(iv) $A' \cap B' = \{-3, 0, 1, 2, 5, 7, 8\} \cap \{-3, 0, 1, 2, 3, 4, 6\}$

$A' \cap B' = \{-3, 0, 1, 2\}$

(v) $\cancel{(B \cup C) = \{5, 7, 8}$.

$(B \cup C)' = \{1, 2, 4, 6\}$

$\therefore (B \cup C)' = \{1, 2, 4, 6\}$

(vi) $A - (B \cup C) = \{4, 6\}$

(vii) $A - (B \cap C) = \{-1, 3, 4, 6\}$

(2) If $K = \{a, b, d, e, f\}$, $L = \{b, e, d, g\}$ and $M = \{a, b, e, d, h\}$.
then find the following:

(i) $K \cup (L \cap M)$ (ii) $K \cap (L \cup M)$

(iii) $(K \cup L) \cap (K \cup M)$ (iv) $(K \cap L) \cup (K \cap M)$

and verify distributive laws.

⇒ Given that $K = \{a, b, d, e, f\}$, $L = \{b, e, d, g\}$
and $M = \{a, b, e, d, h\}$.

(i) Now, $L \cap M = \{b, e, d, g\} \cap \{a, b, e, d, h\}$.

$= \{b, e, d\}$

$\therefore K \cup (L \cap M) = \{a, b, d, e, f\} \cup \{b, e, d\}$

$= \{a, b, e, d, e, f\}$

(ii) $(L \cup M) = \{b, e, d, g\} \cup \{a, b, e, d, h\}$

$= \{a, b, e, d, g, h\}$

Now, $K \cap (L \cup M) = \{a, b, d, e, f\} \cap \{a, b, e, d, g, h\}$

$= \{a, b, d\}$

(iii) $K \cup L = \{a, b, d, e, f\} \cup \{b, c, d, g\}$
$= \{a, b, c, d, e, f, g\}$

and $K \cup M = \{a, b, d, e, f\} \cup \{a, b, c, d, h\}$
$= \{a, b, c, d, e, f, h\}$

Now, $(K \cup L) \cap (K \cup M)$
$= \{a, b, c, d, e, f, g\} \cap \{a, b, c, d, e, f, h\}$
$= \{a, b, c, d, e, f\}$

(iv) $K \cap L = \{a, b, d, e, f\} \cap \{b, c, d, g\}$
$= \{b, d\}$

and $K \cap M = \{a, b, d, e, f\} \cap \{a, b, c, d, h\}$
$= \{a, b, d\}$

Now, $(K \cap L) \cup (K \cap B)$
$= \{b, d\} \cup \{a, b, d\}$
$= \{a, b, d\}$

Now, from (i) and (iii), we get,

$\therefore K \cup (L \cap M) = (K \cup L) \cap (K \cup M)$

and from (ii) and (iv), we get

$\therefore K \cap (L \cup M) = (K \cap L) \cup (K \cap M)$

It is verify that the Distributive laws ~~in~~ of intersection and Union of Sets.

(3) If $A = \{x : x \in \mathbb{Z}, -2 < x \leq 4\}$, $B = \{x : x \in W; x \leq 5\}$, $C = \{-4, -1, 0, 2, 3, 4\}$, then verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

$\Rightarrow$ Given that $A = \{x : x \in \mathbb{Z}, -2 < x \leq 4\}$

$A = \{-1, 0, 1, 2, 3, 4\}$

$B = \{x : x \in W; x \leq 5\}$

$B = \{0, 1, 2, 3, 4, 5\}$ and $C = \{-4, -1, 0, 2, 3, 4\}$.

Now, $B \cap C = \{0, 1, 2, 3, 4, 5\} \cap \{-4, -1, 0, 2, 3, 4\}$

$= \{0, 2, 3, 4\}$

$\therefore A \cup (B \cap C) = \{-1, 0, 1, 2, 3, 4\} \cup \{0, 2, 3, 4\}$

$A \cup (B \cap C) = \{-1, 0, 1, 2, 3, 4\}$. ---- (i)

Now, $A \cup B = \{-1, 0, 1, 2, 3, 4\} \cup \{0, 1, 2, 3, 4, 5\}$

$= \{-1, 0, 1, 2, 3, 4, 5\}$

and $A \cup C = \{-1, 0, 1, 2, 3, 4\} \cup \{-4, -1, 0, 2, 3, 4\}$.

