(ii) $(-p+2q+3r)^2$

$= (-p)^2 + (2q)^2 + (3r)^2 + 2\cdot(-p)\cdot 2q + 2\cdot 2q\cdot 3r + 2\cdot 3r\cdot(-p)$

$= p^2 + 4q^2 + 9r^2 - 4pq + 12qr - 6pr$

$= p^2 + 4q^2 + 9r^2 - 4pq + 12qr - 6pr.$

(iii) $(2p+3)(2p-4)(2p-5)$

[We know that
$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$]

$= (2p)^3 \cancel{2p\cdot 2p} + (3-4-5)(2p)^2 + (-12+20-15)(2p) + 60$

$= 8p^3 + (-6)4p^2 + (-7)2p + 60$

$= 8p^3 - 24p^2 - 14p + 60$

(iv) $(3a+1)(3a-2)(3a+4)$

$= (3a)^3 + (1-2+4)(3a)^2 + (-2-8+4)3a + (-8)$

$= 27a^3 + (3)9a^2 + (-6)3a - 8$

$= 27a^3 + 27a^2 - 18a - 8$

(2) Using algebraic identity, find the coefficients of x^2, x and constant term without actual expansion.

$\Rightarrow$ (i) $(x+5)(x+6)(x+7)$ (ii) $(2x+3)(2x-5)(2x-6)$.

$\Rightarrow$ (i) $(x+5)(x+6)(x+7)$

$= \cancel{(x)^3 + (5+6+7)(x)^2 + (30+42+35)x + 210}$

$= \cancel{x^3 + 18x^2 + 107x + 210}$

$\therefore$ Now, coefficient of x^2 is $= (5+6+7) = 18$

" " x is $(30+42+35) = 107$

" " constant is $= 5\times 6\times 7$

$= 30 \times 7$

$= 210.$

(ii) $(2x+3)(2x-5)(2x-6)$

$= (2x)^3 + (3-5-6)(2x)^2 + (-15+30-18)2x + 90$

$= 8x^3 + (-8)4x^2 + (-3)2x + 90$

$= 8x^3 - 32x^2 - 6x + 90$

$\therefore$ coefficient of x^2 is $(3-5-6)(2)^2$

$= (-8)4$

$= -32$

coefficient of x is $(-15+30-18)2$

$= (-3)2$

$= -6$

coefficient of constant is $3 \times (-5) \times (-6)$

$= (-15 \times -6)$

$= 90$

(3) If $(x+a)(x+b)(x+c) = x^3 + 14x^2 + 59x + 70$, find the value of

(i) $a+b+c$ (ii) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ (iii) $a^2+b^2+c^2$ (iv) $\frac{a}{bc} + \frac{b}{ac} + \frac{c}{ab}$

$\Rightarrow$ now, $(x+a)(x+b)(x+c) = x^3 + 14x^2 + 59x + 70$

or, $(x)^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc = x^3 + 14x^2 + 59x + 7$

both side compare, we get,

$a+b+c = 14$, $ab+bc+ca = 59$, $abc = 70$.

(i) $a+b+c = 14$.

(ii) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

$= \frac{bc+ac+ab}{abc} = \frac{59}{70}$

(iii) $a^2+b^2+c^2$

$= (a+b+c)^2 - 2(ab+bc+ca)$

$= (14)^2 - 2 \cdot 59$

$= 196 - 118$

$= 78$

(iv) $\frac{a}{bc} + \frac{b}{ac} + \frac{c}{ab}$

$= \frac{a^2+b^2+c^2}{abc}$

$= \frac{78}{70}$ [by (iii), $a^2+b^2+c^2 = 78$]

$= \frac{78}{70}$.

④ Expand:

(i) $(3a-4b)^3$ (ii) $(x+\frac{1}{y})^3$

= (i) $(3a-4b)^3$

$=(3a)^3-3(3a)^2\cdot 4b+3\cdot 3a\cdot(4b)^2+(4b)^3$ $\left[\because (x-y)^3 = x^3-3x^2y+3xy^2-y^3\right]$

$=27a^3-3\cdot 9a^2\cdot 4b+3\cdot 3a\cdot 16b^2+64b^3$

$=27a^3-\cancel{27a}\ 108a^2b+144ab^2+64b^3$

$=27a^3-64b^3-108a^2b+144ab^2$

(ii) $(x+\frac{1}{y})^3$

$=(x)^3+3\cdot(x)^2\cdot\frac{1}{y}+3\cdot x\cdot\left(\frac{1}{y}\right)^2+\left(\frac{1}{y}\right)^3$

