = OA$ = radius, and $\angle AOC = 90^\circ$

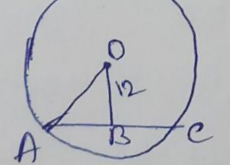
$\therefore \angle OAC = \angle OCA$.

$$\therefore \angle OAC = \angle OCA = \frac{1}{2}(180^\circ - 90^\circ) = \frac{90^\circ}{2} = 45^\circ.$$

Thus, $\angle OAC = \angle OCA = 45^\circ$.

(4) A chord is 12 cm away from the centre of the circle of radius 15 cm. Find the length of the chord.

⇒



OB = 12 cm

OA = radius = 15 cm.

NOW, ΔAOB, $(AB)^2 + (OB)^2 = (OA)^2$

$$(AB)^2 + (12)^2 = (15)^2$$

$$(AB)^2 = 225 - 144 = 81$$

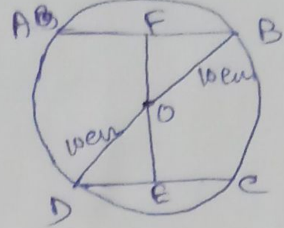
$$AB = 9 \text{ cm}.$$

$\therefore AC = 9 \times 2$ cm $= 18$ cm.

Thus the length of the chord is 18 cm.

(5) In a circle, AB and CD are two parallel chords with centre O and radius 10 cm such that AB = 16 cm, and CD = 12 cm. determine the distance between the two chords?

⇒



AB = 16 cm
BF = 8 cm.
OB = OD = 10 cm.
CD = 12 cm
DE = 6 cm.

In $\triangle OBF$

$(OB)^2 = (OF)^2 + (BF)^2$

$(10)^2 = (OF)^2 + (8)^2$

$(OF)^2 = 100 - 64$

$(OF)^2 = 36$

$OF = 6$

and $\triangle ODE$.

$(OD)^2 = (OE)^2 + (DE)^2$

$(10)^2 = (OE)^2 + (6)^2$

$(OE)^2 = 100 - 36 = 64$

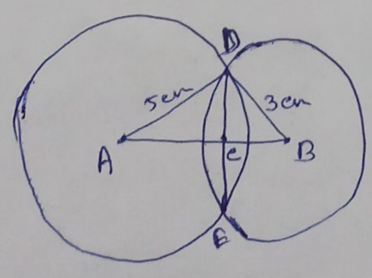
$(OE) = 8$

∴ Now, EF is the distane between the two chords.

$\therefore EF = EO + OF$

$= 6 + 8 = 14$ cm.

(6) Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.



AD = 5 cm
BD = 3 cm
AB = 4 cm
Let AC = x cm
CB = (4 − x) cm

$\triangle ACD$,

$(AC)^2 + (DC)^2 = (AD)^2$

$(DC)^2 = (5)^2 - (x)^2$

$(DC)^2 = 25 - x^2$ ---(i)

ΔDCB,

$(DC)^2 + (CB)^2 = (BD)^2$

$(DC)^2 + (4-x)^2 = (3)^2$

$25 - \cancel{x^2} + 16 - 8x + \cancel{x^2} = 9$ [by equation (i)]

$25 + 16 - 9 = 8x$

$25 + 7 = 8x$

$32 = 8x$

$8x = 32$

$x = 4$

Now, by equation (i), $x = 4$ putting,

$(DC)^2 = 25 - (4)^2$

$(DC)^2 = 25 - 16 = 9$

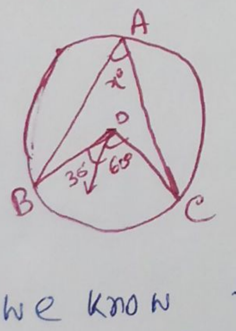
$DC = 3$

then, $DE = 2 \times DC = 2 \times 3 = 6$ cm.

common chord $= 6$ cm.

7. Find the value of x° in the following figures:-

(i)



$\Rightarrow$ we know that

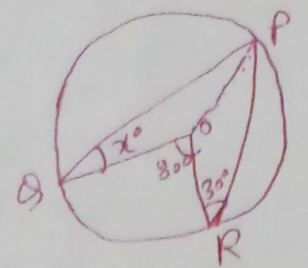
$\angle BOC = 2\angle BAC$

$\angle 90^\circ = 2\angle BAC$ [$\because \angle BOC = 30^\circ + 60^\circ = 90^\circ$]

$\angle BAC = \frac{90^\circ}{2} = 45^\circ$

$x^\circ = 45^\circ$

(ii)



we know, that $\angle QOR = 2\angle QPR$

$80^\circ = 2\angle QPR$

$\angle QPR = 40^\circ$

$\because OR = OQ = OP.$

Now, $\angle ORP = \angle OPR$

$\therefore \angle OPR = 30^\circ,$

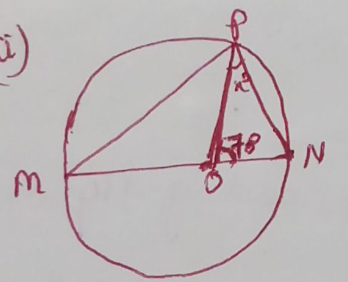
then, $\angle QOP = 40^\circ - 30^\circ = 10^\circ$

Now, Equal sides ~~with~~ equal angles.