$= \{-4, -1, 0, 1, 2, 3, 4\}$

$\therefore$ Now, $(A \cup B) \cap (A \cup C)$

$= \{-1, 0, 1, 2, 3, 4, 5\} \cap \{-4, -1, 0, 1, 2, 3, 4\}$

$= \{-1, 0, 1, 2, 3, 4\}$

$\therefore (A \cup B) \cap (A \cup C) = \{-1, 0, 1, 2, 3, 4\}$ --- (ii)

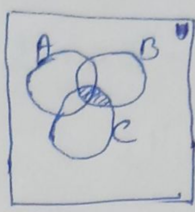
From (i) and (ii), we get.

$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

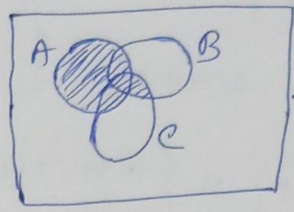
[verifyed].

(4) Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ using venn diagrams.

⇒ Now, $B \cap C$

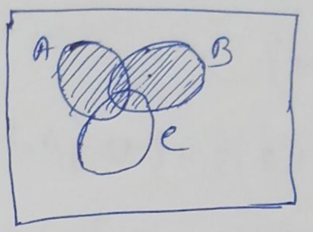


$\therefore A \cup (B \cap C)$

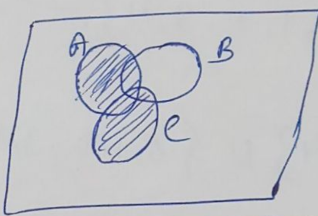


---- (i)

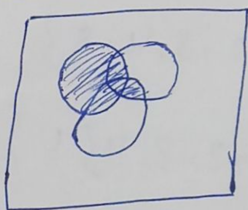
$\therefore A \cup B$



and $A \cup C$



$\therefore (A \cup B) \cap (A \cup C)$



—— (ii)

from (i) and (ii) venn diagrams, we get.

$$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

(5) If $A = \{b, c, e, g, h\}$, $B = \{a, c, d, g, i\}$ and $C = \{a, d, e, g, h\}$, then show that

$$A - (B \cap C) = (A - B) \cup (A - C).$$

⇒ Given that $A = \{b, c, e, g, h\}$

$B = \{a, c, d, g, i\}$ and $C = \{a, d, e, g, h\}$.

Now, $B \cap C = \{a, c, d, g, i\} \cap \{a, d, e, g, h\}$

$= \{a, d, g\}$

$\therefore A-(B\cap C) = \{b, c, e, g, h\} - \{a, d, g\}$

$A-(B\cap C) = \{b, c, e, h\}$ - - - (i)

Now, $(A-B) = \{b, c, e, g, h\} - \{a, c, d, g, i\}$

$= \{b, e, h\}$

and $(A-C) = \{b, c, e, g, h\} - \{a, d, e, g, h\}$

$= \{b, c\}$

Now, $(A-B)\cup(A-C) = \{b, e, h\} \cup \{b, c\}$

$(A-B)\cup(A-C) = \cancel{\{b, e, h\}}\{b, c, e, h\}$ - - (ii)

From (i) and (ii), we get,

$A-(B\cap C) = (A-B)\cup(A-C)$. $\boxed{\text{proved}}$

(6) If $A = \{x : x = 6n, n \in W \text{ and } n < 6\}$, $B = \{x : x = 2n, n \in N, \text{and } 2 < n \leq 9\}$ and $C = \{x : x = 3n, n \in N \text{ and } 4 \leq n < 10\}$, then show that $A-(B\cap C) = (A-B)\cup(A-C)$.