$=x^3+3x^2\cdot\frac{1}{y}+3x\cdot\frac{1}{y^2}+\frac{1}{y^3}$

$=x^3+\frac{1}{y^3}+\frac{3x^2}{y}+\frac{3x}{y^2}$

⑤ Evaluate the following by using identities:

(i) $(98)^3$ (ii) $(1001)^3$

= (i) $(98)^3$

$=(100-2)^3$

$=(100)^3-3(100)^2\cdot 2+3\cdot 100\cdot(2)^2-(2)^3$

$=1000000-3\cdot 10000\cdot 2+3\cdot 100\cdot 4-8$

$=1000000-60000+1200-8$

$\cancel{=1000000}=1001200-60008$

$=941192$

(ii) $(1001)^3$

$=(1000+1)^3$

$=(1000)^3+3\cdot(1000)^2\cdot(1)+3\cdot 1000(1)^2+(1)^3$

$=1000000000+3\cdot 1000000+3\cdot 1000+1$

$=1000000000+3000000+3000+1$

$=1003003001$.

~~= 1030301 = 1030301~~

(6) If $(x+y+z)=9$ and $(xy+yz+zx)=26$, then find the value of $x^2+y^2+z^2$.

$\Rightarrow$ $x^2+y^2+z^2$

$=(x+y+z)^2-2(xy+yz+zx)$

$=(9)^2-2\cdot 26$ $\left[\because x+y+z=9 \text{ and } xy+yz+zx=26\right]$

$=81-52$

$=29$

(7) Find $27a^3+64b^3$ if $3a+4b=10$ and $ab=2$

$\Rightarrow$ $27a^3+64b^3$

$=(3a)^3+(4b)^3$ $\left[\because x^3+y^3=(x+y)^3-3xy(x+y)\right]$

$=(3a+4b)^3-3\cdot 3a\cdot 4b(3a+4b)$

$=(10)^3-36\cdot 2(10)$ $\left[\because 3a+4b=10 \text{ and } ab=2\right]$

$=1000-720$

$=280$

(8) Find x^3-y^3, if $x-y=5$ and $xy=14$.

$\Rightarrow$ x^3-y^3

$=(x-y)^3+3\cdot xy\cdot(x-y)$

$=(5)^3+3\cdot 14\cdot 5$ $\left[\because x-y=5 \text{ and } xy=14\right]$

$=125+210$

$=335$

(9) If $a + \frac{1}{a} = 6$, then find the value of $a^3 + \frac{1}{a^3}$.

$\Rightarrow$ $a^3 + \frac{1}{a^3}$

$= (a)^3 + (\frac{1}{a})^3$

$= (a + \frac{1}{a})^3 - 3 a \cdot \frac{1}{a} (a + \frac{1}{a})$

$= (6)^3 - 3 \cdot 6 \quad [\because a + \frac{1}{a} = 6]$

$= 216 - 18$

$=$ ~~288~~ 198

(10) If $x^2 + \frac{1}{x^2} = 23$, then find the value of $x + \frac{1}{x}$ and $x^3 + \frac{1}{x^3}$.

$\Rightarrow$ $x^2 + \frac{1}{x^2} = 23$

or, $(x + \frac{1}{x})^2 - 2 \cdot \cancel{x} \cdot \frac{1}{\cancel{x}} = 23$

or, $(x + \frac{1}{x})^2 = 23 + 2$

or, $(x + \frac{1}{x})^2 = 25$

or, $(x + \frac{1}{x})^2 = (\pm 5)^2$

$\therefore x + \frac{1}{x} = \pm 5$

now, $x^3 + \frac{1}{x^3}$

$= (x + \frac{1}{x})^3 - 3 x \cdot \frac{1}{x} (x + \frac{1}{x})$

$= (\pm 5)^3 - 3 (\pm 5)$

$= (+5)^3 - 3(+5)$ or $(-5)^3 - 3(-5)$

$= 125 - 15$ or $-125 + 15$

$= 110$ or -110

$= \pm 110$

Thus, the value of $x + \frac{1}{x} = \pm 5$ and $x^3 + \frac{1}{x^3} = \pm 110$

(11) If $\left(y-\frac{1}{y}\right)^3 = 27$, then find the value of $y^3 - \frac{1}{y^3}$.

