Exercise - 4.2

① The angles of a quadrilateral are in the ratio 2 : 4 : 5 : 7. Find all the angles.

⇒ Let the quadrilateral angles are $2x, 4x, 5x$ and $7x$.

we know that sum of angles of a quadrilateral is 360°.

then, $2x + 4x + 5x + 7x = 360$

$$18x = 360$$

$$x = \frac{360}{18}$$

$$x = 20$$

then, the quadrilateral angles are 40°, 80°, 100° and 140°.

② In a quadrilateral ABCD, ∠A = 72° and ∠C is the supplementary of ∠A. The other two angles are $2x - 10$ and $x + 4$. Find the value of x and the measure of all the angles.

⇒ given that ∠A = 72°

then, ∠C is the supplementary of ∠A.

$$\therefore \angle C + \angle A = 180°$$

$$\angle C + 72° = 180°$$

$$\angle C = 180 - 72$$

$$\angle C = 108°$$

we know that sum of angles of a quadrilateral is 360°.

then, $72^\circ + 108^\circ + 2x - 10 + x + 4 = 360^\circ$

$180^\circ + 3x - 6 = 360$

$3x - 6 = 180^\circ$

$3x = 180 + 6$

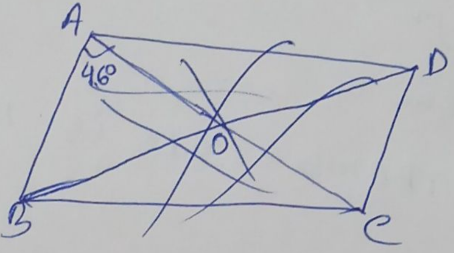
$3x = 186$

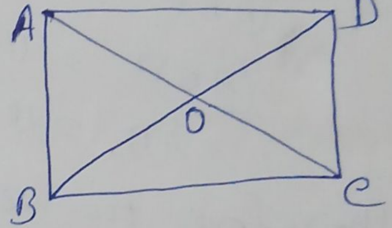
$x = 62$

Thus, the quadrilateral angles are, 72°, 108°, 114° and 66°.

(3) ABCD is a rectangle whose diagonals AC and BD intersect at O. If $\angle OAB = 46^\circ$, find $\angle OBC$.

=>





given, $\angle OAB = 46^\circ$

we know that $OA = OB$

then, $\angle OBA = 46^\circ$

we know that rectangle every angle is 90°.

Now, $\angle ABC = 90^\circ$

$\angle OBA + \angle OBC = 90^\circ$

$46^\circ + \angle OBC = 90^\circ$

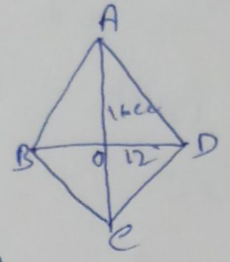
$\angle OBC = 90^\circ - 46^\circ$

$\angle OBC = 44^\circ$

Thus, the angle $\angle OBC = 44^\circ$.

(4) The lengths of the diagonals of a Rhombus are 12 cm and 16 cm. Find the side of the rhombus.

⇒



Given that

AC = 16 cm

and BD = 12 cm.

Now, $OA = \frac{16}{2} = 8$ cm

and $OB = \frac{12}{2} = 6$ cm.

Now, △ABO.

$(AO)^2 + (OB)^2 = (AB)^2$

$(8)^2 + (6)^2 = (AB)^2$

$(AB)^2 = 64 + 36$

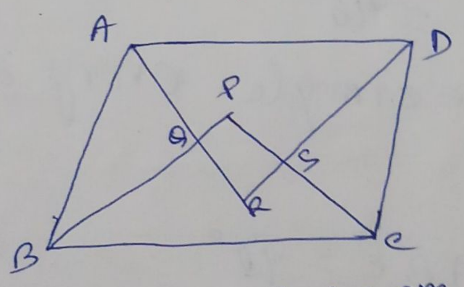
$(AB)^2 = (10)^2$

$AB = 10$ cm

Therefore the ~~ro~~ rhombus side is 10 cm.

(5) Show that the bisectors of angles of a parallelogram from a rectangle.

⇒



Now, ABCD is a parallelogram AR, BP, CP, DR are bisectors of angles are ∠A, ∠B, ∠C, ∠D.

Now, AD ∥ BC and AB is transversal.

we know that interior angles on the same side of transversal are supplementary.

