Exercise - 3·3

① Check whether $p(x)$ is a multiple of $g(x)$ or not.
$p(x) = x^3 - 5x^2 + 4x - 3$; $g(x) = x - 2$.

⇒ Given that $p(x) = x^3 - 5x^2 + 4x - 3$

Now, $g(x) = x - 2$

$g(x) = 0$

$x - 2 = 0$

$x = 2$

now, $p(x) = x^3 - 5x^2 + 4x - 3$

$\therefore p(2) = (2)^3 - 5.(2)^2 + 4.2 - 3$

$= 8 - 20 + 8 - 3$

$= -7. \neq 0.$

$\therefore p(2) \neq 0$

Thus, $p(x)$ is not a multiple of $g(x)$.

② By remainder theorem, find the remainder when, $p(x)$ is divided by $g(x)$ where,

(i) $p(x) = x^3 - 2x^2 - 4x - 1$; $g(x) = x + 1$

(ii) $p(x) = 4x^3 - 12x^2 + 14x - 3$; $g(x) = 2x - 1$

(iii) $p(x) = x^3 - 3x^2 + 4x + 50$; $g(x) = x - 3$.

⇒ (i) $p(x) = x^3 - 2x^2 - 4x - 1$,

$g(x) = x + 1$,

Now, $g(x) = 0$

$x + 1 = 0$

$x = -1$

By remainder theorem, $p(x)$ is divided by $g(x)$, then remainder $p(-1)$.

$\therefore p(-1) = (-1)^3 - 2.(-1)^2 - 4.(-1) - 1$

$= -1 - 2 + 4 - 1$

$= 0$

$\therefore$ Thus, remainder is 0.

(ii) $p(x) = 4x^3 - 12x^2 + 14x - 3$

$g(x) = 2x - 1$.

Now, $g(x) = 0$

$2x - 1 = 0$

$x = \frac{1}{2}$

By remainder therorem, $p(x)$ is divided by $g(x)$ then remainder is $p(\frac{1}{2})$.

$\therefore p(\frac{1}{2}) = 4 \cdot (\frac{1}{2})^3 - 12 \times (\frac{1}{2})^2 + 14 \cdot \frac{1}{2} - 3$

$= 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + 14 \times \frac{1}{2} - 3$

$= \frac{1}{2} - 3 + 7 - 3$

$= \frac{1}{2} + 1 = \frac{3}{2}$

Thus, remainder is $3/2$.

(iii) $p(x) = x^3 - 3x^2 + 4x + 50$,

$g(x) = x - 3$

Now, $g(x) = 0$

$x - 3 = 0$

$x = 3$

By remainder theorem, $p(x)$ is divided by $g(x)$ then, remainder is $p(3)$.

Now, $p(3) = (3)^3 - 3 \cdot (3)^2 + 4 \cdot 3 + 50$

$= 27 - 27 + 12 + 50$

$= 62$

Thus, remainder is 62.

(3) Find the remainder when $3x^3 - 4x^2 + 7x - 5$ is divided by $(x+3)$.

$\Rightarrow$ Let $p(x) = 3x^3 - 4x^2 + 7x - 5$

and $g(x) = x + 3$.

Now, $g(x) = 0$

$x \cancel{+} 3 = 0 \quad \therefore x = -3$

By remainder theorem, $p(x)$ is divided by $g(x)$, then, remainder is $p(-3)$.

$\therefore p(-3) = 3(-3)^3 - 4(-3)^2 + 7.(-3) - 5$

$= -81 \cancel{+} 36 - 21 - 5$

$= -143$

Thus, required remainder is -143.

(4) What is the remainder when $x^{2018} + 2018$ is divided by $x-1$.

$\Rightarrow$ Let $f(x) = x^{2018} + 2018$ and $g(x) = x-1$

Now, $g(x) = 0$

$x - 1 = 0$

$x = 1$

By remainder theorem, $f(x)$ is divided by $g(x)$, then remainder is $f(1)$

$\therefore f(1) = (1)^{2018} + 2018$

$= 1 + 2018 = 2019$

Thus, the remainder is 2019.

(5) For what value of k is the polynomial for what value of k is the polynomial $2x^3 - kx^2 + 3x + 10$ exactly divisible by $(x-2)$.

$p(x) = 2x^3 - kx^2 + 3x + 10$

$\Rightarrow$ Let $g(x) = x - 2$

Now, $g(x) = 0$

$x - 2 = 0$

$x = 2$.

Now, $p(x)$ is exactly divisible by $g(x)$.

then, $p(2) = 0$

$2(2)^3 - k(2)^2 + 3 \cdot 2 + 10 = 0$

$16 - 4k + 6 + 10 = 0$

$4k = 32$

$k = 8$

$\therefore$ Thus, the value of $k = 8$.

⑥ If two polynomials $2x^3 + ax^2 + 4x - 12$ and $x^3 + x^2 - 2x + a$ leave the same remainder when divided by $(x-3)$, find the value of a and also find the remainder.

$\Rightarrow$ Let $f(x) = 2x^3 + ax^2 + 4x - 12$

$P(x) = x^3 + x^2 - 2x + a$

and $g(x) = x - 3$

Now, $g(x) = 0$

$x - 3 = 0$

$x = 3$

By remainder theorem, $f(x)$ is divided by $g(x)$ then, remainder is $f(3)$.

