Chapter 10: Circles

Exercise 10.1

1. Fill in the blanks:

→ i) The common point of tangent and the circle is called ——.

ii) A circle may have —— parallel tangents.

iii) A tangent to a circle intersects it in —— point.

iv) A line intersecting a circle in two points is called a ——.

v) The angle between tangent at a point P on circle and radius through the point is ——

⇒ i) The common point of tangent and the circle is called point of contact.

ii) A circle may have two parallel tangents.

iii) A tangent to a circle intersects it in one point.

iv) A line intersecting a circle in two points is called a ~~segm~~ secant.

v) The angle between tangent at a point 'P' on circle and radius through the point is 90°.

2. How many tangents can a circle have?

→ A tangent to the circle is defined as, it is the line which is intersecting a circle at a single point. As there are infinite number of points on the circle we can draw infinite number of tangents to the circle.

Exercise 10.2

1) If PT is a tangent at T to a circle whose centre is O and OP = 17cm, OT = 8cm. Find the length of the tangent segment PT.

Solution:- Given that, OT = radius = 8 cm

OP = 17 cm

To find the length of the tangent segment PT:

From fig, T is a point of contact.

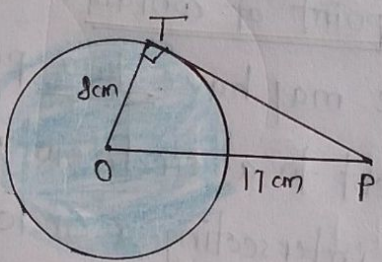
And at point of contact tangent and radius are always perpendicular.

$\therefore \angle OTP = 90^\circ$

In ΔOTP, By Paythagoras theorem,

$8^2 + PT^2 = 17^2$

$17^2 - 8^2 = PT^2$



$PT = \sqrt{289-64}$

$PT = \sqrt{225}$

$\boxed{PT = 15\text{ cm}}$

Thus, the length of the tangent segment is found to be 15cm.

2) Find the length of a tangent drawn to a circle with radius 5cm, from a point 13cm from the centre of a circle.

Solution:- Given that, radius of a circle = 5 cm

Let us consider a circle with centre point 'O'.

From fig, OP = 5 cm, OQ = 13 cm

To find length of tangent PQ:

from fig, In OPQ, $\angle OPQ = 90°$

By Paythagoras theorem,

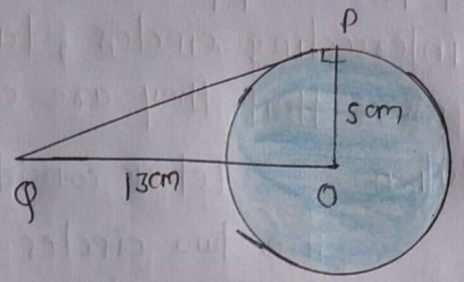
$OQ^2 = OP^2 + PQ^2$

$13^2 = 5^2 + PQ^2$

$PQ^2 = 169 - 25$

$PQ^2 = 144$

$\boxed{PQ = 12\text{ cm}}$



Thus, here the length of the tangent is found to be 12cm.

3) A point 'P' is 26 cm away from 'O' of circle and the length PT of the tangent drawn from P to the circle is 10cm. Find the radius of the circle.

Solution:- Given that, OP = 26 cm

PT = length of tangent = 10cm

Here, to find the radius of circle = OT = ?

from fig, at point of contact, the radius & tangent are perpendicular to each other.

$\therefore \angle OTP = 90°$

Now, In ΔOTP, By Paythagoras theorem,

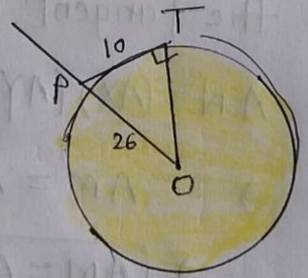
$OP^2 = OT^2 + PT^2$

$26^2 = OT^2 + 10^2$

$OT^2 = 676 - 100$

$OT^2 = 576$

$\boxed{OT = 24\text{ cm}}$



Thus, here the radius of circle is OT = 24 cm.

4) If from any point on the common chord of two intersecting circles, tangents be drawn to the circles, prove that they are equal.

