Exercise - 3.1

1. which of the following expressions are polynomials. If not give reason:

(i) $\frac{1}{x^2}+3x-4$ (ii) $x^2(x-1)$ (iii) $\frac{1}{x}(x+5)$

(iv) $\frac{1}{x^{-2}}+\frac{1}{x^{-1}}+7$ (v) $\sqrt{5}x^2+\sqrt{3}x+\sqrt{2}$ (vi) $m^2-\sqrt[3]{m}+7m-10$

⇒ (i) $\frac{1}{x^2}+3x-4$

$= x^{-2}+3x-4$

so, Negative integer power.

then, This ~~equation~~ expression is not polynomial.

(ii) $x^2(x-1)$

$= x^3 - x^2$

Non-negative integer power.

Thus, this ~~equation~~ expression is polynomial.

(iii) $\frac{1}{x}(x+5)$

$= 1 + 5x^{-1}$

$= 5x^{-1}+1$

Negative integer power.

Thus, This ~~equation~~ expression is not polynomial.

(iv) $\frac{1}{x^{-2}}+\frac{1}{x^{-1}}+7$

$= x^2 + x + 7$

Non-negative integer power.

Thus, This ~~equation~~ expression is polynomial.

(v) $\sqrt{5}x^2+\sqrt{3}x+\sqrt{2}$

Non-negative integer power.

Thus, This ~~equation~~ expression is polynomial.

(vi) $m^2-\sqrt[3]{m}+7m-10$

one of the power of m is a fraction $(\frac{1}{3})$

Thus, This ~~equation~~ expression is not polynomial.

(2) Write the coefficient of x^2 and x in each of the following polynomials.

(i) $4 + \frac{2}{5}x^2 - 3x$ (ii) $6 - 2x^2 + 3x^3 - \sqrt{7}x$

(iii) $\pi x^2 - x + 2$ (iv) $\sqrt{3}x^2 + \sqrt{2}x + 0.5$

(v) $x^2 - \frac{7}{2}x + 8$

$\Rightarrow$ (i) $4 + \frac{2}{5}x^2 - 3x$

$= \frac{2}{5}x^2 - 3x + 4.$

Coefficient of x^2 is $\frac{2}{5}$

coefficient of x is -3.

(ii) $6 - 2x^2 + 3x^3 - \sqrt{7}x$

$= 3x^3 - 2x^2 - \sqrt{7}x + 6$

Thus, coefficient of x^2 is -2

" " x is $-\sqrt{7}$

(iii) $\pi x^2 - x + 2$

$= \pi x^2 - x + 2$

Thus, coefficient of x^2 is π

coefficient of x is -1

(iv) $\sqrt{3}x^2 + \sqrt{2}x + 0.5$

$= \sqrt{3}x^2 + \sqrt{2}x + 0.5$

Thus, coefficient of x^2 is $\sqrt{3}$

" " x is $\sqrt{2}$

(v) $x^2 - \frac{7}{2}x + 8$

$= x^2 - \frac{7}{2}x + 8$

Thus, coefficient of x^2 is 1

" " x is $-7/2$

(3) Find the degree of the following polynomials.

(i) $1-\sqrt{2}y^2+y^7$ (ii) $\frac{x^3-x^4+6x^6}{x^2}$

(iii) $x^3(x^2+x)$ (iv) $3x^4+9x^2+27x^6$ (v) $2\sqrt{3}p^4-\frac{8p^3}{\sqrt{3}}+\frac{2p^2}{7}$

⇒ (i) $1-\sqrt{2}y^2+y^7$

$= y^7-\sqrt{2}y^2+1$

The degree of the polynomial $y^7-\sqrt{2}y^2+1$

= the largest exponent in the polynomial.

= 7.

(ii) $\frac{x^3-x^4+6x^6}{x^2}$

$= x-x^2+6x^4$

$= 6x^4-x^2+x$

The degree of the polynomial $\frac{x^3-x^4+6x^6}{x^2}$

= The largest exponent in the polynomial.

