Exercise - 1.2

① Find the cardinal number of the following sets.

(i) $M = \{p, q, r, s, t, u\}$

(ii) $P = \{x : x = 3n+2,\ n \in W \text{ and } x < 15\}$

(iii) $Q = \{y : y = \frac{4}{3n},\ n \in N \text{ and } 2 < n \leq 5\}$

(iv) $R = \{x : x \text{ is an integers},\ x \in Z \text{ and } -5 \leq x < 5\}$

(v) S = The set of all leap years between 1882 and 1906.

⇒ (i) $M = \{p, q, r, s, t, u\}$

Now, Set M contains 6 elements.

Thus, $n(M) = 6$

(ii) $P = \{x : x = 3n+2,\ n \in W \text{ and } x < 15\}$

Now, $n = 0$ then, $x = 3.0+2 = 2$

$n = 1$ then, $x = 3.1+2 = 5$

$n = 2$ then, $x = 3.2+2 = 8$

$n = 3$ then $x = 3.3+2 = 11$

$n = 4$ then $x = 3.4+2 = 14$

∴ Now, $P = \{2, 5, 8, 11, 14\}$

Set P contains 5 elements.

∴ Thus, $n(P) = 5$

(iii) $Q = \{y : y = \frac{4}{3n},\ n \in N \text{ and } 2 < n \leq 5\}$

We know that $N = \{1, 2, 3, \ldots\}$

Now, $n = 3$ then $y = \frac{4}{3.3} = \frac{4}{9}$ $[\because 2 < n \leq 5]$

$n = 4$ then $y = \frac{4}{3.4} = \frac{4}{12} = \frac{1}{3}$

$n = 5$ then $y = \frac{4}{3.5} = \frac{4}{15}$

∴ Now, $Q = \{\frac{4}{9}, \frac{1}{3}, \frac{4}{15}\}$.

Now Set Q contains 3 elements.

Thus, $n(Q) = 3$.

(iv) $R = \{x : x$ is an integers, $x \in \mathbb{Z}$ and $-5 \le x < 5\}$

$\therefore R = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

Now, Set R contains 10 elements.

Thus, $n(R) = 10$

(v) S = The set of all leap years between 1882 and 1906.

Now, 1882 to 1906 leap years are 1884, 1888, 1892, 1896, 1904.

Now, $S = \{1884, 1888, 1892, 1896, 1904\}$

Thus, $n(S) = 5$

② Identify the following sets as finite or infinite.

(i) X = The set of all districts in Tamilnadu.

(ii) Y = The set of all straight lines passing through a point.

(iii) $A = \{x : x \in \mathbb{Z}$ and $x < 5\}$

(iv) $B = \{x : x^2 - 5x + 6 = 0, x \in \mathbb{N}\}$.

⇒ (i) X = The set of all districts in Tamilnadu

we know that Tamilnadu finite numbers of districts.

Set X contains finite numbers elements.

Thus, the set X is finite.

(ii) Y = The set of all straight lines passing through a point.

we know that any straight infinite numbers of point contains.

$\therefore$ Set Y contains infinite numbers of elements.

Thus, the set Y is infinite.

(iii) $A = \{x : x \in \mathbb{Z} \text{ and } x < 5\}$

we know that $\mathbb{Z} = (-\infty, \infty)$

$\therefore A = \{-\infty, \ldots, -2, -1, 0, 1, 2, 3, 4\}$.

$\therefore$ Set A contains infinite numbers ellements.

Thus, Set A is infinite.

(iv) $B = \{x : x^2 - 5x + 6 = 0, x \in \mathbb{N}\}$

Now, $x^2 - 5x + 6 = 0$

$x^2 - 3x - 2x + 6 = 0$

$x(x-3) - 2(x-3) = 0$

$(x-3)(x-2) = 0$

$x - 3 = 0 \quad | \quad x - 2 = 0$

$x = 3 \quad | \quad x = 2$

Now, $B = \{3, 2\}$.

Set B contains finite elements.

Thus, Set B is finite.

③ Which of the following sets are equivalent or unequal or equal sets?

(i) A = The set of vowels in the English alphabets.

B = The set of all letters in the word "VOWEL"

(ii) $C = \{2, 3, 4, 5\}$ $\quad D = \{x : x \in W, 1 < x < 5\}$

(iii) X = {x: x is a letter in the word "LIFE"}

$Y = \{F, I, L, E\}$

(iv) G = {x : x is a prime number and $3 < x < 23$}

H = {x : x is a divisor of 18}

(i) A = The set of vowels in the English alphabets.

