

// Exercise - 3.5 //

① Factorise the following expressions:

(i) $2a^2 + 4a^2b + 8a^2c$

(ii) $ab - ac - mb + mc$

⇒ (i) $2a^2 + 4a^2b + 8a^2c$

$$= 2a^2(1 + 2b + 4c)$$

$$= 2a^2(1 + 2b + 4c)$$

(ii) $ab - ac - mb + mc$

$$= ab - mb - ac + mc$$

$$= b(a - m) - c(a - m)$$

$$= (a - m)(b - c)$$

$$= (a - m)(b - c)$$

② Factorize the following:

(i) $x^2 + 4x + 4$

(ii) $3a^2 - 24ab + 48b^2$

(iii) $x^5 - 16x$

(iv) $m^2 + \frac{1}{m^2} - 23$

(v) $6 - 216x^2$

(vi) $a^2 + \frac{1}{a^2} - 18$

⇒ (i) $x^2 + 4x + 4$

$$= x^2 + 2x + 2x + 4$$

$$= x(x + 2) + 2(x + 2)$$

$$= (x + 2)(x + 2)$$

$$= (x + 2)^2$$

(ii) $3a^2 - 24ab + 48b^2$

~~$$= 3a^2 + 18ab - 8ab + 48b^2$$~~

$$= 3(a^2 - 8ab + 16b^2)$$

$$= 3\{a^2 - 2 \cdot a \cdot 4b + (4b)^2\}$$

$$= 3(a - 4b)^2$$

(iii) $x^5 - 16x$

$$= x(x^4 - 16)$$

$$= x((x^2)^2 - (4)^2)$$

$$= x(x^2 + 4)(x^2 - 4)$$

$$= x(x^2 + 4)((x)^2 - (2)^2)$$

$$= x(x^2 + 4)(x + 2)(x - 2)$$

$$= x(x + 2)(x - 2)(x^2 + 4)$$

(iv) $m^2 + \frac{1}{m^2} - 23$

$$= (m + \frac{1}{m})^2 - 2 \cdot m \cdot \frac{1}{m} - 23$$

$$= (m + \frac{1}{m})^2 - 2 - 23$$

$$= (m + \frac{1}{m})^2 - 25$$

$$= (m + \frac{1}{m})^2 - (5)^2$$

$$= (m + \frac{1}{m} + 5)(m + \frac{1}{m} - 5)$$

$$(v) 6 - 216x^2$$

$$= 6(1 - 36x^2)$$

$$= 6[(1)^2 - (6x)^2]$$

$$= 6(1+6x)(1-6x)$$

$$(vi) a^2 + \frac{1}{a^2} - 18$$

$$= (a + \frac{1}{a})^2 + 2a \cdot \frac{1}{a} - 18$$

$$= (a - \frac{1}{a})^2 - 16$$

$$= (a - \frac{1}{a})^2 - (4)^2$$

$$= (a - \frac{1}{a} + 4)(a - \frac{1}{a} - 4)$$

③ Factorize the following

$$(i) 4x^2 + 9y^2 + 25z^2 + 12xy + 30yz + 20xz$$

$$(ii) 25x^2 + 4y^2 + 9z^2 - 20xy + 12yz - 30xz$$

$$\Rightarrow (i) 4x^2 + 9y^2 + 25z^2 + 12xy + 30yz + 20xz$$

$$= (2x)^2 + (3y)^2 + (5z)^2 + 2 \cdot 2x \cdot 3y + 2 \cdot 3y \cdot 5z + 2 \cdot 5z \cdot 2x$$

$$= (2x + 3y + 5z)^2 \left[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \right]$$

$$= (2x + 3y + 5z)^2$$

$$(ii) 25x^2 + 4y^2 + 9z^2 - 20xy + 12yz - 30xz$$

$$= (5x)^2 + (-2y)^2 + (-3z)^2 + 2 \cdot 5x \cdot (-2y) + 2(-2y)(-3z) + 2 \cdot 5x \cdot (-3z)$$

$$= (5x - 2y - 3z)^2$$

④ Factorize the following:

$$(i) 8x^3 + 125y^3 \quad (ii) 27x^3 - 8y^3 \quad (iii) a^6 - 64$$

$$\Rightarrow (i) 8x^3 + 125y^3$$

$$= (2x)^3 + (5y)^3$$

$$= (2x + 5y) \left((2x)^2 - 2x \cdot 5y + (5y)^2 \right)$$

$$= (2x + 5y) (4x^2 - 10xy + 25y^2)$$

$$(ii) 27x^3 - 8y^3$$

$$= (3x)^3 - (2y)^3$$

$$= (3x - 2y) \left((3x)^2 + 3x \cdot 2y + (2y)^2 \right)$$

$$= (3x - 2y) (9x^2 + 6xy + 4y^2)$$

$$(ii) a^6 - 64$$

$$= (a)^6 - (2)^6$$

$$= (a^3)^2 - (2^3)^2$$

$$= (a^3 - 2^3)(a^3 + 2^3)$$

$$= (a-2)(a^2+2a+4)(a+2)(a^2-2a+4)$$

$$= (a+2)(a-2)(a^2+2a+4)(a^2-2a+4)$$

5) Factorise the following :-

$$(i) x^3 + 8y^3 + 6xy - 1$$

$$(ii) l^3 - 8m^3 - 27n^3 - 18lmn$$

$$\Rightarrow (i) x^3 + 8y^3 + 6xy - 1$$

$$= (x)^3 + (2y)^3 + (-1)^3 + 3x \cdot 2y \cdot (-1)$$

$$= (x)^3 + (2y)^3 + (-1)^3 + 6xy$$

$$= (x+2y-1) \left\{ (x)^2 + (2y)^2 + (-1)^2 - x \cdot 2y - 2y \cdot (-1) - (-1) \cdot x \right\}$$

$$= (x+2y-1)(x^2 + 4y^2 + 1 - 2xy + 2y + x)$$

$$(ii) l^3 - 8m^3 - 27n^3 - 18lmn$$

$$= (l)^3 + (-2m)^3 + (-3n)^3 - 3 \cdot (l) \cdot (-2m) \cdot (-3n)$$

$$= (l)^3 + (-2m)^3 + (-3n)^3 - 18lmn$$

$$= (l-2m-3n) \left\{ (l)^2 + (-2m)^2 + (-3n)^2 - l \cdot (-2m) - (-2m) \cdot (-3n) - (-3n) \cdot l \right\}$$

$$= (l-2m-3n)(l^2 + 4m^2 + 9n^2 + 2lm - 6mn + 3ln)$$