# Exercise - 2.7

1. Rationalise the denominator

(i) $\frac{1}{\sqrt{50}}$ (ii) $\frac{5}{3\sqrt{5}}$ (iii) $\frac{\sqrt{75}}{\sqrt{18}}$ (iv) $\frac{3\sqrt{5}}{\sqrt{6}}$

⇒ (i) $\frac{1}{\sqrt{50}}$

$= \frac{1}{\sqrt{5\times5\times2}}$

$= \frac{1}{5\sqrt{2}}$

$= \frac{\sqrt{2}}{5\times2}$

$= \frac{\sqrt{2}}{10}$

(ii) $\frac{5}{3\sqrt{5}}$

$= \frac{5\times\sqrt{5}}{3\times5}$

$= \frac{5\sqrt{5}}{15}$

$= \frac{\sqrt{5}}{3}$

(iii) $\frac{\sqrt{75}}{\sqrt{18}}$

$= \frac{\sqrt{5\times5\times3}}{\sqrt{2\times3\times3}}$

$= \frac{5\sqrt{3}}{3\sqrt{2}}$

$= \frac{5\sqrt{3}\times\sqrt{2}}{3\times2}$

$= \frac{5\sqrt{6}}{6}$

(iv) $\frac{3\sqrt{5}}{\sqrt{6}}$

$= \frac{3\sqrt{5}\times\sqrt{6}}{\sqrt{6}\times\sqrt{6}}$

$= \frac{3\sqrt{30}}{6}$

$= \frac{\sqrt{30}}{2}$

2. Rationalise the denominator and simplify

(i) $\frac{\sqrt{48}+\sqrt{32}}{\sqrt{27}-\sqrt{18}}$ (ii) $\frac{5\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(iii) $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$ (iv) $\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}} - \frac{\sqrt{5}}{\sqrt{6}-2}$

⇒ (i) $\frac{\sqrt{48}+\sqrt{32}}{\sqrt{27}-\sqrt{18}}$

$= \frac{4\sqrt{3}+4\sqrt{2}}{3\sqrt{3}-3\sqrt{2}}$

$= \frac{4}{3}\times\frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$

$= \frac{4}{3}\times\frac{5+2\sqrt{6}}{1}$

$= \frac{4}{3}(5+2\sqrt{6})$.

(ii) $\frac{5\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{(5\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$= 15-5\sqrt{6}+\sqrt{6}-2$

$= 13-4\sqrt{6}$

$= 13-4\sqrt{6}$.

(iii) $\dfrac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

$= \dfrac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{(3\sqrt{5}-2\sqrt{6})(3\sqrt{5}+2\sqrt{6})}$

$= \dfrac{6\sqrt{30}+4\times6-3\times5-2\sqrt{30}}{9\times5+6\sqrt{30}-6\sqrt{30}-4\times6}$

$= \dfrac{9+4\sqrt{30}}{45-24}$

$= \dfrac{9+4\sqrt{30}}{21}$

$= \dfrac{1}{21}(9+4\sqrt{30})$

(iv) $\dfrac{\sqrt{5}}{\sqrt{6}+2} - \dfrac{\sqrt{5}}{\sqrt{6}-2}$

$= \sqrt{5}\left(\dfrac{1}{\sqrt{6}+2} - \dfrac{1}{\sqrt{6}-2}\right)$

$= \sqrt{5}\left(\dfrac{\sqrt{6}-2-\sqrt{6}-2}{(\sqrt{6}+2)(\sqrt{6}-2)}\right)$

$= \sqrt{5} \times \dfrac{-4}{6-4}$

$= \sqrt{5} \times \dfrac{-4}{2}$

$= \sqrt{5} \times -2$

$= -2\sqrt{5}$

(3) Find the value of a and b if $\dfrac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$

$\Rightarrow$ Given that

$$\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7}+b$$

$$\Rightarrow \frac{(\sqrt{7}-2)^2}{(\sqrt{7}+2)(\sqrt{7}-2)} = a\sqrt{7}+b$$

$$\Rightarrow \frac{11-4\sqrt{7}}{7-4} = a\sqrt{7}+b$$

$$\Rightarrow \frac{11-4\sqrt{7}}{3} = a\sqrt{7}+b$$

$$\Rightarrow -\frac{4}{3}\sqrt{7}+\frac{11}{3} = a\sqrt{7}+b$$

$\Rightarrow$ Now, both side compare.

$\therefore a = -\dfrac{4}{3}$ and $b = \dfrac{11}{3}$

Thus, the value of $a = -\dfrac{4}{3}$

and $b = \dfrac{11}{3}$

(4) If $x = \sqrt{5} + 2$, then find the value of $x^2 + \frac{1}{x^2}$.

$\Rightarrow$

$\cancel{x^2 + \frac{1}{x^2} = (\sqrt{5}+2)^2 + \frac{1}{(\sqrt{5}+2)^2}}$

Given that

$$x = \sqrt{5} + 2$$

Now, $\frac{1}{x} = \frac{1}{\sqrt{5}+2} = \frac{\sqrt{5}-2}{(\sqrt{5}+2)(\sqrt{5}-2)} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$

$\therefore \left(x + \frac{1}{x}\right) = \sqrt{5} + 2 + \sqrt{5} - 2 = 2\sqrt{5}$ --- (i)

Now, $x^2 + \frac{1}{x^2} = (x)^2 + \left(\frac{1}{x}\right)^2$

$$= \left(x + \frac{1}{x}\right)^2 - 2x \times \frac{1}{x}$$

$$= (2\sqrt{5})^2 - 2 \quad [\text{by equation (i)}]$$

$$= 20 - 2 = 18$$

Therefore, $x^2 + \frac{1}{x^2} = 18$.