Exercise - 3.2

(1) Find the value of the polynomial $f(y) = 6y - 3y^2 + 3$ at

(i) $y = 1$ (ii) $y = -1$ (iii) $y = 0$

⇒ Given that $f(y) = 6y - 3y^2 + 3$

(i) at $y = 1$ then

$$f(1) = 6 \cdot 1 - 3 \cdot (1)^2 + 3$$
$$= 6 - 3 + 3$$
$$= 6.$$

$\therefore f(1) = 6.$

(ii) at $y = -1$ then $f(-1) = 6 \cdot (-1) - 3(-1)^2 + 3$

$$= -6 - 3 + 3$$
$$= -6$$

$\therefore f(-1) = -6$

(iii) at $y = 0$ then $f(0) = 6 \cdot 0 - 3 \cdot (0)^2 + 3$

$$= 3$$

$\therefore f(0) = 3.$

(2) If $p(x) = x^2 - 2\sqrt{2}x + 1$, find $p(2\sqrt{2})$.

⇒ Given that $p(x) = x^2 - 2\sqrt{2}x + 1$

at $x = 2\sqrt{2}$ then $p(2\sqrt{2})$

$$= (2\sqrt{2})^2 - 2\sqrt{2} \cdot 2\sqrt{2} + 1$$
$$= (2\sqrt{2})^2 - (2\sqrt{2})^2 + 1$$
$$= 1.$$

$\therefore p(2\sqrt{2}) = 1$

(3) Find the zeros of the polynomial in each of the following:

(i) $p(x) = x - 3$ (ii) $p(x) = 2x + 5$ (iii) $q(y) = 2y - 3$

(iv) $f(z) = 8z$ (v) $p(x) = ax$ when $a \neq 0$

(vi) $h(x) = ax + b$, $a \neq 0$, $a, b \in R$.

→ (i) Given that $p(x) = x - 3$

$\therefore p(3) = 3 - 3 = 0$

since $p(3) = 0$, $x = 3$ is the zero of $p(x)$.

(ii) Given that $p(x) = 2x + 5$

$= 2\left(x + \frac{5}{2}\right)$

$\therefore p\left(-\frac{5}{2}\right) = 2\left(-\frac{5}{2} + 5/2\right) = 2 \cdot 0 = 0$

since $p(-5/2) = 0$, $x = -5/2$ is the zero of $p(x)$.

(iii) Given that $q(y) = 2y - 3$

$= 2(y - 3/2)$

$\therefore q\left(\frac{3}{2}\right) = 2\left(\frac{3}{2} - 3/2\right) = 2 \cdot 0 = 0$

since $q(3/2) = 0$, $y = 3/2$ is the zero of $q(y)$.

(iv) Given that $f(z) = 8z$.

$\therefore f(0) = 8 \cdot 0 = 0$

since $f(0) = 0$, $z = 0$ is the zero of $f(z)$.

(v) Given that $p(x) = ax$ when $a \neq 0$

$\therefore p(0) = a \cdot 0 = 0$.

since $p(0) = 0$ $\therefore x = 0$ is the zero of $p(x)$.

(vi) Given that $h(x) = ax + b$, $a \neq 0$, $a, b \in \mathbb{R}$

$\therefore h(x) = ax + b$

$= a(x + b/a)$

$\therefore h\left(-\frac{b}{a}\right) = a\left(-\frac{b}{a} + \frac{b}{a}\right) = a \cdot 0 = 0$

since $h\left(-\frac{b}{a}\right) = 0$, $x = -\frac{b}{a}$ is the zero of $h(x)$.

(4) Find the roots of the polynomial equations:

(i) $5x - 6 = 0$ (ii) $x + 3 = 0$

(iii) $10x + 9 = 0$ (iv) $9x - 4 = 0$.

(i) Now, given polynomial equation.

$\therefore 5x - 6 = 0$

or, $5x = 6$

or $x = 6/5$

Thus, $x = 6/5$

(ii) $x + 3 = 0$

or, $x = -3$

Thus, $x = -3$

(iii) $10x + 9 = 0$

or, $10x = -9$

or, $x = -9/10$

Thus, $x = -\frac{9}{10}$

(iv) $9x - 4 = 0$

or, $9x = 4$

or, $x = \frac{4}{9}$

Thus, $x = \frac{4}{9}$.

(5) Verify whether the following are zeros of the polynomial indicated against them, or not.

(i) $p(x) = 2x - 1$, $x = \frac{1}{2}$ (ii) $p(x) = x^3 - 1$, $x = 1$

(iii) $p(x) = ax + b$, $x = \frac{-b}{a}$ (iv) $p(x) = (x+3)(x-4)$, $x = 4, x = -3$

⇒ (i) $p(x) = 2x - 1$,

Then, $p(\frac{1}{2}) = 2 \cdot \frac{1}{2} - 1$

$= 1 - 1 = 0$

$\therefore x = \frac{1}{2}$ is the zero of $p(x)$.

(ii) $p(x) = x^3 - 1$

Then, $p(1) = (1)^3 - 1 = 1 - 1 = 0$

$\therefore$ $x = 1$ is the zero of $p(x)$.