= 4

(iii) $x^3(x^2+x)$

$= x^5+x^4$

The degree of the polynomial $x^3(x^2+x)$

= The largest exponent in the polynomial.

= 5

(iv) $3x^4+9x^2+27x^6$

$= 27x^6+3x^4+9x^2$

The degree of the polynomial $3x^4+9x^2+27x^6$

= The largest exponent in the polynomial

= 6

(v) $2\sqrt{5}\,p^4 - \frac{8p^3}{\sqrt{3}} + \frac{2p^2}{7}$

$= 2\sqrt{5}\,p^4 - \frac{8}{\sqrt{3}}p^3 + \frac{2}{7}p^2$

The degree of the polynomial $2\sqrt{5}\,p^4 - \frac{8}{\sqrt{3}}p^3 + \frac{2}{7}p^2$

= The largest exponent in the polynomial.

= 4

(4) Rewrite the following polynomial in standard form.

(i) $x - 9 + \sqrt{7}x^3 + 6x^2$ (ii) $\sqrt{2}x^2 - 7/2\,x^4 + x - 5x^3$

(iii) $7x^3 - \frac{6}{5}x^2 + 4x - 1$ (iv) $y^2 + \sqrt{5}y^3 - 11 - 7/3\,y + 9y^4$.

⇒ (i) $x - 9 + \sqrt{7}x^3 + 6x^2$

~~$= 6x^2 +$~~

$= \sqrt{7}x^3 + 6x^2 + x - 9$

This polynomial in standard form (Desending order) is $\sqrt{7}x^3 + 6x^2 + x - 9$.

This polynomial in ~~As~~ standard form (Aseending order) is $-9 + x + 6x^2 + \sqrt{7}x^3$.

(ii) $\sqrt{2}x^2 - 7/2\,x^4 + x - 5x^3$

This polynomial standard form is

Desending order

$-7/2\,x^4 - 5x^3 + \sqrt{2}x^2 + x$

Asending order

$x + \sqrt{2}x^2 - 5x^3 - 7/2\,x^4$

(iii) $7x^3 - \frac{6}{5}x^2 + 4x - 1$

This polynomial standard form is

Desending order

$7x^3 - 6/5\,x^2 + 4x - 1$

Asending order

$-1 + 4x - 6/5\,x^2 + 7x^3$

(iv) $y^2 + \sqrt{5}y^3 - 11 - 7/3\, y + 9y^4$

This polynomial standard form is

Desending order

$9y^4 + \sqrt{5}y^3 + y^2 - 7/3\, y - 11$

Asceending order

$-11 - 7/3\, y + y^2 + \sqrt{5}y^3 + 9y^4$

(5) Add the following polynomials and find the degree of the resultant polynomial.

(i) $p(x) = 6x^2 - 7x + 2$ $\quad$ $q(x) = 6x^3 - 7x + 15$

(ii) $h(x) = 7x^3 - 6x + 1$ $\quad$ $f(x) = 7x^2 + 17x - 9$

(iii) $f(x) = 16x^4 - 5x^2 + 9$ $\quad$ $g(x) = -6x^3 + 7x - 15$

⇒ (i) Given that

$p(x) = 6x^2 - 7x + 2$ $\quad$ $q(x) = 6x^3 - 7x + 15$

$p(x) + q(x) = 6x^2 - 7x + 2 + 6x^3 - 7x + 15$

$= 6x^3 + 6x^2 - 14x + 17$

The degree of polynomial $p(x) + q(x)$

= The largest exponent in the polynomial.

= 3

(ii) Given that

$h(x) = 7x^3 - 6x + 1$ and $f(x) = 7x^2 + 17x - 9$

$h(x) + f(x) = 7x^3 - 6x + 1 + 7x^2 + 17x - 9$

$= 7x^3 + 7x^2 + 11x \cancel{+} 8$

$= 7x^3 + 7x^2 + 11x - 8$

The degree of polynomial $h(x) + f(x)$

= The largest exponent in the polynomial.

