Exercise - 1.4

(1) If $P = \{1,2,5,7,9\}$, $Q = \{2,3,5,9,11\}$ and $R = \{3,4,5,7,9\}$ and $S = \{2,3,4,5,8\}$, then find (i) $(P \cup Q) \cup R$ (ii) $(P \cap Q) \cap S$ (iii) $(Q \cap S) \cap R$.

⇒ Given that

$P = \{1,2,5,7,9\}$, $Q = \{2,3,5,9,11\}$

$R = \{3,4,5,7,9\}$ and $S = \{2,3,4,5,8\}$

(i) $P \cup Q = \{1,2,5,7,9\} \cup \{2,3,5,9,11\}$

$= \{1,2,3,5,7,9,11\}$

now, $(P \cup Q) \cup R = \{1,2,3,5,7,9,11\} \cup \{3,4,5,7,9\}$

$= \{1,2,3,4,5,7,9,11\}$.

(ii) $P \cap Q = \{1,2,5,7,9\} \cap \{2,3,5,9,11\}$

$= \{2,5,9\}$

now, $(P \cap Q) \cap S = \{2,5,9\} \cap \{2,3,4,5,8\}$

$= \{2,5\}$

(iii) $Q \cap S = \{2,3,5,9,11\} \cap \{2,3,4,5,8\}$

$= \{2,3,5\}$

now, $(Q \cap S) \cap R = \{2,3,5\} \cap \{3,4,5,7,9\}$

$= \{3,5\}$

(2) Test for the commutative property of union and intersection of the sets.

$P = \{x : x$ is a real number between 2 and 7$\}$ and

$Q = \{x : x$ is a irrational number between 2 and 7$\}$.

$\Rightarrow$ Given that

$P = \{x : x$ is a real number between 2 and 7$\}$

$$P = \{3, 4, 5, 6\}$$

and $Q = \{x : x$ is a irrational number between 2 and 7$\}$

$$Q = \{\sqrt{3}, \sqrt{5}, \sqrt{6}\}$$

Now, $P \cup Q = \{3, 4, 5, 6\} \cup \{\sqrt{3}, \sqrt{5}, \sqrt{6}\}$

$$P \cup Q = \{3, 4, 5, 6, \sqrt{3}, \sqrt{5}, \sqrt{6}\} \quad \text{---- (i)}$$

and $Q \cup P = \{\sqrt{3}, \sqrt{5}, \sqrt{6}\} \cup \{3, 4, 5, 6\}$

$$Q \cup P = \{\sqrt{3}, \sqrt{5}, \sqrt{6}, 3, 4, 5, 6\} \quad \text{-- (ii)}$$

from (i) and (ii), we get

$$P \cup Q = Q \cup P.$$

Now, $P \cap Q = \{3, 4, 5, 6\} \cap \{\sqrt{3}, \sqrt{5}, \sqrt{6}\}$

$$P \cap Q = \{\ \} \quad \text{--- (iii)}$$

and $Q \cap P = \{\sqrt{3}, \sqrt{5}, \sqrt{6}\} \cap \{3, 4, 5, 6\}$

$$Q \cap P = \{\ \} \quad \text{---- (iv)}$$

from (iii) and (iv), we get,

$$P \cap Q = Q \cap P.$$

It is verified that commutative property of Vnion and intersection of the sets.

③ If $A = \{p, q, r, s\}$, $B = \{m, n, q, s, t\}$ and $C = \{m, n, p, q, s\}$, then verify the associative property of union of sets.

⇒ Given that $A = \{p, q, r, s\}$, $B = \{m, n, q, s, t\}$
and $C = \{m, n, p, q, s\}$.

Now, $B \cup C = \{m, n, q, s, t\} \cup \{m, n, p, q, s\}$
$= \{m, n, q, s, t, p\}$

$\therefore A \cup (B \cup C) = \{p, q, r, s\} \cup \{m, n, p, q, s, t\}$

$A \cup (B \cup C) = \{p, q, r, s, m, n, t\}$ --- (i)

Now, $A \cup B = \{p, q, r, s\} \cup \{m, n, q, s, t\}$
$A \cup B = \{p, q, r, s, m, n, t\}$

$\therefore (A \cup B) \cup C = \{p, q, r, s, t, m, n\} \cup \{m, n, p, q, s\}$

$(A \cup B) \cup C = \{p, q, r, s, m, n, t\}$ -- (ii)

From (i) and (ii), we get
$A \cup (B \cup C) = (A \cup B) \cup C$.

It is verify that the associative property of union of sets.

④ verify the associative property of intersection of sets for $A = \{-11, \sqrt{2}, \sqrt{5}, 7\}$, $B = \{\sqrt{3}, \sqrt{5}, 6, 13\}$ and $C = \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\}$.

⇒ Given that, $A = \{-11, \sqrt{2}, \sqrt{5}, 7\}$
$B = \{\sqrt{3}, \sqrt{5}, 6, 13\}$
and $C = \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\}$.

Now, $B \cap C = \{\sqrt{3}, \sqrt{5}, 6, 13\} \cap \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\}$

$= \{\sqrt{3}, \sqrt{5}\}$

$\therefore A \cap (B \cap C) = \{-11, \sqrt{2}, \sqrt{5}, 7\} \cap \{\sqrt{3}, \sqrt{5}\}$

$A \cap (B \cap C) = \{\sqrt{5}\}$. ---- (i)

Now, $A \cap B = \{-11, \sqrt{2}, \sqrt{5}, 7\} \cap \{\sqrt{3}, \sqrt{5}, 6, 13\}$

$= \{\sqrt{5}\}$

$\therefore (A \cap B) \cap C = \{\sqrt{5}\} \cap \{\sqrt{2}, \sqrt{3}, \sqrt{5}, 9\}$

$(A \cap B) \cap C = \{\sqrt{5}\}$ --- (ii)

From (i) and (ii), we get,

$A \cap (B \cap C) = (A \cap B) \cap C$.

It is verify that the associative property of intersection of sets.

5. If $A = \{x : x = 2^n, n \in W \text{ and } n < 4\}$, $B = \{x : x = 2n, n \in N \text{ and } n \leq 4\}$ and $C = \{0, 1, 2, 5, 6\}$, then verify the associative property of intersection of sets.

$\Rightarrow$ Given that $A = \{x : x = 2^n, n \in W \text{ and } n < 4\}$

$\therefore A = \{1, 2, 4, 8\}$

$B = \{x : x = 2n, n \in N \text{ and } n \leq 4\}$

$B = \{2, 4, 6, 8\}$ and $C = \{0, 1, 2, 5, 6\}$.

Now, $B \cap C = \{2, 4, 6, 8\} \cap \{0, 1, 2, 5, 6\}$

$= \{2, 6\}$

$\therefore A \cap (B \cap C) = \{1, 2, 4, 8\} \cap \{2, 6\}$

$A \cap (B \cap C) = \{2\}$ --- (i)