$\angle OQP = \angle OPQ = 10^\circ$

Thus, $x^\circ = 10^\circ$

(iii)



$\Rightarrow ON = OP =$ radius.

$\therefore \angle ONP = \angle OPN = x^\circ$

$\therefore \triangle OPN,$

$x^\circ + 70^\circ + x^\circ = 180^\circ$

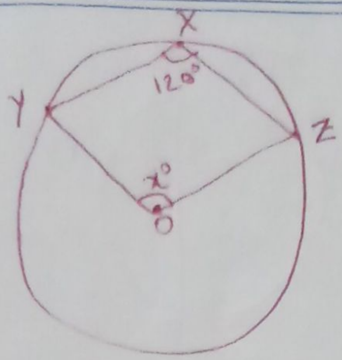
$2x^\circ = 180^\circ - 70^\circ$

$2x^\circ = 110^\circ$

$x^\circ = 55^\circ$

Thus the value of $x^\circ = 55^\circ.$

(iv)



$\Rightarrow \angle YXZ = \frac{1}{2}$ (reflex $\angle YOZ$)

$120^\circ = \frac{1}{2}(360^\circ - x^\circ)$

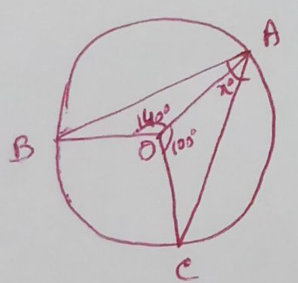
$240^\circ = 360^\circ - x^\circ$

$x^\circ = 360^\circ - 240^\circ$

$x^\circ = 120^\circ$

Thus, the value of $x^\circ = 120^\circ$.

(v)



$\Rightarrow$ Now, $OB = OC = OA$ = radius.

$\angle OBA = \angle OAB$ and $\angle OCA = \angle OAC$

ΔOBA, $\therefore \angle OAB = \frac{1}{2}(180^\circ - 140^\circ)$

$\angle OAB = \frac{1}{2} \times 40^\circ$

$\angle OAB = 20^\circ$ --- (i)

and ΔOCA, $\angle OAC = \frac{1}{2}(180^\circ - 100)$

$\angle OAC = \frac{1}{2} \times 80^\circ = 40^\circ$ --- (ii)

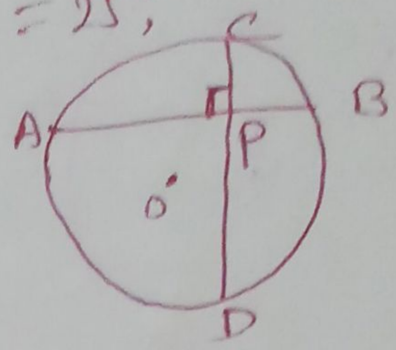
Now, $\angle BAC = \angle OAB + \angle OAC$

$= 20^\circ + 40^\circ$ [by equation (i) and (ii)]

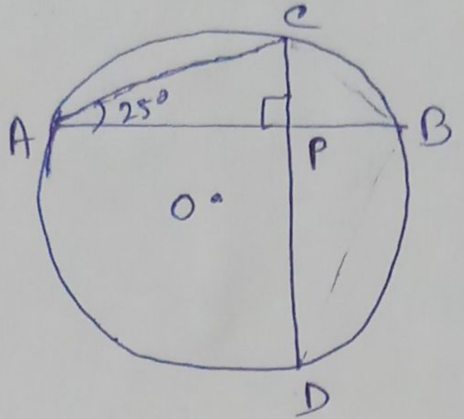
$= 60^\circ$

$\therefore \angle BAC = x^\circ = 60^\circ$.

(8) In the given figure, ∠CAB = 25°, find ∠BDC, ∠DBA and ∠COB



⇒



ΔAPC, ∠ACP

$= 180° - (∠CAD + ∠CPA)$

$= 180° - (25° + 90°)$

$= 180° - 115°$

$= 65°$

Now, ∠BAC = ∠BDC

∠BDC = 25°

and ∠PCA = ∠DBA

∠DBA = 65°

Now, ∠COB = 2∠DAC

∠COB = 2 × 25°

= 50°.