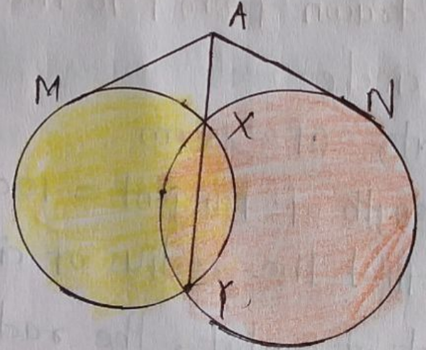
solution:- Let us consider here,

The two circles intersects each other in two points X and Y as shown in fig. below.

And hence, we can say that XY is the common chord.

Suppose, 'A' is any point on the common chord as shown in fig and AM, AN are the tangents drawn from point A to the circles.

Now, we have to prove that: $AM = AN$



from fig, AM is the tangent and AXY is the secant

$$\therefore AM^2 = (AX)(AY) \quad \text{—— ①}$$

Also, AN is the tangent and AXY is a secant.

$$\therefore AN^2 = (AX)(AY) \quad \text{—— ②}$$

from ① & ② $\Rightarrow$ $AM^2 = AN^2$

$$\Rightarrow \boxed{AM = AN}$$

Thus, we can say that, the tangents drawn from any point on the common chord of two intersecting circles are equal.

Hence proved.

5) If the quadrilateral sides touch the circle, prove that the sum of pair of opposite sides is equal to the sum of the other pair.

Solution:-

Let us consider a quadrilateral ABCD which touches a circle having centre point 'O' at points E, F, G and H respectively as shown in fig.

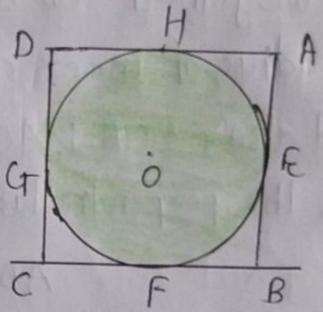
But, we already know that

The tangents drawn from same external points to the circle are equal in length always.

Let us consider:

1) Tangents from point 'A' are AH and AE

$\Rightarrow AH = AE$ — ①



2) Tangents from point 'B' are EB & BF.

$\Rightarrow BF = EB$ — ②

3) Tangents from point 'C' are CF & GC

$\Rightarrow CF = GC$ — ③

4) tangents from point 'D' are DG & DH

$\Rightarrow DG = DH$ — ④

By taking addition of ①, ② & ③, ④.

$(AH + BF + CF + DH) = (AE + EB + CG + DG)$

$(AH + DH) + (BF + FC) = (AE + EB) + (CG + DG)$

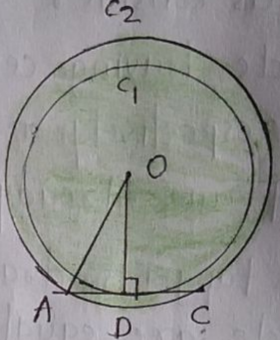
$\Rightarrow AD + BC = AB + DC$ $\because$ from fig.

Thus, we can say that, the sum of one pair of opposite sides is equal to other.

Hence proved.

6) Out of the two concentric circles, the radius of the outer circle is 5cm and the chord AC of length 8cm is a tangent to the inner circle. Find the radius of inner circle.

→ Here, we consider that,
The two concentric circles C_1 & C_2 with their point of centre 'O'.
As shown in fig, AC is the chord which touches inner circle 'C_1' at point 'D'.



- Now, we joined 'OD' which is the radius of inner circle.

And also, $OD \perp AC$ from figure.
Here, the perpendicular line OD bisects the chord AC.

$\therefore \quad AD = DC = 4cm$

In ΔAOD, $\quad \angle ADO = 90°$

$\Rightarrow (OA)^2 = AD^2 + DO^2$ $\because$ By Paythagoras theorem

$DO^2 = 5^2 - 4^2 = 25 - 16 = 9$

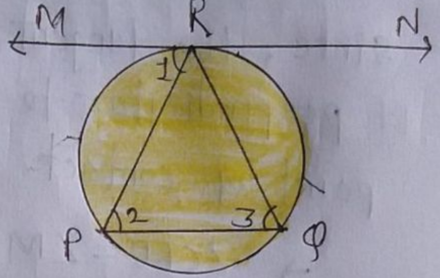
$\Rightarrow \boxed{DO = 3cm}$

Therefore, the radius of the inner circle is found to be 3cm.