= 3

(iii) Given that

$f(x) = 16x^4 - 5x^2 + 9$ and $g(x) = -6x^3 + 7x - 15$

$f(x) + g(x) = 16x^4 - 5x^2 + 9 + (-6x^3 + 7x - 15)$

$= 16x^4 - 5x^2 + 9 - 6x^3 + 7x - 15$

$= 16x^4 - 6x^3 - 5x^2 + 7x - 6$

The degree of polynomial of $f(x) + g(x)$

= The largest exponent in the polynomial.

$= 4$.

6) Subtract the second polynomial from the first polynomial and find the degree of the resultant polynomial.

(i) $p(x) = 7x^2 + 6x - 1$ $q(x) = 6x - 9$

(ii) $f(y) = 6y^2 - 7y + 2$ $g(y) = 7y + y^3$

(iii) $h(z) = z^5 - 6z^4 + z$ $f(z) = 6z^2 + 10z - 7$.

⇒ (i) Given that

$p(x) = 7x^2 + 6x - 1$ and $q(x) = 6x - 9$

$p(x) - q(x) = 7x^2 + 6x - 1 - 6x + 9$

$= 7x^2 + 8$

The degree of polynomial $p(x) - q(x)$

= The largest exponent in the polynomial.

$= 2$.

(ii) Given that

$f(y) = 6y^2 - 7y + 2$ and $g(y) = 7y + y^3$

$f(y) - g(y) = 6y^2 - 7y + 2 - 7y - y^3$

$= -y^3 + 6y^2 - 14y + 2$

The degree of polynomial $f(y) - g(y)$

= The largest exponent in the polynomial.

$= 3$

(iii) Given that

$h(z) = z^5 - 6z^4 + z$ and $f(z) = 6z^2 + 10z - 7$

$h(z) - f(z) = z^5 - 6z^4 + z - 6z^2 - 10z + 7$

$= z^5 - 6z^4 - 6z^2 - 9z + 7$

The degree of polynomial $h(z) - f(z)$

= The largest exponent ~~of~~ in the polynomial.

= 5

(7) What should be added to $2x^3 + 6x^2 - 5x + 8$ to get $3x^3 - 2x^2 + 6x + 15$?

⇒ Let add be the polynomial $p(x)$.

Now, $(2x^3 + 6x^2 - 5x + 8) + p(x) = 3x^3 - 2x^2 + 6x + 15$

$p(x) = (3x^3 - 2x^2 + 6x + 15) - (2x^3 + 6x^2 - 5x + 8)$

$p(x) = 3x^3 - 2x^2 + 6x + 15 - 2x^3 - 6x^2 + 5x - 8$

$p(x) = x^3 - 8x^2 + 11x + 7$

Thus, the required polynomial is $x^3 - 8x^2 + 11x + 7$.

(8) What must be subtracted from $2x^4 + 4x^2 - 3x + 7$ to get $3x^3 - x^2 + 2x + 1$?

⇒ Let subtracte be the polynomial $q(x)$.

Now, $(2x^4 + 4x^2 - 3x + 7) - q(x) = 3x^3 - x^2 + 2x + 1$

$q(x) = (2x^4 + 4x^2 - 3x + 7) - (3x^3 - x^2 + 2x + 1)$

$q(x) = 2x^4 + 4x^2 - 3x + 7 - 3x^3 + x^2 - 2x - 1$

$q(x) = 2x^4 - 3x^3 + 5x^2 - 5x + 6$

Thus, the required polynomial is

$2x^4 - 3x^3 + 5x^2 - 5x + 6$.

(2) Multiply the following polynomial and find the degree of the resultant polynomial.

(i) $p(x) = x^2 - 9$ $\quad q(x) = 6x^2 + 7x - 2$

(ii) $f(x) = 7x + 2$ $\quad g(x) = 15x - 9$

(iii) $h(x) = 6x^2 - 7x + 1$ $\quad f(x) = 5x - 7$.