$\therefore A = \{A, E, I, O, U\}$

$\therefore n(A) = 5$

and B = The set of all letters in the word "VOWEL"

$\therefore B = \{V, O, W, E, L\}$

$\therefore n(B) = 5.$

Now, $n(A) = n(B)$

So, ~~A~~ Set A and B are equivalent sets.

(ii) $C = \{2, 3, 4, 5\}$ and $D = \{x : x \in W, 1 < x < 5\}$

$n(C) = 4$

$D = \{2, 3, 4\}$

$n(D) = 3$

Now, $n(C) \neq n(D)$

So, Set C and D are un-equal sets

(iii) $X = \{x : x$ is a letter in the word "LIFE"$\}$

$X = \{L, I, F, E\}$

$n(X) = 4$

and $Y = \{F, I, L, E$

$n(Y) = 4$

Now, $n(X) = n(Y)$ and same elements.

So, Set X and Y are equal sets.

(iv) $G = \{x : x$ is a prime number and $3 < x < 23\}$

$G = \{5, 7, 11, 13, 17, 19\}$.

$\therefore n(G) = 6$

and $H = \{x : x$ is a divisor of $18\}$

$H = \{1, 2, 3, 6, 9, 18\}$

$n(H) = 6$

Now, $n(G) = n(H)$

So, Set G and H are equivalent sets.

(4) Identify the following sets ~~are~~ as null set or singleton set.

(i) $A = \{x : x \in N, 1 < x < 2\}$

(ii) B = The set of all even natural numbers which are not divisible by 2

(iii) $C = \{0\}$.

(iv) D = The set of all triangles having four sides.

⇒ (i) $A = \{x : x \in N, 1 < x < 2\}$

Now, $N = \{1, 2, 3, \dots\}$

Now, 1 to 2 between no natural numbers.

$\therefore A = \{\ \}$.

Thus, A is a null set.

(ii) B = The set of all even natural numbers which are not divisible by 2

We know that every even natural numbers divisible by 2

So, $B = \{\ \}$.

Thus, B is a null set

(iii) $C = \{0\}$.

$\therefore n(C) = 1$

So, C is a singleton set.

(iv) D = The set of all triangles having four sides.

We know that every triangle ~~havi~~ contains 3 sides.

So, $D = \{\ \}$.

Thus, D is a null set.

⑤ State which pairs of sets are disjoint or overlapping?

(i) $A = \{f, i, a, s\}$ and $B = \{a, n, f, h, s\}$

(ii) $C = \{x : x$ is a prime number, $x > 2\}$ and

$D = \{x : x$ is an even prime number$\}$.

(iii) $E = \{x : x$ is a factor of $24\}$ and

$F = \{x : x$ is a multiple of $3, x < 30\}$

⇒ (i) Given that $A = \{f, i, a, s\}$ and $B = \{a, n, f, h, s\}$

Now, $A \cap B = \{f, i, a, s\} \cap \{a, n, f, h, s\}$

$= \{f, a, s\}$

Now, $A \cap B = \{f, a, s\}$

$\therefore A \cap B \neq \phi$

So, A and B sets are overlapping.

(ii) $C = \{x : x$ is a prime number, $x > 2\}$

$C = \{3, 5, 7, 11, 13, 17, 19, \dots\}$

and $D = \{x : x$ is an even prime number$\}$

$D = \{2\}$.

Now, $C \cap D = \{3, 5, 7, 11, 13, 17, 19 \dots\} \cap \{2\}$

$= \{\ \}$

Now, $C \cap D = \phi$

Thus, set C and D are disjoint sets.

(iii) $E = \{x : x$ is a factor of $24\}$

$E = \{1, 2, 3, 4, 6, 8, 12, 24\}$

and $F = \{x : x$ is a multiple of $3, x < 30\}$

$F = \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$

Now, $E \cap F$

$= \{1, 2, 3, 4, 6, 8, 12, 24\} \cap \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$

$= \{3, 6, 12, 24\}$

Now, $E \cap F = \{3, 6, 12, 24\}$

So, $E \cap F \neq \phi$

Thus, Set E and F are overlapping sets.