7) A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that 'R' bisects the arc PRQ.

→ Here, given that,

The chord PQ is parallel to tangent at R.



Now, we have to prove that:

R bisects the arc PRQ.

Now, from fig.

PQ || tangent drawn at point 'R'

Hence, from figure,

$\angle MRP = \angle RPQ$ ∵ alternate internal angles

and $\angle MRP = \angle RQP$

Because, angle between tangent and chord is equal to angle made by chord in alternate segment.

$\Rightarrow \angle RPQ = \angle RQP$

And hence, the sides opposite to the equal angles are also equal. $\Rightarrow PR = QR$

Hence, we can say that 'R' bisects the arc PRQ.

8) Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

→ Given that,

From figure, AB is a diameter of the circle.

And we drawn a tangent from point 'A' as shown in fig.

Now, we are drawing a chord CD parallel to the tangent MAN.

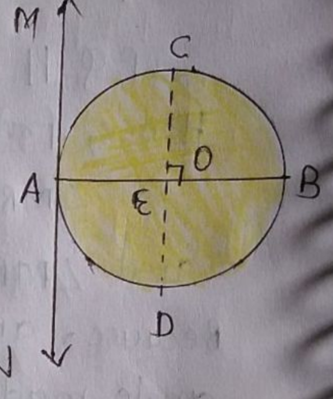
from fig. CD is the chord and OA is the radius of the circle.

$\therefore \angle MAO = 90°$

Since, the tangent at any point of circle is always perpendicular to the radius of circle. at that point of contact.

$\Rightarrow \angle CEO = \angle MAO$ since they are corresponding angles

$\Rightarrow \angle CEO = 90°$



Hence, we can say that,

OE is bisecting CD.

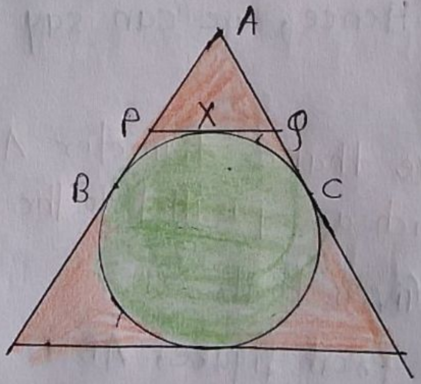
Simillarly, we can say that the diameter AB bisects all the chords which are parallel to the tangent at point A.

9) If AB, AC, PQ are the tangents in the figure, and AB = 5cm, find the perimeter of ΔAPQ.

→ • Here, given that

• AB, AC and PQ are tangents as shown in figure.

• Also, AB = 5 cm



Perimeter of ΔPAQ,

$P = AP + PQ + QA$

$= AP + (PX + QX) + QA$

But, the two tangents drawn from external point to the circle are equal in length.

Thus, here from fig

$AB = AC = 5cm$ (at point A)

Also, $PX = PB$ (at point P)

$QX = QC$ (at point Q)

The total perimeter is given by,

$P = AP + AQ + (PB + QC)$

$= (AP + PB) + (AQ + QC)$

$= AB + AC$

$= 5 + 5$

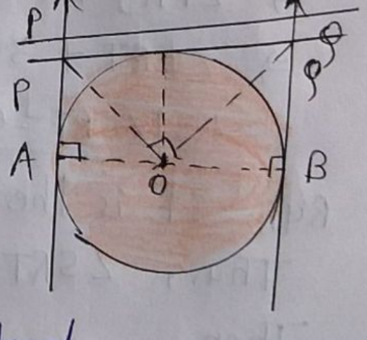
$\boxed{P = 10cm}$

Thus, the total perimeter of triangle APQ is 10cm.

10) Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at centre.

→

- Let us consider, a circle with centre 'O' and two parallel tangents touches circle at points A and B respectively as shown in fig.
- The points A and B are the two end points of diameter AB.
- Now, we drawn a tangent through point 'M' which touches other two tangents at points P & Q respectively,



from fig. to prove: $\angle POQ = 90°$.

The fig. shows that ABQP is a quadrilateral.