⇒ Given that

$A = \{x : x = 6n, n \in W \text{ and } n < 6\}$

$A = \{0, 6, 12, 18, 24, 30\}$

$B = \{x : x = 2n, n \in N \text{ and } 2 < n \leq 9\}$

$B = \{6, 8, 10, 12, 14, 16, 18\}$

and $C = \{x : x = 3n, n \in N \text{ and } 4 \leq n < 10\}$

$C = \{12, 15, 18, 21, 24, 27\}$

Now, $B\cap C = \{6, 8, 10, 12, 14, 16, 18\} \cap \{12, 15, 18, 21, 24, 27\}$

$B\cap C = \{12, 18\}$

$\therefore A-(B\cap C) = \{0, 6, 12, 18, 24, 30\} - \{12, 18\}$

$A-(B\cap C) = \{0, 6, 24, 30\}$ - - - - (i)

Now, $A - B = \{0, 6, 12, 18, 24, 30\} - \{6, 8, 10, 12, 14, 16, 18\}$

$= \{0, 24, 30\}$

and $A - C = \{0, 6, 12, 18, 24, 30\} - \{12, 15, 18, 21, 24, 27\}$

$= \{0, 6, 30\}$

Now, $(A-B) \cup (A-C) = \{0, 24, 30\} \cup \{0, 6, 30\}$

$(A-B) \cup (A-C) = \{0, 6, 24, 30\}$ ---- (ii)

From (i) and (ii), we get.

$A - (B \cap C) = (A-B) \cup (A-C)$ [proved]

(7) If $A = \{-2, 0, 1, 3, 5\}$, $B = \{-1, 0, 2, 5, 6\}$ and $C = \{-1, 2, 5, 6, 7\}$, then show that

$A - (B \cup C) = (A-B) \cap (A-C)$.

$\Rightarrow$ Given that $A = \{-2, 0, 1, 3, 5\}$

$B = \{-1, 0, 2, 5, 6\}$ and $C = \{-1, 2, 5, 6, 7\}$

Now, $B \cup C = \{-1, 0, 2, 5, 6\} \cup \{-1, 2, 5, 6, 7\}$

$= \{-1, 0, 2, 5, 6, 7\}$.

$\therefore A - (B \cup C) = \{-2, 0, 1, 3, 5\} - \{-1, 0, 2, 5, 6, 7\}$

$A - (B \cup C) = \{-2, 1, 3\}$ --- (i)

Now, $A - B = \{-2, 0, 1, 3, 5\} - \{-1, 0, 2, 5, 6\}$

$A - B = \{-2, 1, 3\}$

$A - C = \{-2, 0, 1, 3, 5\} - \{-1, 2, 5, 6, 7\}$

$= \{-2, 0, 1, 3\}$

Now, $(A-B) \cap (A-C) = \{-2, 1, 3\} \cap \{-2, 0, 1, 3\}$

$(A-B) \cap (A-C) = \{-2, 1, 3\}$ ---- (ii)

From (i) and (ii), we get

$A - (B \cup C) = (A-B) \cap (A-C)$. [Proved]

(8) If $A = \{y : y = \frac{a+1}{2}, a \in W \text{ and } a \le 5\}$, $B = \{y : y = \frac{2n-1}{2}, n \in W \text{ and } n < 5\}$ and $C = \{-1, -\frac{1}{2}, 1, \frac{3}{2}, 2\}$, then show that $A - (B \cup C) = (A - B) \cap (A - C)$.

$\Rightarrow$ Given that

$$A = \{y : y = \frac{a+1}{2}, a \in W \text{ and } a \le 5\}$$

$$A = \{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\}$$

$$\therefore B = \{y : y = \frac{2n-1}{2}, n \in W \text{ and } n < 5\}$$

$$B = \{-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\}$$

and $C = \{-1, -\frac{1}{2}, 1, \frac{3}{2}, 2\}$.

Now, $B \cup C = \{-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\} \cup \{-1, -\frac{1}{2}, 1, \frac{3}{2}, 2\}$

$$= \{-\frac{1}{2}, -1, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 7/2\}$$

