Exercise - 2.6

(1) Simplify the following using addition and subtraction properties of surds:

(i) $5\sqrt{3}+18\sqrt{3}-2\sqrt{3}$ (ii) $4\sqrt[3]{5}+2\sqrt[3]{5}-3\sqrt[3]{5}$

(iii) $3\sqrt{75}+5\sqrt{48}-\sqrt{243}$ (iv) $5\sqrt[3]{40}+2\sqrt[3]{625}-3\sqrt[3]{320}$

$\Rightarrow$ (i) $5\sqrt{3}+18\sqrt{3}-2\sqrt{3}$

$= 23\sqrt{3}-2\sqrt{3}$

$= 21\sqrt{3}$

(ii) $4\sqrt[3]{5}+2\sqrt[3]{5}-3\sqrt[3]{5}$

$= 6\sqrt[3]{5}-3\sqrt[3]{5}$

$= 3\sqrt[3]{5}$

(iii) $3\sqrt{75}+5\sqrt{48}-\sqrt{243}$

$= 3\sqrt{5\times5\times3}+5\sqrt{4\times4\times3}-\sqrt{9\times9\times3}$

$= 3\times5\times\sqrt{3}+5\times4\times\sqrt{3}-9\times\sqrt{3}$

$= 15\sqrt{3}+20\sqrt{3}-9\sqrt{3}$

$= 35\sqrt{3}-9\sqrt{3}$

$= 26\sqrt{3}$

(iv) $5\sqrt[3]{40}+2\sqrt[3]{625}-3\sqrt[3]{320}$

$= 5\sqrt[3]{2\times2\times2\times5}+2.\sqrt[3]{5\times5\times5\times5}-3\sqrt[3]{4\times4\times4\times5}$

$= 5\times2\times\sqrt[3]{5}+2\times5\times\sqrt[3]{5}-3\times4\times\sqrt[3]{5}$

$= 10\sqrt[3]{5}+10\sqrt[3]{5}-12\sqrt[3]{5}$

$= 20\sqrt[3]{5}-12\sqrt[3]{5}$

$= 8\sqrt[3]{5}$

(2) Simplify the following using multiplication and division properties of surds:

(i) $\sqrt{3} \times \sqrt{5} \times \sqrt{2}$ (ii) $\sqrt{35} \div \sqrt{7}$ (iii) $\sqrt[3]{27} \times \sqrt[3]{8} \times \sqrt[3]{125}$

(iv) $(7\sqrt{a} - 5\sqrt{b})(7\sqrt{a} + 5\sqrt{b})$ (v) $\left[\sqrt{\frac{225}{729}} - \sqrt{\frac{25}{144}}\right] \div \sqrt{\frac{16}{81}}$

$\Rightarrow$ (i) $\sqrt{3} \times \sqrt{5} \times \sqrt{2}$

$= \sqrt{3 \times 5 \times 2}$

$= \sqrt{30}$

(ii) $\sqrt{35} \div \sqrt{7}$

$= \sqrt{\frac{35}{7}}$

$= \sqrt{5}$

(iii) $\sqrt[3]{27} \times \sqrt[3]{8} \times \sqrt[3]{125}$

$= \sqrt[3]{3 \times 3 \times 3} \times \sqrt[3]{2 \times 2 \times 2} \times \sqrt[3]{5 \times 5 \times 5}$

$= 3 \times 2 \times 5$

$= 30$

(iv) $(7\sqrt{a} - 5\sqrt{b})(7\sqrt{a} + 5\sqrt{b})$

$= (7)^2\sqrt{a^2} + 35\cancel{\sqrt{ab}} - 35\cancel{\sqrt{ab}} - (5)^2\sqrt{b^2}$

$\cancel{= 7a - 25b}$

$= 49a - 25b$

(v) $\left[\sqrt{\frac{225}{729}} - \sqrt{\frac{25}{144}}\right] \div \sqrt{\frac{16}{81}}$

$= \dfrac{\left(\frac{15}{27} - \frac{5}{12}\right)}{\frac{4}{9}}$

$= \dfrac{9 \times \left(\frac{60 - 45}{108}\right)}{4}$

$= \dfrac{9 \times \frac{15}{108}}{4}$

$= \dfrac{\cancel{9} \times \cancel{15}\,5}{\cancel{108}\,\cancel{12}\,4 \times 4}$

$= \frac{5}{16}$

(3) If $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$, $\sqrt{10} = 3.162$, then find the values of the following correct to 3 places of decimals.