So, $\angle A + \angle B = 90° + 90° = 180°$

$\angle A + \angle B + \angle P + \angle Q = 360°$

$\Rightarrow \angle P + \angle Q = 360° - 180° = 180°$

But at point P & Q,

$\angle APO = \angle OPQ = \frac{1}{2} \angle P$

$\angle BQO = \angle PQO = \frac{1}{2}\angle Q$

$\Rightarrow 2\angle OPQ + 2\angle PQO = 180°$

$\boxed{\angle OPQ + \angle PQO = 90°}$

Now, In ΔOPQ,

$\angle OPQ + \angle PQO + \angle POQ = 180°$

$90° + \angle POQ = 180°$

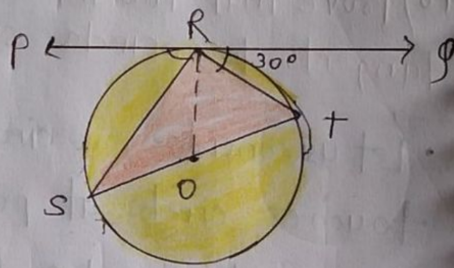
$\angle POQ = 180° - 90° = 90°$

Thus, $\boxed{\angle POQ = 90°}$

11) In fig. below, PQ is tangent at point R of the circle with centre 'O'. If $\angle TRQ = 30°$, find $\angle PRS$.

→

From fig.,



$\angle TRQ = 30°$

Also, at point R, $OR \perp RQ$

Thus, $\angle ORQ = 90°$

$\Rightarrow \angle TRQ + \angle ORT = 90°$

$\Rightarrow \angle ORT + 30° = 90°$

$\boxed{\angle ORT = 60°}$

But, ST is the diameter here.

Thus, $\angle SRT = 90°$ which is the angle substended in a semicircle.

Then, $\angle ORT + \angle SRO = 90°$

$\angle SRO + \angle PRS = 90°$

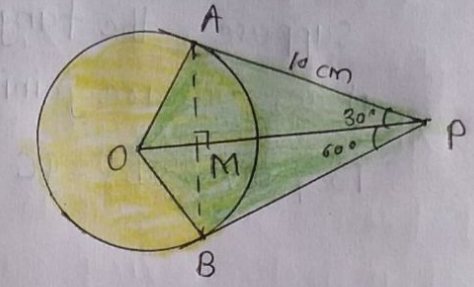
$\therefore \angle PRS = 90° - 30° = 60°$

Thus, $\boxed{\angle PRS = 60°}$

12) If PA and PB are tangents from an outside point P, such that PA = 10 cm and $\angle APB = 60°$. Find the length of chord AB.

→ from fig,

$AP = 10cm$ and $\angle APB = 60°$



We all know that,
A line drawn from centre to point where external tangents to the circle are drawn divides or bisects the angle made by tangents at that point.

So, $\angle APO = \angle OPB = \frac{1}{2}(60°) = 30°$

And, AB chord is bisected perpendicularly.

$\therefore AB = 2AM$

In ΔAMP, $\sin 30° = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AM}{AP}$

$AM = AP \sin 30°$

$AP/2 = 10/2 = 5cm$ $\qquad \because AB = 2AM$

Thus, $AP = 2AM = 10 cm$

And $AB = 2AM = 10 cm$

Another Method:

In ΔAMP, $\angle AMP = 90°$, $\angle APM = 30°$

$\angle AMP + \angle APM + \angle MAP = 180°$

$90° + 30° + \angle MAP = 180°$

$\Rightarrow \boxed{\angle MAP = 60°}$

In ΔPAB, $\angle MAP = \angle BAP = 60°$, $\angle APB = 60°$

$\Rightarrow \angle PBA = 60°$

Thus, ΔAPB is a equilateral triangle.

$\boxed{AB = AP = 10cm}$

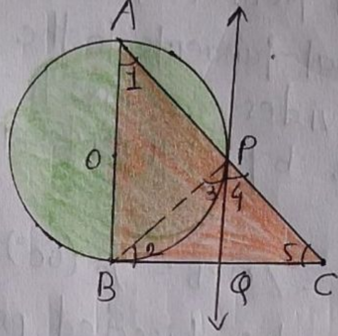
13) In a right triangle ABC in which $\angle B = 90°$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisects BC.

→ Let us consider a circle with centre 'O'.

Suppose, the tangent at 'P' meets BC at 'Q'.

Now, we are joining BP.

