(i) $(4x^3+6x^2-23x+18) \div (x+3)$

$$
\begin{array}{r|l}
 & 4x^2-6x-5 \\
\hline
x+3 & 4x^3+6x^2-23x+18 \\
 & 4x^3+12x^2 \\
 & (-)\quad(-) \\
\hline
 & -6x^2-23x+18 \\
 & -6x^2-18x \\
 & (+)\quad(+) \\
\hline
 & -5x+18 \\
 & -5x-15 \\
 & (+)\quad(+) \\
\hline
 & 33
\end{array}
$$

$\therefore$ quotient is $4x^2-6x-5$

and remainder is 33

(ii) $(8y^3-16y^2+16y-15) \div (2y-1)$

$$
\begin{array}{r|l}
 & 4y^2-6y+5 \\
\hline
2y-1 & 8y^3-16y^2+16y-15 \\
 & 8y^3-4y^2 \\
 & (-)\quad(+) \\
\hline
 & -12y^2+16y-15 \\
 & -12y^2+6y \\
 & (+)\quad(-) \\
\hline
 & 10y-15 \\
 & 10y-5 \\
 & (-)\quad(+) \\
\hline
 & -10
\end{array}
$$

$\therefore$ quotient is $4y^2-6y+5$ and remainder is -10.

(iii) $(8x^3-1) \div (2x-1)$

$$
\begin{array}{r|l}
 & 4x^2+2x+1 \\
\hline
(2x-1) & 8x^3-1 \\
 & 8x^3-4x^2 \\
 & (-)\quad(+) \\
\hline
 & +4x^2-1 \\
 & +4x^2-2x \\
 & (-)\quad(+) \\
\hline
 & 2x-1 \\
 & 2x-1 \\
\hline
 & 0
\end{array}
$$

$\therefore$ quotient is $4x^2+2x+1$ and remainder is 0

(iv) $(-18z + 14z^2 + 24z^3 + 18) \div (3z + 4)$

$$
\begin{array}{r|l}
 & 8z^2 - 6z + 2 \\
\hline
3z + 4 & 24z^3 + 14z^2 - 18z + 18 \\
 & 24z^3 + 32z^2 \\
 & (-) \quad (-) \\
\hline
 & -18z^2 - 18z + 18 \\
 & -18z^2 - 24z \\
 & (+) \quad (+) \\
\hline
 & 6z + 18 \\
 & 6z + 8 \\
 & (-) \quad (-) \\
\hline
 & 10
\end{array}
$$

$\therefore$ The quotient is $8z^2 - 6z + 2$ and remainder is 10.

(2) The area of a rectangle is $x^2 + 7x + 12$. if its breadth is $(x+3)$, then find its length.

$\Rightarrow$ Area $= x^2 + 7x + 12$

breadth $= x + 3$

Now, Area $=$ breadth $\times$ length

length $=$ (Area) $\div$ breadth.

$\therefore (x^2 + 7x + 12) \div (x + 3)$

$$
\begin{array}{r|l}
 & x + 4 \\
\hline
x + 3 & x^2 + 7x + 12 \\
 & x^2 + 3x \\
 & (-) \quad (-) \\
\hline
 & 4x + 12 \\
 & 4x + 12 \\
 & (-) \quad (-) \\
\hline
 & 0
\end{array}
$$

$\therefore$ length $= x + 4$.

Thus, the rectangle length is $(x+4)$.

(3) The base of a parallelogram is $(5x+4)$. Find its height, if the area is $25x^2-16$.

$\Rightarrow$ Area $= 25x^2 - 16$

$= (5x)^2 - (4)^2$

$= (5x+4)(5x-4)$

base $= 5x+4$.

Area $=$ base $\times$ height

height $= \dfrac{\text{Area}}{\text{base}}$

height $= \dfrac{(5x+4)(5x-4)}{5x+4}$

height $= (5x-4)$.

Thus, the parallelogram height is $(5x-4)$

(4) The sum of $(x+5)$ observations is (x^3+125). Find the mean of the observations.

$\Rightarrow$ The sum $= x^3 + 125$

$= (x)^3 + (5)^3$

$= (x+5)(x^2-5x+25)$

mean $= \dfrac{\text{sum of number}}{\text{number}}$

$= \dfrac{x^3+125}{x+5}$

$= \dfrac{(x+5)(x^2-5x+25)}{(x+5)}$

$= x^2 - 5x + 25$

Thus, the mean is $x^2 - 5x + 25$.