$f(3) = 2\cdot(3)^3 + a\cdot(3)^2 + 4\cdot 3 - 12$

$= 54 + 9a + 12 - 12$

$= 9a + 54$

By remainder theorem, $P(x)$ is divided by $g(x)$, then remainder is $P(3)$.

$\therefore P(3) = (3)^3 + (3)^2 - 2\cdot 3 + a$

$= 27 + 9 - 6 + a$

$= a + 30.$

Now, two remainder are same.

$\therefore f(3) = P(3)$

$9a + 54 = a + 30$

$8a = -24$

$a = -3$

$\therefore$ The remainder $= 9a + 54$

$= 9 \times (-3) + 54$

$= -27 + 54$

$= 27.$

Thus, the value of $a = -3$ and remainder 27.

7. Determine whether $(x-1)$ is a factor of the following polynomials:

(i) $x^3+5x^2-10x+4$ (ii) x^4+5x^2-5x+1

⇒ (i) Let $p(x) = x^3+5x^2-10x+4$

and $g(x) = x-1$

$g(x) = 0$

$x-1 = 0$

$x = 1.$

By factor theorem, $(x-1)$ is factor $p(x)$, if $p(1) = 0$

$\therefore p(1)$

$= (1)^3+5(1)^2-10\cdot(1)+4$

$= 1+5-10+4$

$= 10-10 = 0$

$\therefore p(1) = 0$

Therefore, $(x-1)$ is a factor of $x^3+5x^2-10x+4$.

(ii) Let $p(x) = x^4+5x^2-5x+1$

By factor theorem $(x-1)$ is factor $p(x)$, if $p(1) = 0$.

$\therefore p(1)$

$= (1)^4+5(1)^2-5\cdot(1)+1$

$= 1+5-5+1$

$= 2 \neq 0.$

Thus, $(x-1)$ is not a factor of x^4+5x^2-5x+1.

8. Using factor theorem; Show that $(x-5)$ is a factor of the polynomial $2x^3-5x^2-28x+15$.

⇒ Let $p(x) = 2x^3-5x^2-28x+15$

and $g(x) = x-5$

$\therefore g(x) = 0$

$x-5 = 0$

$x = 5$

By factor theorem, $(x-5)$ is a factor $p(x)$, if $p(5)=0$

$\therefore p(5)$

$= 2\cdot(5)^3 - 5(5)^2 - 28(5) + 15$

$= 250 - 125 - 140 + 15$

$= 265 - 165$

$= 0$

$\therefore$ Now, $p(5) = 0$.

Thus, $(x-5)$ is a factor of the polynomial $2x^3 - 5x^2 - 28x + 15$.

(9) Determine the value of m, if $(x+3)$ is a factor of $x^3 - 3x^2 - mx + 24$.

$\Rightarrow$ Let $p(x) = x^3 - 3x^2 - mx + 24$.

$g(x) = x + 3$

$\therefore g(x) = 0$

$x + 3 = 0$

$x = -3$

Now, $(x+3)$ is a factor of $p(x)$.

then, $p(-3) = 0$

$(-3)^3 - 3(-3)^2 - m(-3) + 24 = 0$

$-27 - 27 + 3m + 24 = 0$

$-30 + 3m = 0$

$3m = 30$

$m = 10$

Thus, the value of $m = 10$.

(10) If both $(x-2)$ and $(x-\frac{1}{2})$ are the factors of $ax^2 + 5x + b$, then show that $a = b$.

$\Rightarrow$ Let $p(x) = ax^2 + 5x + b$.

Now, $(x-2)$ is a factor of $p(x)$.

then, $p(2) = 0$

$a(2)^2 + 5\cdot(2) + b = 0$

$4a + b + 10 = 0 \cdots\cdots$ (i)

Now, $(x-\frac{1}{2})$ is a factor of $p(x)$.

then, $p(\frac{1}{2})=0$

$a\cdot(\frac{1}{2})^2+5\cdot(\frac{1}{2})+b=0$

$\frac{a}{4}+\frac{5}{2}+b=0$

$a+4b+10=0$ ...... (ii)

Now, (i)×1 − (ii)×4, we get,

$4a+b+10=0$
$4a+16b+40=0$
$(-)\quad(-)\quad(-)$

$-15b-30=0$

$b=-2$

from (i), $b=-2$ putting, we get,

$4a+b+10=0$

$4a+(-2)+10=0$

$4a-2+10=0$

$4a=-8$

$a=-2$

Now, $a=-2=b$.

$\therefore a=b$. [proved]

(11) If $(x-1)$ divides the polynomial $kx^3-2x^2+25x-26$ without remainder, then find the value of k.

⇒ Let $P(x)=kx^3-2x^2+25x-26$

$g(x)=(x-1)$, $\therefore g(x)=0$
$x-1=0$
$x=1$

Now, $(x-1)$ divides the polynomial $p(x)$ without remainder.

then, $p(1)=0$

$k(1)^3-2(1)^2+25(1)-26=0$

$k-2+25-26=0$

$k-2-1=0$

$k-3=0$

$k=3$ Thus, the value of $k=3$.