$\therefore \angle A + \angle B = 180°$

$\frac{\angle A}{2} + \frac{\angle B}{2} = \frac{180°}{2}$

$\angle QAB + \angle QBA = 90^\circ$ --- (i)

Now, ΔAQB;

$\angle QAB + \angle QBA + \angle AQB = 180^\circ$

$\angle QAB + \angle QBA = 180^\circ$ [by equation (i)]

$90^\circ + \angle AQB = 180^\circ$

$\angle AQB = 90^\circ$

Now, AR and BP intersect Q point.

then, $\angle AQB = \angle PQR$

$\therefore \angle PQR = 90^\circ$

Similary, we can prove that

$\angle PSR = 90^\circ, \angle QPS = 90^\circ, \angle QRS = 90^\circ$.

Now, $\angle PQR = \angle PSR = 90^\circ$

and $\angle QPS = \angle QRS = 90^\circ$

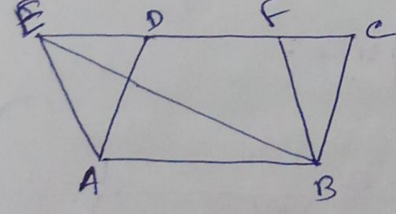
So, PQRS is a parallelogram.

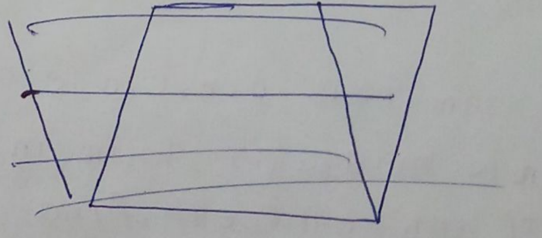
then, $\angle PQR = \angle PSR = \angle QPS = \angle QRS = 90^\circ$

Therefore PQRS is a rectangle. [proved]

(6) If a triangle and a parallelogram lie on the same base and between the same parallels, then prove that the area of the triangle is equal to half of the area of parallelogram.

⇒





A parallelogram ABCD and a triangle ~~ABD~~ ABE on the base same AB.

$AB \parallel DC$

then $EC \parallel AB$.

Now, draw a line ~~BF~~ BF Parallel to AE.

$\therefore$ BF || AE.

Now, AB || EF and BF || AE

then, ABEF is a parallelogram.

then, ~~parall~~
~~□ABEF = □~~

then parallelogram ABCD and parallelogram ABEF on the same base AB and same parallel lines AB and Ee.

$\therefore$ Area (ABEF) = Area (ABCD) - - -(i)

Now, parallelogram ABEF.

BE is the diagonal

So, $\triangle ABE \cong \triangle BEF$

then Area ($\triangle$ABE) = Area ($\triangle$BEF)

Now, Area ($\triangle$ABE) = Area ($\triangle$BEF)

= $\frac{1}{2}$ Area (ABEF)

$\therefore$ Area ($\triangle$ABE) = $\frac{1}{2}$ Area (ABEF)

Area ($\triangle$ABE) = $\frac{1}{2}$ Area (ABCD) [by equation(i)]

[proved].

⑦ Iron rods a, b, c, d, e, and f are making a design in a bridge as shown in the figure.

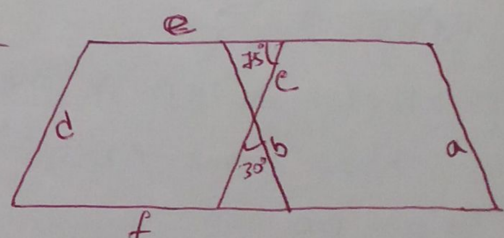
If a||b, c||d, e||f, find the marked angles between

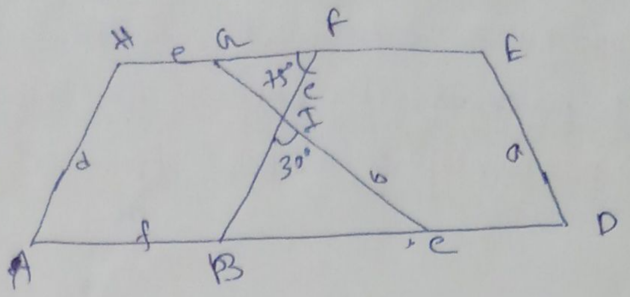
(i) b and c

(ii) d and e

(iii) d and f

(iv) c and f





now, AD || HG.

∠BFG = 75° then ∠BFG = ∠FBC

∴ then ∠FBC = 75°.

Now, ∠BIC = 30°

then, ∠BIC = ∠GIF [∵ vertically opposite angles]

∴ ∠GIF = 30°.