⇒ (i) Given that

$p(x) = x^2 - 9$ and $q(x) = 6x^2 + 7x - 2$

$p(x) \times q(x) = (x^2 - 9)(6x^2 + 7x - 2)$

$= 6x^4 + 7x^3 - 2x^2 - 54x^2 - 63x + 18$

$= 6x^4 + 7x^3 - 56x^2 - 63x + 18$

The degree of the polynomial $p(x)\,q(x)$.

= The largest exponent in the polynomial.

$= 4$

(ii) Given that

$f(x) = 7x + 2$ and $g(x) = 15x - 9$

Now, $f(x) \times g(x)$

$= (7x + 2)(15x - 9)$

$= 105x^2 - 63x + 30x - 18$

$= 105x^2 - 33x - 18$

The degree of the polynomial $f(x)\,g(x)$

= The largest exponent in the polynomial.

$= 2$

(iii) Given that $h(x) = 6x^2 - 7x + 1$ and $f(x) = 5x - 7$.

Now, $h(x) \times f(x) = (6x^2 - 7x + 1)(5x - 7)$

$= 30x^3 - 42x^2 - 35x^2 + 49x + 5x - 7$

$= 30x^3 - 77x^2 + 54x - 7$.

The degree of the polynomial $h(x)\,f(x)$

= The largest exponent in the polynomial.

$= 3$.

(10) The cost of a chocolate is Rs. $(x+y)$ and Amir bought $(x+y)$ chocolates. Find the total amount paid by him in terms of x and y. If $x=10$, $y=5$ find the amount paid by him.

⇒ Given that,

The cost of a chocolate $= (x+y)$

The number of " $= (x+y)$

Total amount paid by Amir

$= (x+y)(x+y)$

$= x^2+xy+xy+y^2$

$= x^2+y^2+2xy.$

Now, Given that $x=10$, $y=5$;

$= (10)^2+(5)^2+2\cdot10\cdot5$

$= 100+25+100$

$= 225.$

Thus, the Amir paid by Rs: 225.

(11) The length of a rectangle is $(3x+2)$ units and it's breadth is $(3x-2)$ units. Find its area in terms of x. What will be the area if $x = 20$ units.

⇒ Given that

rectangle length $= (3x+2)$ units

" breadth $= (3x-2)$ units.

" Area = length × breadth

$= (3x+2)(3x-2)$ sq. units

$= 9x^2-4$ sq. units.

Given that $x = 20$ units.

$$\text{Area} = 9x^2 - 4 \text{ sq. units.}$$
$$= 9 \times (20)^2 - 4 \text{ sq units}$$
$$= 9 \times 400 - 4 \text{ sq units}$$
$$= 3600 - 4 \text{ sq units}$$
$$= 3596 \text{ sq. units.}$$

Thus, the rectangle area 3596 sq units.

(12) $p(x)$ is a polynomial of degree 1 and $q(x)$ is a polynomial of degree 2. What kind of the polynomial $p(x) \times q(x)$ is?

⇒ Given $p(x)$ is a polynomial of degree 1.

So, Let $p(x) = a_1x + b_1$

Given $q(x)$ is a polynomial of degree 2.

Let $q(x) = a_2x^2 + b_2x + c_1$

Now, $p(x) \times q(x)$
$$= (a_1x + b_1)(a_2x^2 + b_2x + c_1)$$
$$= (a_1a_2x^3 + a_1b_2x^2 + a_1c_1x + a_2b_1x^2 + b_1b_2x + b_1c_1)$$
$$= a_1a_2x^3 + (a_1b_2 + a_2b_1)x^2 + (a_1c_1 + b_1b_2)x + b_1c_1$$

∴ This polynomial cubic polynomial or polynomial of degree 3.

Thus, the polynomial $p(x) \times q(x)$ is cubic polynomial or degree 3.