(6) If $S = \{$square, rectangle, circle, rhombus, triangle$\}$, list the elements of the following subset of S.

(i) The set of shapes which have 4 equal sides.

(ii) The set of shapes which have radius.

(iii) The set of shapes in which the sum of all interior angles is 180°.

(iv) The set of shapes which have 5 sides.

⇒ (i) The set of shapes which have 4 equal sides.

We know that square and rhombus contains 4 equal sides.

Thus, subset of S = $\{$square, rhombus$\}$.

(ii) The set of shapes which have radius.

We know that circle has radius.

Thus, subset of S = $\{$circle$\}$.

(iii) The set of shapes in which the sum of all interior angles is 180°.

We know that ~~triangles~~ triangle the sum of all interior angles is 180.

Thus, subset of S = $\{$triangle$\}$.

(iv) The set of shapes which have 5 sides.

$\therefore$ Subset of $S = \{\ \} = \phi$

(7) If $A = \{a, \{a,b\}\}$, write all the subsets of A.

$\Rightarrow$ Given $A = \{a, \{a,b\}\}$.

Thus, All the subsets of A are $\{\ \}$, $\{a\}$, $\{a,b\}$ and $\{a, \{a,b\}\}$.

(8) Write down the power set of the following sets;

(i) $A = \{a,b\}$, (ii) $B = \{1,2,3\}$, (iii) $D = \{p,q,r,s\}$

(iv) $E = \phi$

$\Rightarrow$ (i) $A = \{a,b\}$.

all the subsets of A are ϕ, $\{a\}$, $\{b\}$, $\{a,b\}$.

$\therefore$ power set $P(A) = \{\phi, \{a\}, \{b\}, \{a,b\}\}$

(ii) $B = \{1,2,3\}$

all the subsets of B are ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$ $\{2,3\}$ and $\{1,2,3\}$

power set $P(B) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

(iii) $D = \{p,q,r,s\}$

all the subsets of D are ϕ, $\{p\}$, $\{q\}$, $\{r\}$, $\{s\}$, $\{p,q\}$, $\{p,r\}$ $\{p,s\}$, $\{q,r\}$, $\{q,s\}$, $\{r,s\}$, $\{p,q,r\}$, $\{p,r,s\}$, $\{q,r,s\}$, $\{p,q,r,s\}$.

$$\text{Power set } P(D) = \left\{\begin{array}{l}\phi, \{p\}, \{q\}, \{r\}, \{s\}, \{p,q\}, \{p,r\}, \{p,s\} \\ \{q,r\}, \{q,s\}, \{r,s\}, \{p,q,r\}, \{p,r,s\} \\ \{q,r,s\}, \{p,q,r,s\}.\end{array}\right\}$$

(iv) $E = \phi$

The subset of E is ϕ

power set $P(E) = \phi$.

(9) Find the number of subsets and the number of proper subsets of the following sets,

(i) $W = \{red, blue, yellow\}$

(ii) $X = \{x^2 : x \in N, x^2 \leq 100\}$.

$\Rightarrow$ (i) Given that

$W = \{red, blue, yellow\}$

$\therefore n(W) = 3$

The number of subsets $= n[P(W)]$

$= 2^3 = 8$

The number of proper subsets

$= n[P(W)] - 1$

$= 2^3 - 1$

$= 8 - 1$

$= 7$

(ii) Given that

$X = \{x^2 : x \in N, x^2 \leq 100\}$

$X = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

$\therefore n(X) = 10$

The number of subsets $= n[P(X)]$

$= 2^{10}$

$= 1024$

The number of proper subsets

$= n[P(X)] - 1$

$= 2^{10} - 1$

$= 1024 - 1$

$= 1023$.

(10) (i) If $n(A) = 4$, find $n[P(A)]$.

(ii) If $n(A) = 0$, find $n[P(A)]$.

(iii) If $n[P(A)] = 256$, find $n(A)$.

⇒ (i) Given that $n(A) = 4$

we know, that $n[P(A)]$

$= 2^{n(A)}$

$= 2^4 = 16$

(ii) Given that $n(A) = 0$.

then, $n[P(A)]$

$= 2^{n(A)} = 2^0 = 1$

(iii) Given that

$n[P(A)] = 256$

Let $n(A) = x$

Now, $2^x = 256$

$2^x = 2^8$

$x = 8$

Thus, $n(A) = 8$.