(i) b and c makes angles = 30°.

(ii) AB || HF and AH || BF then, ∴ ABFH is a parralelogram.

sum of the adjacent angles of a parrallelogram is 180°

∴ d and e makes angle = ~~180° - 75°~~ ∠AHF.

then, ∠AHF + ∠BFH = 180°

∠AHF + 75° = 180°

∠AHF = 180° - 75° = 105°

∴ d and e makes angles = 105°

(iii) similarely, ∠HAB + ∠AHF = 180°

∠HAB + 105° = 180°

∠HAB = 108° - 105° = 75°

then, d and f makes angle = ∠HAB = 75°

(iv) similarly, ∠HFB + ∠ABF = 180°

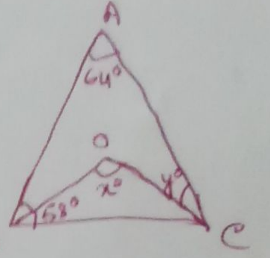
75° + ∠ABF = 180°

∠ABF = 180° - 75°

∠ABF = 105°

Thus, e and f makes angles = ∠ABF = 105°.

(8) In the given figure, $\angle A = 64^\circ$, $\angle ABC = 58^\circ$. If BO and CO are the bisectors of $\angle ABC$ and $\angle ACB$ respectively of $\triangle ABC$, find x° and y°.



⇒ given figure,

$\angle A = 64^\circ$ and $\angle ABC = 58^\circ$

then, $\angle BOC = \frac{1}{2}\angle ABC$ [∵ BO bisectors of $\angle ABC$]

$\therefore \angle BOC = \frac{1}{2} \times 58^\circ = 29^\circ$

Now, $\triangle ABC$,

$\angle A + \angle B + \angle C = 180^\circ$

$64^\circ + 58^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 122^\circ$

$\angle C = 58^\circ$

then, $\angle OCB = \frac{1}{2}\angle C$ [∵ CO bisectors of $\angle C$]

$\angle OCB = \angle OCA = \frac{1}{2} \times 58^\circ = 29^\circ$

$\angle OCA = y^\circ = 29^\circ$.

Now, $\triangle BOC$,

$\angle OBC + \angle x^\circ + \angle OCB = 180^\circ$

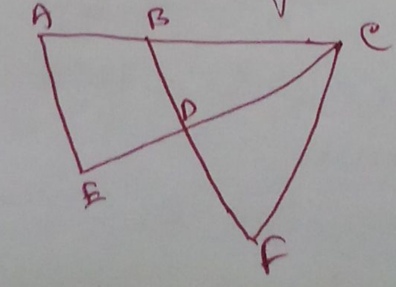
$29^\circ + \angle x^\circ + 29^\circ = 180^\circ$

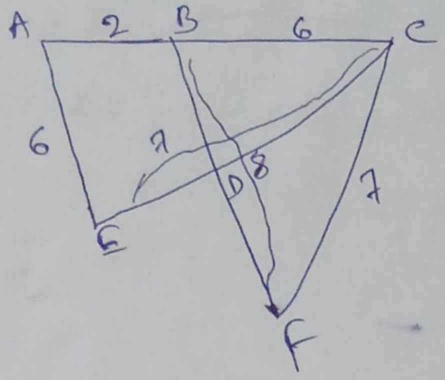
$\angle x^\circ = 180^\circ - 58^\circ$

$\angle x^\circ = 122^\circ$

Thus, angles $\angle x^\circ = 122^\circ$ and $\angle y^\circ = 29^\circ$.

(9) In the given figure, if AB = 2, BC = 6, AE = 6, BF = 8, CE = 7, and CF = 7, compute the ratio of the area of quadrilateral ABDE to the area of $\triangle CDF$. (Use congruent property of triangles).





NOW, $\triangle AEC$ and $\triangle BCF$

$AC = AB + BC = 2 + 6 = 8$

$\therefore AC = 8$ BF.

$AE = 6 = BC$

$EC = 7 = CF$

$\therefore \triangle AEC \cong \triangle BCF$.

then, Area ($\triangle AEC$) = Area ($\triangle BCF$).