$\Rightarrow$ Now, $\left(y-\frac{1}{y}\right)^3 = 27$

~~or, $\left(y^3-\frac{1}{y^3}\right) - 3y.\frac{1}{y}\left(y-\frac{1}{y}\right) = 27$~~

$\left(y-\frac{1}{y}\right)^3 = (3)^3$

$\left(y-\frac{1}{y}\right) = 3 \quad \cdots (i)$

Now, $y^3 - \frac{1}{y^3}$

$= \left(y-\frac{1}{y}\right)^3 + 3\,y.\frac{1}{y}\left(y-\frac{1}{y}\right)$

$= (3)^3 + 3.3$ [by equation (i)]

$= 27 + 9$

$= 36$

(12) Simplify : (i) $(2a+3b+4c)(4a^2+9b^2+16c^2-6ab-12bc-8ca)$

(ii) $(x-2y+3z)(x^2+4y^2+9z^2+2xy+6yz-3xz)$

$\Rightarrow$ (i) $(2a+3b+4c)(4a^2+9b^2+16c^2-6ab-12bc-8ca)$

$= (2a+3b+4c)\{(2a)^2+(3b)^2+(4c)^2-2a.3b-3b.4c-4c.2a\}$

$= (2a)^3+(3b)^3+(4c)^3-3.2a.3b.4c$ $\left[\because x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)\right]$

$= 8a^3+27b^3+64c^3-72abc$

(ii) $(x-2y+3z)(x^2+4y^2+9z^2+2xy+6yz-3xz)$

$= (x-2y+3z)\{(x)^2+(-2y)^2+(3z)^2-x.(-2y)-(-2y).3z-3z.x\}$

$= (x)^3+(-2y)^3+(3z)^3-3.x.(-2y).3z$ $\left[\because x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)\right]$

$= x^3-8y^3+27z^3+18xyz$

$= x^3-8y^3+27z^3+18xyz.$

(13) By using identity evaluate the following:

(i) $7^3 - 10^3 + 3^3$ (ii) $1 + \frac{1}{8} - \frac{27}{8}$

$\Rightarrow$ (i) $7^3 - 10^3 + 3^3$

$= (7-10+3)\{(7)^2 + (-10)^2 + (3)^2 - 7\cdot(-10) - (-10)\cdot 3 - 7\cdot 3\} + 3\cdot 7\cdot(-10)\cdot 3$

$= 0\cdot(49 + 100 + 9 + 70 + 30 - 21) - 630$

$= -630$

(ii) $1 + \frac{1}{8} - \frac{27}{8}$

$= (1)^3 + (\frac{1}{2})^3 + (-\frac{3}{2})^3$

$= (1 + \frac{1}{2} - \frac{3}{2})\{(1)^2 + (\frac{1}{2})^2 + (-\frac{3}{2})^2 - 1\cdot\frac{1}{2} - \frac{1}{2}\cdot(-\frac{3}{2}) - (-\frac{3}{2})\cdot 1\} + 3\cdot 1\cdot\frac{1}{2}\cdot(-\frac{3}{2})$

$= (\frac{3}{2} - \frac{3}{2})(1 + \frac{1}{8} + \frac{9}{4} - \frac{1}{2} + \frac{3}{4} + \frac{3}{2}) - \frac{9}{4}$

$= 0\,(1 + \frac{1}{8} + \frac{9}{4} - \frac{1}{2} + \frac{3}{4} + \frac{3}{2}) - \frac{9}{4}$

$= 0 - \frac{9}{4}$

$= -\frac{9}{4}$

(14) If $2x - 3y - 4z = 0$, then find $8x^3 - 27y^3 - 64z^3$

$\Rightarrow 8x^3 - 27y^3 - 64z^3$

$= (2x)^3 + (-3y)^3 + (-4z)^3$

$= (2x - 3y - 4z)\{(2x)^2 + (3y)^2 + (-4z)^2 - 2x\cdot(-3y) - (-3y)\cdot(-4z) - (-4z)\cdot 2x\}$
$+ 3\cdot 2x\cdot(-3y)(-4z)$

$= 0\cdot\{4x^2 + 9y^2 + 16z^2 + 6xy - 12yz + 8zx\} + 72xyz$

$[\because 2x - 3y - 4z = 0]$

$= 0 + 72xyz$

$= 72xyz$