To prove: $BQ = QC$



Here,

$\angle ABC = 90°$

Because, tangent at any point of circle is perpendicular to radius through the point of contact.

In ΔABC, $\angle 1 + \angle 5 = 90°$ $\because \angle ABC = 90°$

and also, $\boxed{\angle 3 = \angle 1}$

Since, angle between tangent and the chord equals angle made by the chord in alternate segment.

Thus, $\angle 3 + \angle 5 = 90°$ —— ①

And, the angle in semicircle is of 90°.

$\therefore \angle APB = 90°$

$\therefore \angle 3 + \angle 4 = 90°$ —— ② $\because \angle APB + \angle BPC = 180°$

equn ① & ② ⇒ $\angle 3 + \angle 5 = \angle 3 + \angle 4$

⇒ $\boxed{\angle 5 = \angle 4}$

But, we know that,

The sides opposite to equal angle are also equal.

⇒ $\boxed{PQ = QC}$

Also, QP = QB

Tangents drawn from an internal point to a circle are equal.

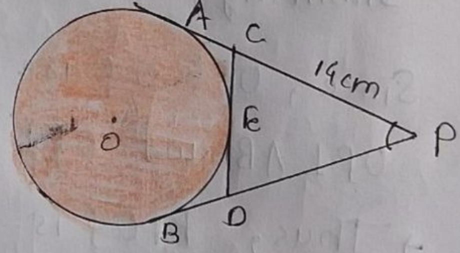
$\Rightarrow \boxed{QB = QC}$

Hence proved.

14) From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at point E which intersect and PA = 14 cm, find the perimeter of ΔPCD.

→ Given that,

- PA and PB are the tangents drawn from point 'P' outside the circle having centre 'O'.
- Again, CD is another tangent which touches the circle at point E and to the tangents AP & PB at points C & D respectively.



Here, PA = 14 cm

PA & PB are the tangents to the circle from point 'P'.

Thus, PA = PB = 14 cm

Also, CA & CE are the tangents from point 'C' to the circle.

$\therefore CA = CE$ —— ①

Also, DB & DE are the tangents from point 'D' to the circle.

$DB = DE$ —— ②

Thus, perimeter of ΔPCD,

$= PC + PD + CD$

$= PC + PD + CE + DE$

$= PC + CE + PD + DE$

$= PC + CA + PD + DB$ $\quad \because$ from ① & ②

$= PA + PB$

$P(\Delta PCD) = 14 + 14 = 28$ cm

15) In the figure, ABC is a right triangle right angled at B such that $BC = 6\,cm$ and $AB = 8\,cm$. find the radius of the its incircle.

→ From fig,

- In right ΔABC, $\angle B = 90°$

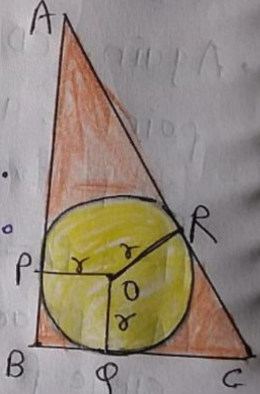
  Also, $BC = 6\,cm$, $AB = 8\,cm$

- Let us consider, 'r' is the radius of incircle having center 'O'.
- Also, the circle touches the sides AB, BC and CA at points P, Q & R respectively.

Since, AP & AR are tangents to the circle at point 'A'.

$\Rightarrow AP = AR$

Similarly, $CR = CQ$ and $BQ = BP$.



Since, OP & OQ are the radii of the circle.

$OP \perp AB$ and $OQ \perp BC$ and also $\angle B = 90°$

$\Rightarrow$ Thus, BPOQ is a square only.

So, $AR = AP = AB - PB = 8 - r$

and $CR = CQ = BC - BQ = 6 - r$

But, $AC^2 = AB^2 + BC^2$

$= (8)^2 + (6)^2$

$= 64 + 36$

$AC^2 = 100 \Rightarrow \boxed{AC = 10\,cm}$

Thus, $AR + CR = 10$

$8 - r + 6 - r = 10$

$14 - 2r = 10$

$\Rightarrow 2r = 14 - 10 = 4$

$\boxed{r = 2}$

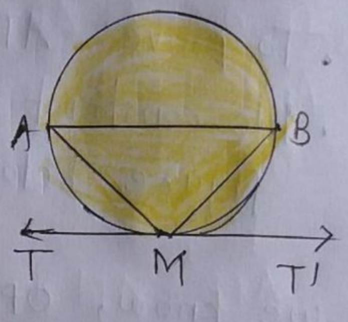
Hence, the radius of the incircle is found to be 2 cm.