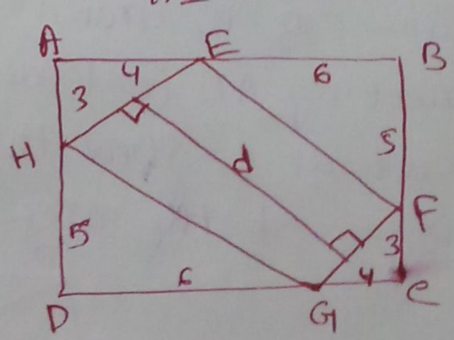
then, Area($\triangle AEC$) − Area($\triangle BDC$) = Area($\triangle BCF$) − Area($\triangle BDC$)

Area (AEDB) = Area ($\triangle CDF$)

$$\frac{\text{Area (AEDB)}}{\text{Area } (\triangle CDF)} = \frac{1}{1}$$

Thus, the required ratio is 1 : 1.

(10) In the figure, ABCD is a rectangle and EFGH is a parallelogram. Using the measurments given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?



⇒ given ABCD rectangle.

Area = (6 + 4) × (3 + 5)

= 10 × 8

= 80

Now, $\triangle AHE$ then,

$(AH)^2 + (AE)^2 = (HE)^2$

$(HE)^2 = (4)^2 + (3)^2 = 16 + 9 = 25 = (5)^2$

$\therefore HE = 5$.

Now, Area of ΔAEH + Area of ΔBEF + Area of ΔFCG + Area of ΔHDG

$= \frac{1}{2} \times 3 \times 4 + \frac{1}{2} \times 6 \times 5 + \frac{1}{2} \times 3 \times 4 + \frac{1}{2} \times 6 \times 5$

$= 6 + 15 + 6 + 15$

$= 30 + 12$

$= 42.$

Now, Area of ~~parralegram~~ parallelgram EFGH

= Area of rectangle ABCD − 4 triangle area

$= 80 - 42$

$= 38.$

Now, ~~(or) Length x britta~~

HE × EF = area of ~~parralegram~~ parallelogram EFGH

$5 \times EF = 38$

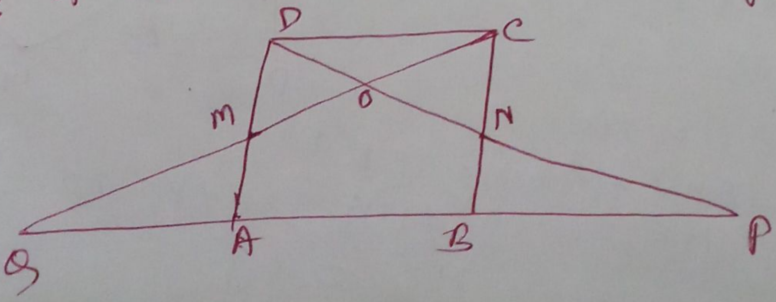
$EF = \frac{38}{5}$

$d = \frac{38}{5}$ [$\because EF = d$]

$d = 7.6$

Thus, the lenght of $d = 7.6$.

(11) In parallelogram ABCD of the accompanying diagram, line DP is drawn bisecting BC at N and meeting AB (extended) at P. From vertex C, line CQ is drawn bisecting side AD at M and meeting AB (extended) at Q. Lines DP and CQ meet at O. Show that the area of triangle QPO is $\frac{9}{8}$ of the area of the parallelogram ABCD.



⇒ Now, area of Δ OQP = Area of Δ QAM + Area of Δ BNP
+ Area of AMONB . . . (i)

Now, Δ AQM $\cong$ Δ MDE,
then, area of Δ AQM = area of Δ MDE . . . (i)

and Δ DCN $\cong$ Δ BNP,
then, area of Δ DCN = area of Δ BNP . . . (ii)

Now, area of Δ OQP
= Area of Δ MDE + area of Δ DCN + area of AMONB [by equation (i) and (ii)]
= Area of Δ MDE + area of Δ DOC + Area of Δ ONE + Area of Δ DOC
= Area of Δ MOD + area of Δ DOC + Area of Δ ONE +
+ Area of AMONB.
= Area of Δ MOD + Area of Δ DOE + Area of Δ NOC + Area of
AMONB + Area of Δ DOC
= Area of ABCD + Area of Δ DOE
= Area of ABCD + $\frac{1}{4}$ Area of MNCD [$\because$ MNDE is a parallelogram]
= Area of ABCD + $\frac{1}{4} \times \frac{1}{2}$ Area of ABCD [$\because$ ABCD = 2MNCD]
= Area of ABCD + $\frac{1}{8}$ Area of ABCD
= $\left(1 + \frac{1}{8}\right)$ × Area of ABCD
= $\frac{8+1}{8}$ Area of ABCD
= $\frac{9}{8}$ Area of ABCD

Thus, the area of triangle QPO
= $\frac{9}{8}$ of the area of the parallelogram ABCD.

[proved].