$\therefore A - (B \cup C) = \{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\} - \{-\frac{1}{2}, -1, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 7/2\}$

$$A - (B \cup C) = \{3\} \text{ --- (i)}$$

Now, $A - B = \{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\} - \{-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, 7/2\}$

$$= \{1, 2, 3\}$$

$$A - C = \{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\} - \{-1, -\frac{1}{2}, 1, 3/2, 2\}$$

$$= \{\frac{1}{2}, \frac{5}{2}, 3\}$$

Now, $(A - B) \cap (A - C) = \{1, 2, 3\} \cap \{\frac{1}{2}, \frac{5}{2}, 3\}$

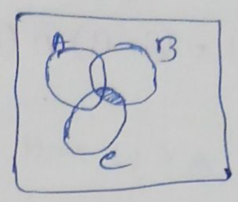
$$(A - B) \cap (A - C) = \{3\} \text{ --- (ii)}$$

from (i) and (ii), we get,

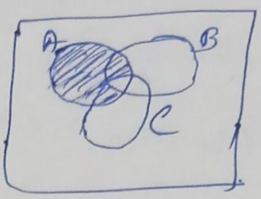
$$A - (B \cup C) = (A - B) \cap (A - C). \quad \boxed{\text{proved}}$$

(9) verify $A-(B\cap C)=(A-B)\cup(A-C)$ using venn diagrams.

$\Rightarrow$ Now, $B\cap C$

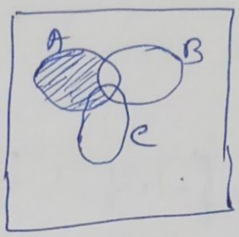


$\therefore A-(B\cap C)$

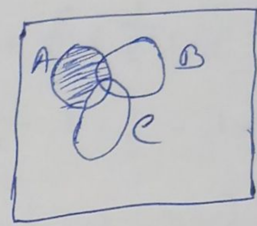


- - - (i)

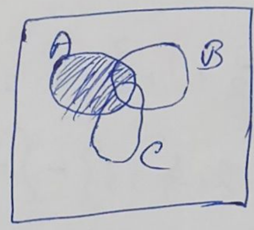
Now, $A-B$



$A-C$



$\therefore (A-B)\cup(A-C)$



- - (ii)

from (i) and (ii) venn diagrams, we get,

$$A-(B\cup C)=(A-B)\cup(A-C).$$

(10) If $U=\{4,7,8,10,11,12,15,16\}$, $A=\{7,8,11,12\}$ and $B=\{4,8,12,15\}$ then verify De morgan's laws for complementation.

$\Rightarrow$ Given that $U=\{4,7,8,10,11,12,15,16\}$

$A=\{7,8,11,12\}$ and $B=\{4,8,12,15\}$

Now, $A\cup B=\{7,8,11,12\}\cup\{4,8,12,15\}$

$=\{4,7,8,11,12,15\}$

$\therefore (A\cup B)'=U-(A\cup B)$

$(A\cup B)'=\{10,16\}$ - - - (i)

Now, $A'=U-A=\{4,10,15,16\}$

$B'=U-B=\{7,10,11,16\}$

Now, $A' \cap B' = \{4, 10, 15, 16\} \cap \{7, 10, 11, 16\}$

$A' \cap B' = \{10, 16\}$ ---- (ii)

from (i) and (ii), we get,

$(A \cup B)' = A' \cap B'$

Now, $A' \cup B' = \{4, 10, 15, 16\} \cup \{7, 10, 11, 16\}$

$A' \cup B' = \{4, 7, 10, 11, 15, 16\}$ ---- (iii)

Now, $A \cap B = \{7, 8, 11, 12\} \cap \{4, 8, 12, 15\}$

$= \{8, 12\}$

$\therefore (A \cap B)' = U - (A \cap B)$

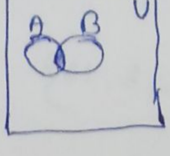
$(A \cap B)' = \{4, 7, 10, 11, 15, 16\}$ ---- (iv)

from (iii) and (iv), we get,

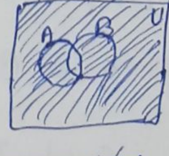
$(A \cap B)' = A' \cup B'$

$\therefore$ It is verifyed that De morgan's laws for complementation.

(11) verify $(A \cap B)' = A' \cup B'$ using venn diagrams.

⇒ Now, $(A \cap B)$ 

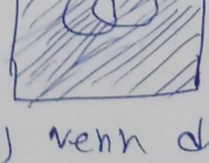
$(A \cap B)' = U - (A \cap B)$



---- (i)

Now, A' 

B' 

Now, $A' \cup B'$ 

---- (ii)

from (i) and (ii) venn diagrams, we get

$\therefore (A \cap B)' = A' \cup B'$ [verifed].