16) Prove that, the tangent drawn at the mid point of an arc of a circle is parallel to the chord joining the end points of the arc.

→

Let us consider, a midpoint of an arc AMB be M and TMT' be the tangent to the circle.

Now, we are joining AB, AM and MB.



Since, arc AM = arc MB

⇒ chord AM = chord MB

In ΔAMB, $AM = MB$

⇒ $\angle MAB = \angle MBA$ —— (1)

Since, equal sides corresponding to the equal angle.

Thus; TMT' is a tangent line.

⇒ $\angle AMT = \angle MBA$.

Since, angles in alternate segment are equal.

Thus, $\angle AMT = \angle MAB$ ∵ from (1)

But, we know $\angle AMT$ and $\angle MAB$ are alternate angles, which is possible only when AB ∥ TMT'.

Hence, the tangent drawn at the midpoint of an arc of a circle is parallel to the chord joining the end points of the arc.

Hence proved.

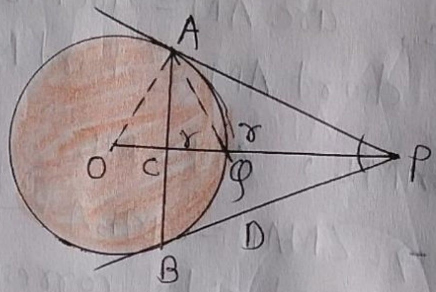
17) From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP = diameter of the circle, show that ΔAPB is equilateral.

→

Given that,

- from a point 'P' outside the circle having centre 'O', PA and PB are the tangents to the circle such that OP is a diameter.

- To prove: APB is an equilateral triangle.
- We construct by joining OP, AQ and OA as shown in figure below.



we know, $OP = 2r$

$\Rightarrow OQ + QP = 2r$

$\Rightarrow OQ = QP = r$

In right triangle OAP,

OP is the hypotenuse & Q is the midpoint.

Then, $OA = AQ = OQ$

since, midpoint of hypotenuse of a right triangle is equidistance from the vertices.

Thus, ΔOAQ is the equilateral triangle.

So, $\angle AOQ = 60°$.

In ΔOAP, $\angle APO = 90° - 60° = 30°$

$\Rightarrow \angle APB = 2\angle APO = 2 \times 30 = 60$

But, $PA = PB$ $\because$ PA & PB tangents to the circle from outer point P.

$\Rightarrow \angle PAB = \angle PBA = 60°$

Thus, ΔAPB is an equilateral triangle.

Hence proved.

18) Two tangents segments PA and PB are drawn to a circle with centre O such that $\angle APB = 120°$. Prove that $OP = 2AP$.

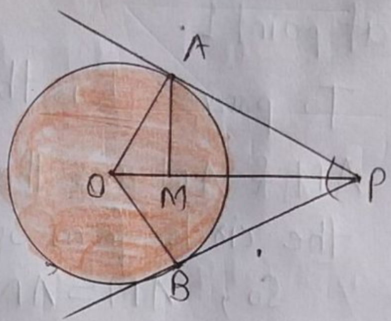
→ Given that,

from a point P, PA & PB are the tangents drawn to the circle having centre 'O'.

And $\angle APB = 120°$.

Now, we joined OP.

To prove: $OP = 2AP$



Now, we are taking mid-point M of OP & joining AM.

Also, joining OA and OB.

from fig. In $\triangle OAP$,

$\angle OPA = \frac{1}{2}(\angle APB) = \frac{1}{2}(120°) = 60°$

$\angle AOP = 90° - 60° = 30°$ ∵ by angle sum property.

Also, M is the mid-point of hypotenuse OP of $\triangle OAP$,

Thus, $MO = MA = MP$

$\therefore \angle OAM = \angle AOM = 30°$ & $\angle PAM = 90° - 30° = 60°$

⇒ $\triangle AMP$ is an equilateral triangle.

$\therefore MA = MP = AP$

But, we know that, M is the midpoint of OP.