(i) $\sqrt{40} - \sqrt{20}$ (ii) $\sqrt{300} + \sqrt{90} - \sqrt{8}$

$\Rightarrow$ (i) $\sqrt{40} - \sqrt{20}$

$= \sqrt{4 \times 5 \times 2} - \sqrt{4 \times 5}$

$= 2\sqrt{10} - 2\sqrt{5}$

$= 2\left(\sqrt{5 \times 2} - \sqrt{5}\right)$

$= 2\left(2.236 \times 1.414 - 2.236\right)$ $\left[\because \text{given, } \sqrt{5} = 2.236 \text{ and } \sqrt{2} = 1.414\right]$

$= 2\{2.236(1.414 - 1)\}$

$= 2(2.236 \times .414)$

$= 2 \times 0.925704$

$= 1.851408$

$= 1.852$

(ii) $\sqrt{300} + \sqrt{90} - \sqrt{8}$

$= \sqrt{3 \times 10 \times 10} + \sqrt{3 \times 3 \times 10} - \sqrt{2 \times 2 \times 2}$

$= 10\sqrt{3} + 3\sqrt{10} - 2\sqrt{2}$

$= 10 \times 1.732 + 3 \times 1.414 \times 2.236 - 2 \times 1.414$

$= 17.32 + 9.485112 - 2.828$

$= 26.805112 - 2.828$

$= 23.977112$

$= 23.978$

(4) Arrange surds in deseending order:-

(i) $\sqrt[3]{5}, \sqrt[9]{4}, \sqrt[6]{3}$ (ii) $\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt{\sqrt{3}}$.

$\Rightarrow$ (i) $\sqrt[3]{5}, \sqrt[9]{4}, \sqrt[6]{3}$

L.C.M of 3, 9, 6 = 18

$\sqrt[3]{5} = (5)^{\frac{1}{3}} = (5)^{\frac{6}{18}} = \sqrt[18]{15625}$

$\sqrt[9]{4} = (4)^{\frac{1}{9}} = (4)^{\frac{2}{18}} = \sqrt[18]{16}$

$\sqrt[6]{3} = (3)^{\frac{1}{6}} = (3)^{\frac{3}{18}} = \sqrt[18]{27}$

The desending order of the surds $\sqrt[3]{5}, \sqrt[9]{4}, \sqrt[6]{3}$

is $\sqrt[3]{5} > \sqrt[6]{3} > \sqrt[9]{4}$

(ii) $\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt{\sqrt{3}}$

$= \sqrt[6]{5}, \sqrt[12]{7}, \sqrt[4]{3}$

L.C.M of 6, 12, 4 = 12

$\sqrt[6]{5} = (5)^{\frac{1}{6}} = (5)^{\frac{2}{12}} = \sqrt[12]{25}$

$\sqrt[12]{7} = (7)^{\frac{1}{12}} = \sqrt[12]{7}$

$\sqrt[4]{3} = (3)^{\frac{1}{4}} = (3)^{\frac{3}{12}} = \sqrt[12]{27}$

The desending order of the surds, $\sqrt[2]{\sqrt[3]{5}}, \sqrt[3]{\sqrt[4]{7}}, \sqrt{\sqrt{3}}$

is $\sqrt[4]{3} > \sqrt[6]{5} > \sqrt[12]{7}$

or $\sqrt{\sqrt{3}} > \sqrt[2]{\sqrt[3]{5}} > \sqrt[3]{\sqrt[4]{7}}$

(5) Can you get a pure surd when you find.
(i) the sum of two surds (ii) the differenee of two surds.
(iii) the product of two surds (iv) the quotient of two surds.
Justify each answer with an example.

⇒ (i) yes,

$5\sqrt[3]{3} + 3\sqrt[3]{3}$

$= 8\sqrt[3]{3}$

(ii) yes,

$8\sqrt{6} - 2\sqrt{6}$

$= (8-2)\sqrt{6}$

$= 6\sqrt{6}$

(iii) yes,

$\sqrt[4]{3} \times \sqrt[4]{6}$

$= \sqrt[4]{3\times6}$

$= \sqrt[4]{18}$

(iv) yes,

$\sqrt{30} \div \sqrt{6}$

$= \frac{\sqrt{30}}{\sqrt{6}}$

$= \frac{\sqrt{2\times3\times5}}{\sqrt{2\times3}}$

$= \frac{\cancel{\sqrt{2}}\times\cancel{\sqrt{3}}\times\sqrt{5}}{\cancel{\sqrt{2}}\times\cancel{\sqrt{3}}} = \sqrt{5}$

(6) Can you get a rational number when you compute
(i) the sum of two surds (ii) the differenee of two surds
(iii) the product of two surds (iv) the quotient of two surds
Justify each answer with an example.

⇒ (i) yes,

$(3+\sqrt{5}) + (3-\sqrt{5})$

$= 3+\sqrt{5}+3-\sqrt{5}$

$= 6$, rational number

(ii) yes,

$(8+\sqrt{3}) - (5+\sqrt{3})$

$= 8+\sqrt{3}-5-\sqrt{3}$

$= 8-5 = 3$ rational number.

(iii) yes,

$(3+\sqrt{7})(3-\sqrt{7})$

$= 9+3\sqrt{7}-3\sqrt{7}-7$

$= 9-7$

$= 2$ rational number

(iv) yes,

$\sqrt{50} \div \sqrt{2}$

$= \frac{\sqrt{50}}{\sqrt{2}} = \frac{\sqrt{5\times5\times2}}{\sqrt{2}}$

$= \frac{5\sqrt{2}}{\sqrt{2}} = 5$ rational number.