(iii) $p(x) = ax + b$,

Then, $p\left(-\frac{b}{a}\right) = a \cdot -\frac{b}{a} + b$

$= -b + b$

$= 0$

$\therefore$ $x = -\frac{b}{a}$ is the zero of $p(x)$.

(iv) $p(x) = (x+3)(x-4)$,

Then, $p(4) = (4+3)(4-4)$

$= 7 \cdot 0 = 0$

$\therefore$ $p(-3) = (-3+3)(-3-4)$

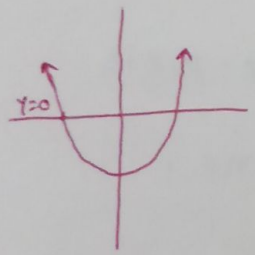
$= 0 \cdot -7$

$= 0$

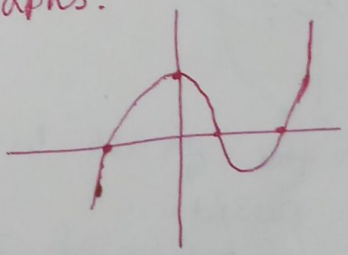
Thus, $x = 4$, and $x = -3$ are zeros of $p(x)$.

(6) Find the number of zeros of the following polynomials represented by their graphs.

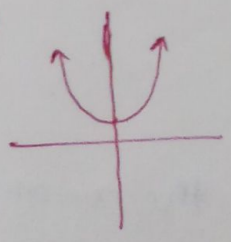
(i)



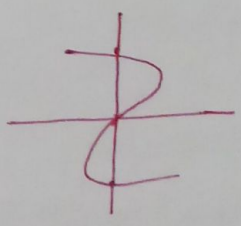
(ii)



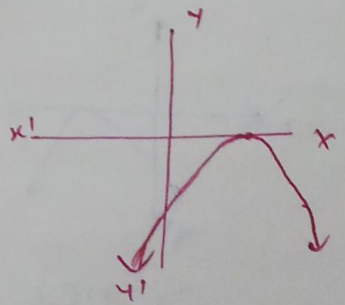
(iii)



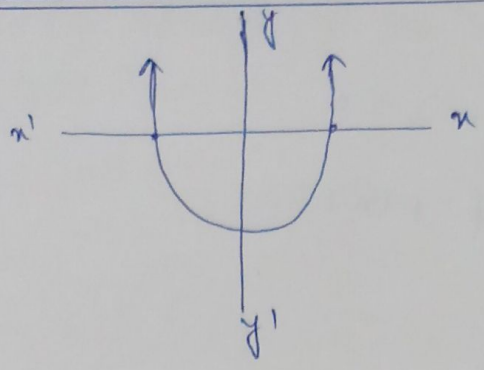
(iv)



(v)



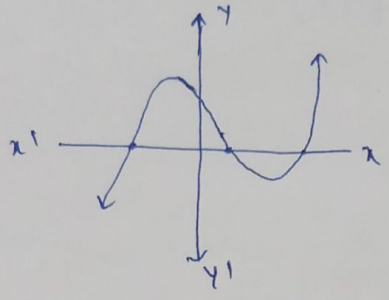
⇒ (i)



Since the graph touches the x-axis at two points

Thus, The number of zeros is two

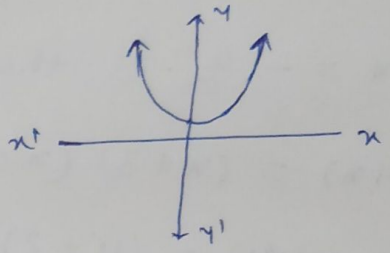
(ii)



Since the graph touches the x-axis 3 points.

Thus, the number of zeros is 3.

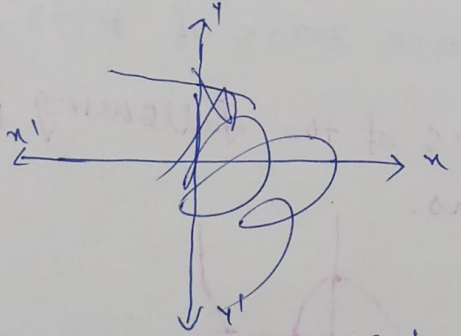
(iii)

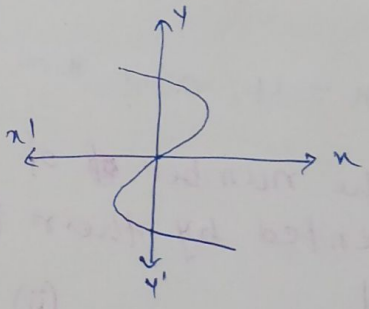


Since the graph no touches the x-axis.

Thus, the number of zeros is zero.

(iv)

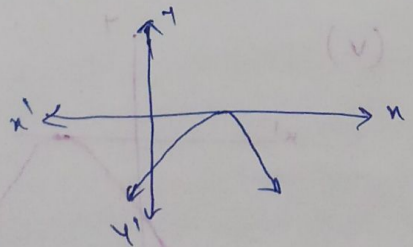




Since the graph touches the x-axis one point.

Thus, the number of zeros is one.

(v)



Since the graph touches the x-axis one point.

Thus, The number of zeros is one.