(5) Find the quotient and remainder for the following using synthetic division:

(i) $(x^3+x^2-7x-3)\div(x-3)$ (ii) $(x^3+2x^2-x-4)\div(x+2)$

(iii) $(3x^3-2x^2+7x-5)\div(x+3)$ (iv) $(8x^4-2x^2+6x+5)\div(4x+1)$

⇒ (i) $(x^3+x^2-7x-3)\div(x-3)$

x^3+x^2-7x-3

$x-3$

Now, $x-3=0$

$x=3$

$$\begin{array}{r|rrrr} 3 & 1 & 1 & -7 & -3 \\ & 0 & 3 & 12 & 15 \\ \hline & 1 & 4 & 5 & 12 \end{array}$$

The quotient is x^2+4x+5 and remainder is 12

(ii) $(x^3+2x^2-x-4)\div(x+2)$

x^3+2x^2-x-4

$x+2$

Now, $x+2=0$

$x=-2$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -1 & -4 \\ & 0 & -2 & 0 & 2 \\ \hline & 1 & 0 & -1 & -2 \end{array}$$

Thus, the quotient is $x-1$ and remainder ~~e~~ is -2.

(iii) $(3x^3-2x^2+7x-5)\div(x+3)$

$3x^3-2x^2+7x-5$

$x+3$

Now, $x+3=0$

$x=-3$

$$\begin{array}{r|rrrr} & 3 & -2 & 7 & -5 \\ -3 & 0 & -9 & 33 & -120 \\ \hline & 3 & -11 & 40 & -125 \end{array}$$

Thus, the quotient is $3x^2-11x+40$ and remainder is -125.

(iv) $(8x^4-2x^2+6x+5) \div (4x+1)$

$$\frac{8x^4+0\cdot x^3 - 2x^2+6x+5}{4x+1}$$

Now, $4x+1=0$

$4x=-1$

$x=-\frac{1}{4}$

$$\begin{array}{r|rrrrr} & 8 & 0 & -2 & 6 & 5 \\ -\frac{1}{4} & 0 & -2 & \frac{1}{2} & 3/8 & -\frac{51}{32} \\ \hline & 8 & -2 & -3/2 & \frac{51}{8} & \frac{109}{32} \end{array}$$

The quotient is $= 8x^3-2x^2-\frac{3}{2}x+\frac{51}{8}$

$= 2x^3-\frac{x^2}{2}-\frac{3}{8}x+\frac{51}{32}$

and remainder is $\frac{109}{32}$.

⑥ If the quotient obtained on dividing $(8x^4-2x^2+6x-7)$ by $(2x+1)$ is $(4x^3+px^2-qx+3)$, then find p, q and also the remainder.

⇒ We know that

Dividend = (Divisor × Quotient) + Remainder.

Let remainder is $g(x)$.

$8x^4-2x^2+6x-7 = (4x^3+px^2-qx+3)(2x+1)+g(x)$.

$\cancel{8x^4}-2x^2+6x-7 = \cancel{8x^4}+2px^3-2qx^2+6x$
$+4x^3+px^2-qx+3+g(x)$

$-2x^2+6x-7 = (2p+4)x^3+(p-2q)x^2+(6-q)x+3+g(x)$.

$-2x^2+6x-7 = (2p+4)x^3+(p-2q)x^2+(6-q)x+3+g(x)$.

compare both side, we get,

$2p+4=0$
$2p=-4$
$p=-2$

$(p-2q)=-2$
$-2-2q=-2$
$-2q=0$
$q=0$

$6=6-q$
$6-6=-q$
$q=0$

$-7=+3+g(x)$
$g(x)=-3-7$
$g(x)=-10$

Thus, the quotient is $=4x^3+px^2-qx+3$
$=4x^3-2x^2+3$

The value of $p=-2$ and $q=0$
and remainder is -10.

(7) If the quotient obtained on dividing $3x^3+11x^2+34x+106$ by $x-3$ is $3x^2+ax+b$, then find a, b and also the remainder.

=> Let remainder is $g(x)$.
Now, Dividend = (Divisor x quotient) + remainder.

$3x^3+11x^2+34x+106 = (3x^2+ax+b)(x-3)+g(x)$
$3x^3+11x^2+34x+106 = 3x^3+ax^2+bx-9x^2-3ax-3b+g(x)$
$3x^3+11x^2+34x+106 = 3x^3+(a-9)x^2+(b-3a)x-3b+g(x)$.

both side comparing, we get

$a-9=11$
$a=11+9$
$a=20$

$b-3a=34$
$b-3\times 20=34$
$b-60=34$
$b=94$

$-3b+g(x)=106$
$-3\times 94+g(x)=106$
$g(x)=106+282$
$g(x)=388$

The quotient is $3x^2+ax+b$
$=3x^2+20x+94$

The value of $a=20$ and $b=94$

∴ remainder is 388.