So, $\boxed{OP = 2MP = 2AP}$

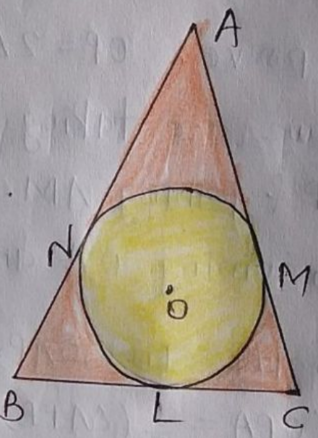
Hence proved.

19) If ΔABC is isosceles with $AB = AC$ and $C(O, r)$ is the incircle of the ΔABC touching BC at L. Prove that L bisects BC.

→ Given that,

In ΔABC, $AB = AC$ and 'O' is the centre of the circle. And 'r' is the radius which touches the side BC of ΔABC at point L.

To prove: L is the mid-point of BC



AM & AN are the tangents to the circle from outer point A.

So, $AM = AN$

But, $AB = AC$

$AB - AN = AC - AM$

$\Rightarrow \boxed{BN = CM}$

BL & BN are the tangents to the circle from outer point B.

So, A $\boxed{BL = BN}$

Simillarly, CL & CM are tangents to the circle from outer point C,

$\boxed{CL = CM}$

But, we know $BN = CM$

Thus, $\boxed{BL = CL}$

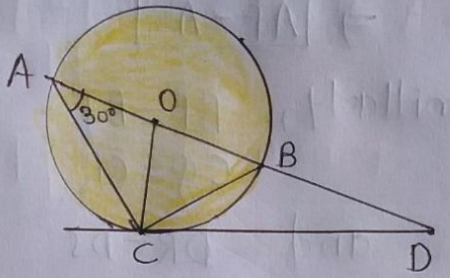
Thus, L is the midpoint of BC.

Hence proved.

20) AB is a diameter and AC is a chord of a circle with centre 'O' such that $\angle BAC = 30°$. The tangent at 'C' intersects AB at a point D. Prove that BC = BD.

→ Here, To prove: BC = BD

we join BC and OC.



Given that, $\angle BAC = 30°$

$\Rightarrow \angle BCD = 30°$

$\angle ACD = \angle ACO + \angle OCD$

$\angle ACD = 30° + 90° = 120°$

{ ∵ OC ⊥er CD & OA = OC = radius

$\Rightarrow \angle OAC = \angle OCA = 30°$ }

from fig. In $\triangle ACD$,

$\angle CAD + \angle ACD + \angle ADC = 180°$

$\Rightarrow 30° + 120° + \angle ADC = 180°$

$\Rightarrow \angle ADC = 180° - 30° - 120° = 30°$

Now, in $\triangle BCD$,

$\angle BCD = \angle BDC = 30°$

$\Rightarrow BC = BD$

Since, sides opposite to equal angles are equal.

Hence proved.

21) In the fig, a circle touches all the four sides of a quadrilateral ABCD with AB = 6 cm, BC = 7 cm and CD = 4 cm. Find AD.

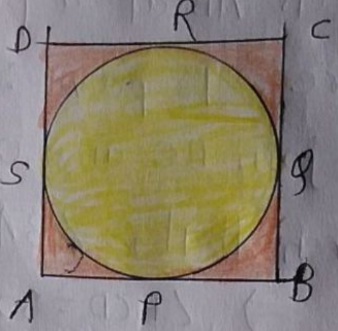
→ Given that,

A circle touches all the four sides of a quadrilateral ABCD with sides AB = 6 cm, BC = 7 cm, CD = 4 cm.

Let, $AD = x$

As AP & AS are the tangents to the circle from point A.

$\Rightarrow \boxed{AP = AS}$



Simillarly, $BP = BQ$

$CQ = CR$

and $DR = DS$

So, In $ABCD$

$AB + CD = AD + BC$ ∵ property of a cyclic quadrilateral

$\Rightarrow 6 + 4 = 7 + x$

$10 = 7 + x$

$x = 10 - 7 = 3$

Thus, $\boxed{AD = 3cm}$

22) Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

→ Given that,

TS is a tangent to the circle with centre 'O' at point P.

and we joined OP here.

To prove: OP is perpendicular to TS which is passing through the centre of the circle.

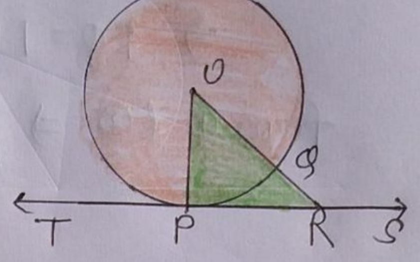
Now, we drawn a line OR which intersects the circle at Q and meets the tangent TS at R.

Now, from fig.

OP = OQ (raddi of the same circle)

Also, OQ < OR

⇒ OP < OR



Simillarly, we can show that OP is less than all lines which can be drawn from O to TS.

⇒ OP is the shortest

OP is $\perp^{er}$ to TS

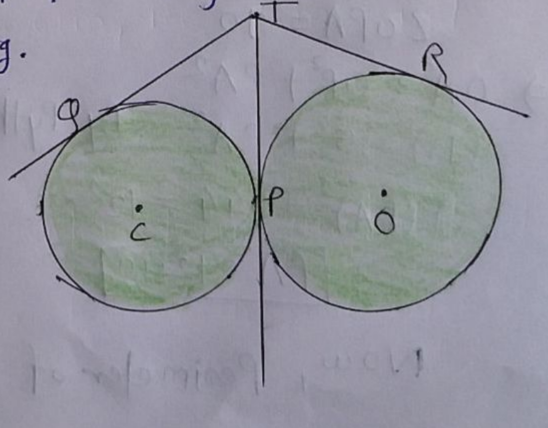
Thus, the perpendicular through point 'P' will pass through the centre of the circle.

Hence proved.

23) Two circles touch externally at a point P. From a point T on the tangent at P, tangents TQ & TR are drawn to the circles with points of contact Q & R respectively. Prove that TQ = TR.

⟶ Here, given that

Two circles with centre 'O' & 'C' touch each others externally at point 'P'. And PT is the common tangent. We drawn the tangents PT, TR and TQ ~~H~~ to the circles from point T as shown in fig.



To prove: TQ = TR

from fig,

TR & TP are two tangents to the circle with centre 'O' from outside point 'T'.

So, TR=TP —— ①

Similarly, from point T,

TQ & TP are two tangents to the circle with centre 'c'.

$\Rightarrow$ TQ=TP —— ②

from ① & ② $\Rightarrow$ $\boxed{TQ=TR}$

Hence proved.

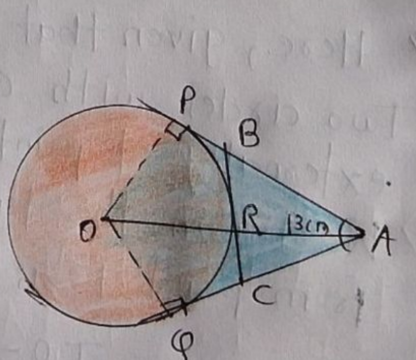
24) A is a point at a distance 13 cm from the centre O of a circle of radius 5cm. AP & AQ are tangents to the circle at P and Q. If a tangent BC is drawn at a point R' lying on the minor arc PQ to intersect AP at B and AQ at C, find the perimeter of the ΔABC.

$\rightarrow$ Given that,

Two tangents are drawn from an external point A to the circle 'O' as shown in fig.

Tangent BC is drawn at a point R and here radius of the circle is 5cm.

To find: perimeter of ΔABC



we know that,

$\angle OPA = 90°$

$\Rightarrow OA^2 = OP^2 + PA^2$

$(13)^2 = 5^2 + PA^2$ $\because$ By Pythagoras Thm

$\Rightarrow (PA)^2 = 144 = 12^2$

$\boxed{PA = 12cm}$

Now, Perimeter of $\Delta ABC = AB + BC + CA$

$$
\begin{aligned}
P(\Delta ABC) &= AB + BC + CA \\
&= (AB+BR) + (RC+CA) \\
&= AB + BP + CQ + CA \\
&= AP + AQ \qquad \because BR = BP \ \& \ RC = CQ \\
&= 2AP \qquad \because AP = AQ \\
&= 2(12)
\end{aligned}
$$

$$\boxed{P(\Delta ABC) = 24\text{cm}}$$

Thus, the perimeter of ΔABC is found to be 24cm.